

## Interacting Electrons in Disordered Wires: Anderson Localization and Low- $T$ Transport

I. V. Gornyi,<sup>1,\*</sup> A. D. Mirlin,<sup>1,2,†</sup> and D. G. Polyakov<sup>1,\*</sup>

<sup>1</sup>*Institut für Nanotechnologie, Forschungszentrum Karlsruhe, 76021 Karlsruhe, Germany*

<sup>2</sup>*Institut für Theorie der kondensierten Materie, Universität Karlsruhe, 76128 Karlsruhe, Germany*

(Received 21 May 2005; published 8 November 2005)

We study the conductivity  $\sigma(T)$  of interacting electrons in a low-dimensional disordered system at low temperature  $T$ . For weak interactions, the weak-localization regime crosses over with lowering  $T$  into a dephasing-induced “power-law hopping.” As  $T$  is further decreased, the Anderson localization in Fock space crucially affects  $\sigma(T)$ , inducing a transition at  $T = T_c$ , so that  $\sigma(T < T_c) = 0$ . The critical behavior of  $\sigma(T)$  above  $T_c$  is  $\ln\sigma(T) \propto -(T - T_c)^{-1/2}$ . The mechanism of transport in the critical regime is many-particle transitions between distant states in Fock space.

DOI: 10.1103/PhysRevLett.95.206603

PACS numbers: 72.20.-i, 71.30.+h, 72.15.Rn, 73.63.-b

In a path breaking paper [1] Anderson demonstrated that a quantum particle may become localized by a random potential. In particular, in noninteracting systems of one-dimensional (1D) or two-dimensional (2D) geometry even weak disorder localizes all states [2], yielding zero conductivity,  $\sigma(T) = 0$ , whatever temperature  $T$ . A nonzero  $\sigma(T)$  may then only occur due to inelastic processes leading to dephasing of electrons: (i) scattering by external excitations (phonons) and (ii) electron-electron ( $e$ - $e$ ) scattering [3]. In either case, at sufficiently high  $T$  the dephasing rate  $\tau_\phi^{-1}$  is high and the localization effects are reduced to a weak-localization (WL) correction to the Drude conductivity. This correction diverges with lowering  $T$  for dimensionality  $d \leq 2$ , signaling the onset of the strong localization (SL) regime. This prompts a question as to how the system conducts at low  $T$ .

For the case of electron-phonon ( $e$ -ph) scattering the conductivity is governed by Mott’s variable-range hopping (VRH) [4],  $\sigma(T) \propto \exp\{-(T_0/T)^\mu\}$  with  $\mu = 1/(d+1)$ . In the presence of a long-range Coulomb interaction, the VRH exponent is modified,  $\mu = \frac{1}{2}$  [5].

But what is the low- $T$  behavior of  $\sigma(T)$  if the  $e$ -ph coupling is negligible and the only source of the inelastic scattering is the  $e$ - $e$  interaction? Our purpose here is to solve this long-standing fundamental problem, which is also of direct experimental relevance; see Refs. [6–8], where the crossover from WL to SL with lowering  $T$  was studied for 1D and 2D systems. For definiteness, we focus on a many-channel 1D system with short-range interaction and discuss generalizations (single-channel wires, 2D systems, Coulomb interaction) at the end.

It was proposed in [9] that the  $e$ - $e$  interaction by itself is sufficient to induce VRH. This idea was widely used for interpretation of experimental [8,10] and numerical [11] results on 2D systems. Further, Ref. [12] used bosonization to study the problem in 1D and concluded that transport is of VRH character. These results are, however, in conflict with the energy-conservation argument [13], forbidding elementary hops in the low- $T$  limit [14]. The situation is particularly interesting for  $d \leq 2$ , where no mobility edge

exists, activation to which might give  $\sigma(T) \neq 0$ . If neither VRH nor activation, then what?

Let us now specify the model. We consider a many-channel weakly disordered wire, so that the relevant length scales satisfy  $k_F^{-1} \ll l \ll \xi$ , where  $k_F$  is the Fermi momentum,  $l$  the mean free path,  $\xi \sim \pi\nu D$  the localization length,  $\nu$  the density of states, and  $D$  the diffusion constant [15,16]. The corresponding energy scales are the Fermi energy  $E_F$ , the elastic scattering rate  $\tau^{-1}$ , and the level spacing in the localization volume,  $\Delta_\xi = 1/\nu\xi$ , with  $E_F \gg \tau^{-1} \gg \Delta_\xi$ . We will assume a short-range  $e$ - $e$  interaction  $U(\mathbf{r} - \mathbf{r}')$  characterized by a dimensionless coupling  $\alpha = \nu\tilde{U}(0)$ , where  $\tilde{U}(\mathbf{q})$  is the Fourier transform of  $U(\mathbf{r})$ . We assume that  $\alpha \ll 1$ , which yields a richer behavior of  $\sigma(T)$  and allows better understanding of underlying physics; the case  $\alpha \sim 1$  is discussed in the end.

At sufficiently high  $T$ ,  $\sigma(T)$  is close to its Drude value  $\sigma_D$ , with quantum corrections related to the weak localization ( $\Delta\sigma_{WL}$ ) and to the interplay of interaction and disorder (Altshuler-Aronov contribution  $\Delta\sigma_{AA}$ ) [3],

$$\frac{|\Delta\sigma_{WL}|}{\sigma_D} \sim \int_{l_\phi} \frac{dq}{\pi\nu Dq^2} \sim \frac{l_\phi}{\xi} \sim \left(\frac{\Delta_\xi}{\alpha^2 T}\right)^{1/3}. \quad (1)$$

Here we used the result for the dephasing rate length  $l_\phi = (D\tau_\phi)^{1/2}$  due to  $e$ - $e$  interaction [3],

$$\tau_\phi^{-1} \sim \alpha^2 T \int_{l_\phi} \frac{dq}{\pi\nu Dq^2} \sim \alpha^2 T \frac{l_\phi}{\xi}. \quad (2)$$

The WL correction grows with lowering  $T$  and finally becomes strong ( $\Delta\sigma_{WL}/\sigma_D \sim 1$ ) when  $l_\phi$  reaches  $\xi$ , or, equivalently, when  $\tau_\phi^{-1} \sim \Delta_\xi$ . This happens at  $T \sim T_1 = \alpha^{-2}\Delta_\xi$ , marking the beginning of the SL regime. The interaction-induced correction  $\Delta\sigma_{AA}/\sigma_D \sim (\alpha^2\Delta_\xi/T)^{1/2}$  remains small at  $T \sim T_1$  and thus is of no relevance in the present context. (For  $\alpha \sim 1$ ,  $\Delta\sigma_{AA}$  is of order  $\sigma_D$  at  $T \sim T_1$  and does not lead to any qualitative changes, either.) The subject of our interest is  $\sigma(T)$  for  $T < T_1$ .

In fact, SL does not necessarily mean  $\sigma(T)$  is exponentially small. Specifically, in the high- $T$  part of the SL

regime the transport mechanism—we call it power-law hopping (PLH)—is analogous to the one identified in [17] for  $e$ -ph scattering, i.e., hopping over length  $\sim \xi$  in time  $\sim \tau_\phi$ . In other words,  $\tau_\phi$  serves in this regime as a lifetime of localized states, which adds an imaginary part  $i/2\tau_\phi$  to the single-particle energies  $\epsilon_\alpha$ . This yields

$$\sigma(T) \sim \sigma_{ac}(\Omega)|_{\Omega=i/\tau_\phi} \sim e^2 \nu \xi^2 / \tau_\phi, \quad (3)$$

where  $\sigma_{ac}(\Omega)$  is the zero- $T$  conductivity of noninteracting electrons at frequency  $\Omega$ . The crucial point here is that  $\tau_\phi$  in this SL regime can still be calculated via Fermi's golden rule, as we are going to show. The lowest-order decay process of a localized state  $|\alpha\rangle$  is the transition to a state  $|\beta\rangle$  accompanied by the transition of another electron  $|\delta\rangle \rightarrow |\gamma\rangle$ , all states being within a distance  $\sim \xi$  and the energy window of width  $T$ , Fig. 1(a). This can be viewed as a decay of the state  $|\alpha\rangle$  to a three-particle state (two electrons and a hole)  $|\beta\gamma\bar{\delta}\rangle$ . The corresponding matrix element of the interaction is

$$V_{\alpha\beta\gamma\delta} \equiv V \sim \alpha \Delta_\xi; \quad |\epsilon_\alpha - \epsilon_\beta|, |\epsilon_\gamma - \epsilon_\delta| \lesssim \Delta_\xi \quad (4)$$

and decays fast for larger energy differences. Indeed, in a metallic grain  $V \sim \alpha \Delta / g$  for energy transfers less than the Thouless energy (inverse time of diffusion through the sample)  $g\Delta$ , where  $g$  is the dimensionless conductance and  $\Delta$  the level spacing [16,18]. Setting  $g \sim 1$ , as appropriate for a sample of length  $\xi$ , we get Eq. (4). The level spacing of relevant three-particle states, to which the original state  $|\alpha\rangle$  is coupled according to (4), reads

$$\Delta_\xi^{(3)} \sim \Delta_\xi^2 / T. \quad (5)$$

Using (4) and (5) and the golden rule, we find

$$\tau_\phi^{-1} \sim |V|^2 / \Delta_\xi^{(3)} \sim \alpha^2 T. \quad (6)$$

Note that this result could also be obtained from (2) if  $\xi^{-1}$  is used as the infrared cutoff, as appropriate for the SL regime. The condition of validity of the golden-rule calculation is  $V \gg \Delta_\xi^{(3)}$ , or, equivalently,  $\tau_\phi^{-1} \gg \Delta_\xi^{(3)}$ . This introduces a new temperature scale  $T_3 = \alpha^{-1} \Delta_\xi$ , so that the PLH regime is restricted to the range  $T_3 \ll T \ll T_1$ . Combining (3) and (6), we get  $\sigma(T)$  in this regime,

$$\sigma(T) \sim e^2 \nu \xi^2 \alpha^2 T \sim e^2 \xi T / T_1. \quad (7)$$

What happens below  $T_3$ ? Simple hops on a distance  $\sim \xi$  are then not sufficient to delocalize electrons. Increasing the distance  $r$  of a hop does not help: the matrix element

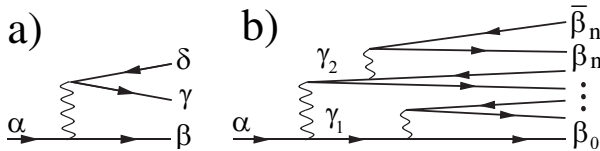


FIG. 1. Diagrams for the golden rule (a) and higher-order decay amplitude (b).

vanishes exponentially with  $r/\xi$ , while the level spacing  $\Delta^{(3)}$  only as a power law. We thus have to analyze higher-order processes by exploring the structure of the theory in the many-body Fock space, similarly to the ideas developed for the problem of a quasiparticle decay in quantum dots [18–20]. The process of  $n$ th order represents a transition with excitation of  $n$  electron-hole pairs,  $|\alpha\rangle \rightarrow |B^{(n)}\rangle \equiv |\beta_0\beta_1\bar{\beta}_1\dots\beta_n\bar{\beta}_n\rangle$  [with the energy difference  $\lesssim \Delta_\xi$  for each pair in view of (4)]; see Fig. 1(b). Let us estimate the dimensionless coupling of  $n$ th order (ratio of the matrix element to the level spacing of final states),  $V^{(n)} / \Delta^{(2n+1)}$ , which is the  $n > 1$  generalization of the ratio  $V / \Delta_\xi^{(3)}$  considered above. In this estimate, it will be sufficient for us to keep track of factors of the type  $n^n$  (or  $n!$ ) and  $(T/T_3)^n$ . Factors of the type  $c^n$ , where  $c \sim 1$ , will be unessential and thus neglected.

Let us assume that the set of states  $B^{(n)}$  is spread over the length  $m\xi$ , with  $\sim n/m$  pairs in each box of length  $\xi$ ; later we optimize with respect to  $m$  (for  $1 \leq m \leq n$ ). The corresponding level spacing can be estimated as

$$\Delta_{m\xi}^{(2n+1)} / \Delta_\xi \sim [(n/m)\Delta_\xi / T]^n. \quad (8)$$

The matrix element  $V_{\alpha;\beta_1,\dots,\beta_{n+1}} \equiv V^{(n)}$  is given by

$$V^{(n)} = \sum_{\text{diagrams}} \sum_{\gamma_1, \dots, \gamma_{n-1}} V_1 \prod_{i=1}^{n-1} \frac{V_{i+1}}{E_i - \epsilon_{\gamma_i}}, \quad (9)$$

where the matrix element  $V_i$  corresponds to the  $i$ th interaction line,  $\gamma_i$  are the virtual states corresponding to internal lines, and  $E_i$  the corresponding energy variable which can be expressed as a linear superposition of  $\epsilon_{\alpha_i}$  using the energy conservation. We need to take into account only those contributions where all states forming each matrix element are within a distance  $\sim \xi$  from each other, so that  $V_i \sim \alpha \Delta_\xi$ . Therefore, the summation over each  $\gamma_i$  is effectively taken over a single localization domain, and we can replace it by taking the “optimal”  $\gamma_i$ , with  $|E_i - \epsilon_{\gamma_i}| \sim \Delta_\xi$ . We thus get

$$V^{(n)} / \Delta_\xi \sim \alpha^n [M_m^{(n)}]^{1/2}, \quad (10)$$

where  $M_m^{(n)}$  is the number of diagrams contributing to the amplitude of the transition  $|\alpha\rangle \rightarrow |B^{(n)}\rangle$ . These contributions have random signs, hence the factor  $[M_m^{(n)}]^{1/2}$ .

To find  $M_m^{(n)}$ , we first calculate the number of topologically different diagrams,  $D^{(n)}$ , which satisfies the recursion

$$D^{(n)} = \sum_{n_1+n_2+n_3=n-1; n_{1,2,3} \geq 0} D^{(n_1)} D^{(n_2)} D^{(n_3)}, \quad (11)$$

with the initial condition  $D^{(0)} = 1$ . It is easy to show that its solution increases only as  $D^{(n)} \sim a^n$ , with  $a \sim 1$ , so that with the required accuracy it can be replaced by unity. Thus  $M_m^{(n)} \sim A_m^{(n)}$ , the number of allowed permutations of the set  $B^{(n)}$  over the final-state lines. Noticing that only electron-hole pairs within the same (or nearby) localization volume

can be interchanged, we estimate  $A_m^{(n)} \sim [(n/m)!]^m \sim (n/m)^n$ . Combining this with (8) and (10), we finally get

$$V^{(n)}/\Delta^{(2n+1)} \sim [\alpha(m/n)^{1/2}T/\Delta_\xi]^n. \quad (12)$$

Let us now analyze the result. First, for given  $n$  the most favorable case is  $m \sim n$ , which corresponds to “ballistic” paths. In such a process, an electron makes a many-body transition over the distance  $n\xi$ , leaving behind a string of  $n$  particle-hole pairs, Fig. 2. Second,  $V^{(n)}/\Delta^{(2n+1)}$  increases with  $n$  at sufficiently high  $T$  and decreases with  $n$  (thus remaining small for all  $n$ ) at low  $T$ . Therefore, at low  $T$  the higher-order processes do not help a localized single-particle state to decay, so that  $\sigma(T)$  is exactly zero. In contrast, at high  $T$ , the increase of the coupling (12) with  $n$  guarantees that the golden-rule calculation for the WL and PLH regimes is not spoiled by the higher-order effects. The temperature  $T_c$  of the transition into the zero-conductivity regime can be immediately estimated from (12),  $T_c \sim \Delta_\xi/\alpha$ . In fact, (12) misses a  $\ln\alpha^{-1}$  factor recovered in a more accurate treatment below, Eq. (14).

What helps us to analyze the critical behavior at the transition is that the structure of the theory, when restricted to the optimal (ballistic) paths, reduces essentially to that of the Anderson model on the Bethe lattice—a tree with a fixed branching number. Indeed, consider the process shown in Fig. 2: an electron hops to an adjacent localization volume, creating an electron-hole pair, then to the next one and so forth. Clearly, the density of final states increases at each step by the same factor

$$K \sim \Delta_\xi/\Delta_\xi^{(3)} \sim T/\Delta_\xi, \quad (13)$$

which is the branching number of the Bethe lattice. The Bethe-lattice character of the problem can also be inferred from the exponential dependence of the coupling (12) on  $n$  at  $m = n$ . This should be contrasted with the opposite limit,  $m = 1$ , corresponding to the case of a quantum dot, when (12) contains an additional  $n^{-n/2}$  factor. The latter is related to a decrease of the effective branching number with  $n$  in this case [19,20]. In other words, the mapping on the Bethe-lattice model, oversimplified for a quantum dot [for which it was proposed in [18]], works perfectly in the case of localized states in a wire (or, generally, in a nonrestricted geometry), where going to higher generations in Fock space can be accompanied by the exploration of new regions in real space.

We can now use the results for the Anderson transition on the Bethe lattice that has been studied extensively [15,19,21–23]. For a large branching number  $K$  the equation for the transition point reads [24]



FIG. 2. Scheme of a high-order ballistic process: the string of particle-hole pairs (black and white circles). The initial state and intermediate virtual states ( $\gamma_i$ ) are shown in gray.

$$\Delta/V = 4 \ln K, \quad (14)$$

where  $V$  is the hopping matrix element, and  $\Delta$  is the mean level spacing of states of generation  $n + 1$  coupled to a given state of generation  $n$ . Using (4) and (13), and  $\Delta = \Delta_\xi^{(3)}$ , we find the transition temperature,

$$T_c \sim \Delta_\xi/\alpha \ln\alpha^{-1}, \quad (15)$$

so that  $T_c \sim T_3/\ln\alpha^{-1}$ . The critical behavior of  $\sigma(T)$  above  $T_c$  is governed by that of the decay rate  $\tau_\phi^{-1}$ , which translates into the imaginary part of the self-energy  $\text{Im} \Sigma$  for the Bethe-lattice problem. The critical behavior of the latter was found in [19,23], yielding

$$\sigma(T) \propto \text{Im} \Sigma \propto \exp\{-c_\alpha[(T - T_c)/T_c]^{-1/2}\}, \quad (16)$$

with  $c_\alpha \sim \ln\alpha^{-1}$  for  $\alpha \ll 1$ . Near the transition, the local density of states on the Bethe lattice acquires an increasingly more sparse “spatial” structure [19,23], so that the transport is governed by processes connecting remote states in Fock space, Fig. 2, which implies a glassy character of the system. When  $T \rightarrow T_c$ , the length of the particle-hole strings diverges. We term the low- $T$  phase “Anderson-Fock glass” (AFG), since its physics is governed by the Anderson localization in Fock space.

The found behavior of  $\sigma(T)$  is illustrated in Fig. 3. If  $\alpha \sim 1$ , all scales become of the same order,  $T_1 \sim T_3 \sim T_c \sim \Delta_\xi$ , and the range of PLH disappears. In realistic systems, weak coupling to phonons will lead to PLH (for  $\alpha \ll 1$ ) and VRH below  $T_c$ , but with a small prefactor, so that the transition at  $T_c$  should be well observable. Note the peculiar character of the transition: not only the exponential critical behavior (16) but also that the ordered (metallic) phase corresponds to  $T > T_c$ . An apparent conflict with the Mermin-Wagner theorem is related to the unconventional (functional) nature of the order parameter for Anderson localization [15,22,23].

Finally, we mention a few extensions of our results [25]:

(i) *1D: Single-channel.*—We have recently shown [26] that the notion of WL and dephasing are also applicable to a disordered Luttinger liquid and calculated  $\sigma(T)$  in the WL regime. The low- $T$  results presented above can then be easily generalized to the single-channel case. An important difference is that  $\tau$  (and thus,  $\xi$ ) is strongly renormalized by interaction,  $\tau(T)/\tau \sim (T/E_F)^\alpha$ .

(ii)  *$d > 1$ .*—Generalization to 2D systems is straightforward, with  $\xi$  depending exponentially on disorder,  $\xi \propto \exp(2\pi^2\nu D)$ . Our consideration is also applicable to 3D Anderson insulators; however, in this case  $\sigma(T < T_c)$  will not be exactly zero but rather determined by the activation above the single-particle mobility edge.

(iii) *Coulomb interaction.*—For 1D and 2D geometry, the transition survives also for  $1/r$  Coulomb interaction, since correlated hops of two electrons separated by a large distance  $r \gg \xi$  will not delocalize them. Indeed, the corresponding matrix elements decrease with  $r$  as  $V_{\alpha\beta\gamma\delta} \propto 1/r^3$ , which is not compensated by the increase  $\propto r^d$  of the

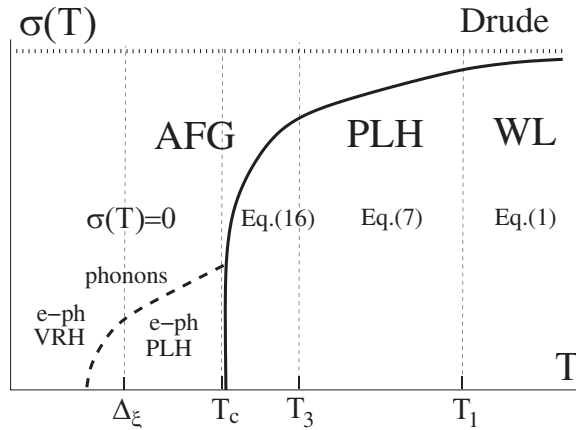


FIG. 3. Schematic behavior of  $\sigma(T)$  on the log-log scale: the weak-localization (WL), power-law hopping (PLH), and Anderson-Fock glass (AFG) regimes, with a localization transition at  $T_c$ . Dashed line: PLH and VRH contribution to  $\sigma(T)$  in the case of a weak coupling to phonons.

density of final states for  $d < 3$ . On the other hand, in 3D such processes will lead to delocalization for any  $T$  [13]; the behavior of  $\sigma(T)$  in this case, as well as the role of the Coulomb-gap effects, requires a separate study.

(iv) *Creep*.—Our approach can be used to analyze the nonlinear conductivity  $\sigma(E)$  at weak electric field  $E$ .

In conclusion, we have studied the conductivity of interacting electrons in a disordered quantum wire; very similar results hold for a 2D system. In contrast to a popular belief, the  $e-e$  interaction is not sufficient to support the VRH transport in the low- $T$  limit. Instead, the system undergoes a localization transition at temperature  $T_c$ , below which  $\sigma(T) = 0$  (assuming vanishing coupling to phonons). We have shown that the conductivity vanishes as  $\ln\sigma(T) \propto -(T - T_c)^{-1/2}$  as  $T \rightarrow T_c$ . Transport in the critical regime is governed by many-particle transitions between distant states in Fock space, corresponding to the formation of long strings of electron-hole pairs. For weak interactions, this Anderson-Fock glass phase is separated from the WL regime by an intermediate temperature regime of power-law hopping.

We thank V. Cheianov, D. Maslov, G. Minkov, T. Nattermann, and B. Shklovskii for valuable discussions. We are particularly grateful to I. Aleiner, B. Altshuler, and D. Basko for very useful discussions and criticism of the earlier version of this work. This helped us to correct an error in counting of the number of diagrams  $M_m^{(n)}$  [which yielded a spurious double exponential tail of  $\sigma(T)$  in the AFG phase], whose elimination gave a true phase transition at  $T = T_c$ , as also found in Ref. [27] for a model of weakly coupled granules. The work was supported by SPP “Quanten-Hall-Systeme” and CFN of the DFG and by RFBR.

\*Also at A. F. Ioffe Physico-Technical Institute, 194021 St. Petersburg, Russia.

†Also at Petersburg Nuclear Physics Institute, 188350 St. Petersburg, Russia.

- [1] P. W. Anderson, *Phys. Rev.* **109**, 1492 (1958).
- [2] E. Abrahams *et al.*, *Phys. Rev. Lett.* **42**, 673 (1979).
- [3] B. L. Altshuler and A. G. Aronov, in *Electron-Electron Interactions in Disordered Systems*, edited by A. L. Efros and M. Pollak (North-Holland, Amsterdam, 1985).
- [4] N. Mott, *Metal-Insulator-Transitions* (Taylor & Francis, London, 1974).
- [5] B. I. Shklovskii and A. L. Efros, *Electronic Properties of Doped Semiconductors* (Springer, New York, 1984).
- [6] M. E. Gershenson *et al.*, *Phys. Rev. Lett.* **79**, 725 (1997); Yu. B. Khavin *et al.*, *Phys. Rev. B* **58**, 8009 (1998).
- [7] S. Y. Hsu and J. M. Valles, *Phys. Rev. Lett.* **74**, 2331 (1995); F. W. van Keuls *et al.*, *Phys. Rev. B* **56**, 13 263 (1997); G. M. Minkov *et al.*, *ibid.* **65**, 235322 (2002).
- [8] S. I. Khondaker *et al.*, *Phys. Rev. B* **59**, 4580 (1999).
- [9] L. Fleishman *et al.*, *Phys. Rev. Lett.* **40**, 1340 (1978).
- [10] I. Shlimak *et al.*, *Solid State Commun.* **112**, 21 (1999); A. Ghosh *et al.*, *Phys. Status Solidi B* **230**, 211 (2002).
- [11] R. Berkovits and B. I. Shklovskii, *J. Phys. Condens. Matter* **11**, 779 (1999); P. H. Song and D. L. Shepelyansky, *Phys. Rev. B* **61**, 15 546 (2000).
- [12] T. Nattermann *et al.*, *Phys. Rev. Lett.* **91**, 056603 (2003).
- [13] L. Fleishman and P. W. Anderson, *Phys. Rev. B* **21**, 2366 (1980).
- [14] Recently, S. V. Malinin *et al.*, *Phys. Rev. B* **70**, 235120 (2004), argued that the VRH of Ref. [12] indeed requires a finite coupling to the phonon bath.
- [15] K. B. Efetov, *Supersymmetry in Disorder and Chaos* (Cambridge University Press, Cambridge, England, 1997).
- [16] A. D. Mirlin, *Phys. Rep.* **326**, 259 (2000).
- [17] A. A. Gogolin, V. I. Melnikov, and É. I. Rashba, *Sov. Phys. JETP* **42**, 168 (1976); A. A. Gogolin and G. T. Zimanyi, *Solid State Commun.* **46**, 469 (1983); A similar transport regime in the context of a dynamical localization was found in D. M. Basko, *Phys. Rev. Lett.* **91**, 206801 (2003).
- [18] B. L. Altshuler *et al.*, *Phys. Rev. Lett.* **78**, 2803 (1997).
- [19] A. D. Mirlin and Y. V. Fyodorov, *Phys. Rev. B* **56**, 13 393 (1997).
- [20] P. G. Silvestrov, *Phys. Rev. Lett.* **79**, 3994 (1997); *Phys. Rev. E* **58**, 5629 (1998).
- [21] R. Abou-Chacra *et al.*, *J. Phys. C* **6**, 1734 (1973).
- [22] M. R. Zirnbauer, *Phys. Rev. B* **34**, 6394 (1986); *Nucl. Phys.* **B265**, 375 (1986); A. D. Mirlin and Y. V. Fyodorov, *Nucl. Phys.* **B366**, 507 (1991).
- [23] A. D. Mirlin and Y. V. Fyodorov, *Phys. Rev. Lett.* **72**, 526 (1994); *J. Phys. I (France)* **4**, 655 (1994).
- [24] Fluctuations in  $V$  do not change Eq. (14) qualitatively, only affecting the numerical coefficient.
- [25] I. V. Gornyi, A. D. Mirlin, and D. G. Polyakov (to be published).
- [26] I. V. Gornyi *et al.*, *Phys. Rev. Lett.* **95**, 046404 (2005).
- [27] D. M. Basko, I. L. Aleiner, and B. L. Altshuler, *cond-mat/0506617*.