# Optical Polarimetry of Random Fields

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(Received 21 June 2005; published 10 November 2005)

The state of polarization of an optical field provides detailed information concerning both the radiation emission processes and the intricate interaction between light and matter. We report here a novel approach for characterizing the polarization properties of electromagnetic fields for which the electric field vector at a point may fluctuate in three dimensions. Using probes which couple all three components of the field, we were able to extract the polarized and unpolarized components of such fields. Our results constitute the proof of concept for what could be called three-dimensional optical polarimetry.

DOI: 10.1103/PhysRevLett.95.203905 PACS numbers: 42.25.Ja, 07.60.Dq, 46.65.+g

The usual analysis of the state of polarization is restricted to beamlike fields. Hence, there are many practical situations that could benefit from understanding the polarization properties of more general electromagnetic fields such as those for which the electric field vector at a point fluctuates in three dimensions.

As the characteristics of a fluctuating field are directly influenced by the properties of the sources that give rise to that field, the direct measurement of the field itself provides greater insight into the underlying source of radiation. It is expected that the ordered portion of a measured three-dimensional field—the polarized local density of states—will relate to both bulk structural and morphological properties of the sources. In this context, three-dimensional polarimetry is directly applicable to the study of fluorescence and multiphoton microscopy. For instance, fluorescence measurements performed in the far-zone provide only limited information about the structural and dynamical properties of the fluorophore centers. Determining the properties of the three-dimensional field inside the emission volume should offer a greater detail regarding the structural and dynamic properties of the fluorophores. Similarly, the identification of the polarized component of a three-dimensional fluctuating field is expected to improve the potential for imaging objects imbedded in dense scattering media.

Another notable application is in near-field optics and nanophotonics where the properties of the optical field reveal the complex light-matter interaction [1]. It has been shown that near-field measurements are usually subject to the transverse component of the electric field [2]. Furthermore, the shape information of subwavelength structures depends greatly on the incident field and is encoded in the full three-dimensional scattered field [3]. 3D polarimetry could therefore provide a direct measurement of the shapes of underlying subwavelength objects without the need for additional assumptions. Furthermore, the full polarimetric measurements suggested here should also improve the spatial resolution of near-field scanning optical microscopy, by exploiting different contrast mechanisms.

So far, a direct measurement of the properties of three-dimensional optical fields has not been possible and one had to rely on measurements performed far from the region of interest and the assumptions that then relate these measurements to the properties of the field of interest. While the direct problem of calculating the properties of the far field for a given source can be straightforward, the inverse problem—which is usually of interest in practice—does not have a unique solution. Progress in solving the inverse problem can be achieved only by either making assumptions regarding the properties of the near field to be determined or by performing additional measurements along different directions of propagation. Nevertheless, having direct access to information about the three-dimensional field seems to be a better option.

In this Letter we first present the general theory of a three-dimensional polarimetric measurement. Then, we describe an experimental setup capable of capitalizing on this theory. Experimental measurements of various three-dimensional fluctuating fields is present, and we conclude with comments pertaining to the possible interpretations of the information present in a polarimetric measurement of a three-dimensional fluctuating field.

Polarimetry deals with the characterization of the polarization properties of a fluctuating electromagnetic field, such as its state of polarization and the degree of polarization. These quantities are specified in terms of the second-order field correlations. We may characterize the second-order correlation properties of a fluctuating electric field by the $3 \times 3$ cross-spectral density matrix at points $\mathbf{r}_1$ and $\mathbf{r}_2$ and frequency $\omega$ (see [4], Sec. 6.6.1)

$$\bar{W}_{ij}(\mathbf{r}_1, \mathbf{r}_2, \omega) = \langle E_i^*(\mathbf{r}_1, \omega) E_j(\mathbf{r}_2, \omega)\rangle, \qquad (1)$$

where the subscripts $i$ and $j$ label the Cartesian components of the (generally complex) electric field. Since we are interested in the state of polarization of the field at some particular point represented by a position vector $\mathbf{r}$ and frequency $\omega$ we only need to consider the matrix (1) for $\mathbf{r}_1 = \mathbf{r}_2 \equiv \mathbf{r}$.

The complete determination of the polarimetric characteristics of a three-dimensional field requires measurements of the field correlations in three mutually perpendicular directions. This could be realized by using, for instance, three orthogonal dipolelike probes which are overlapped spatially and which are detected simultaneously. In the optical domain however, this approach cannot be implemented because an ensemble of three dipoles which can be read independently simply does not exist. This is not, however, the only means by which the polarimetric information can be obtained.

Instead of superposing three dipolelike detectors, one could envision placing in the point of interest a probe which couples all three components of the field and then reemits the radiation. This probe would act as a secondary source for the radiation which will eventually be sensed by a conventional detector, placed away from the point where measurements are made. The result will be a linear combination of the measurements possible with three independent dipoles and, in order to determine all the nine elements of the field-field correlation matrix $\bar{W}_{ij}$, one would need to perform measurements with nine different probe configurations. It is important to emphasize that, due to propagation from the probe to the detector, the field becomes practically transverse having a propagation vector pointed toward the two-dimensional detector.

The entire operation of detecting the intensity $I_D$ by the two-dimensional detector can be formally written as

$$I_D = \mathrm{tr}[\bar{T}^{\dagger}_{2\times3}\bar{\alpha}^{\dagger}_{3\times3}\bar{W}_{3\times3}\bar{\alpha}_{3\times3}\bar{T}_{3\times2}] \qquad (2)$$

where the three-dimensional field correlations included in $\bar{W}$ are detected after being collected by the probe characterized by a "polarizability" $\bar{\alpha}$ and transferred to the detector by a linear system having a transfer function $\bar{T}$. The indices denote the dimensionalities of the corresponding matrices. The equivalent measurement [4] of intensities in the classical two-dimensional polarimetry is $I_D = \mathrm{tr}[\bar{T}^{\dagger}_{2\times2}\bar{W}_{2\times2}\bar{T}_{2\times2}]$, with four independent configurations of $\bar{T}_{2\times2}$ required to fully characterize $\bar{W}_{2\times2}$. In order to perform three-dimensional polarimetry, i.e., determine all the elements of $\bar{W}_{3\times3}$, one has to repeat the measurement with nine independent configurations of $\bar{\alpha}_{3\times3}\bar{T}_{3\times2}$. A simple calculation demonstrates that only four independent configurations can be achieved by altering only elements of $\bar{T}_{3\times2}$, suggesting the necessity for multiple probes.

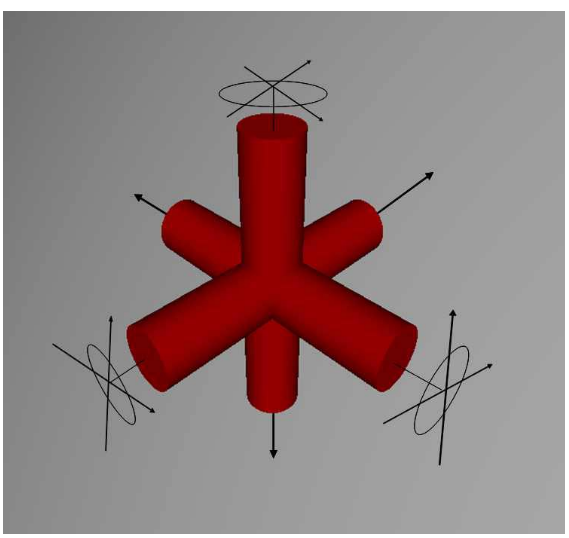

FIG. 1 (color online). Three laser beams are overlapped to produce a three-dimensional field distribution. The polarization properties of this field are fully adjustable by controlling independently the characteristics of the three beams.

In our experiment, we choose an approach where nine different probe configurations $\bar{\alpha}$ are used to generate nine values of the detected intensity $I_D$. The linear independence of these choices allows retrieving all the elements of the field-field correlation matrix $\bar{W}$.

We have produced a field with adjustable properties by overlapping three orthogonal laser beams generated by three independent laser sources as shown in Fig. 1.

The field probe used in this experiment is a single mode silica fiber which is conically shaped at one end and is coupled to a photomultiplier at the other. To realize nine different probe configurations, we make use of the fact that two sharp fibers placed in proximity alter each other's coupling properties. By adjusting the relative position of additional tapered fibers which are placed in the probe's proximity, one can create nine independent probes. It is worth noting that the polarizabilities of these nine probe configurations do not need to be known *a priori* (i.e., we do not need nine specifically shaped probes but just nine independent polarizabilities), because the coupling may be calibrated by using a set of at least nine known generated fields. Once the nine independent configurations have been realized and the calibration made, it is possible to reconstruct all the second-order field correlations, i.e., the nine elements of the $\bar{W}$ matrix, of a given field.

As we just saw, our measurements provide all nine independent elements of the correlation matrix $\bar{W}_{ij}(\mathbf{r}, \mathbf{r}, \omega)$. As this matrix is Hermitian, it can be diagonalized by a unitary transformation, $\bar{D} = \bar{U}^{\dagger}\overline{WU}$. As a unitary transformation serves only to change the basis of polarization states in which the correlations are described, this diagonalization indicates that the second-order correlations at a point of any three-dimensional field can be completely replicated by the incoherent superposition of three orthogonally polarized fields, i.e., fields for which their Jones vectors are orthogonal [5–7]. Evidently, the correlation matrix $\bar{W}$ is diagonal when viewed in this "natural" basis of the field (as opposed to the basis comprised of linear states corresponding to the laboratory coordinate system). Essentially,

$$\bar{W} = \sum_{i=1}^{3} \lambda_i \bar{w}_i, \qquad (3)$$

where the $\lambda_i$ are the eigenvalues of the $\bar{W}$ matrix, which are proportional to the intensity of their field components, and the $\bar{w}_i$ are correlation matrices of unit trace corresponding to these fully polarized fields whose polarization states are mutually orthogonal. As the $\bar{w}_i$ matrices represent fully polarized fields, they can be fully specified by the associated polarization ellipses.

Our three-dimensional polarimetry has been confirmed by measuring a number of three-dimensional field configurations. We illustrate the measured data for four field configurations in Fig. 2. Figure 2 is a graphical representation of the measured data in terms of this decomposition, into the natural basis of uncorrelated, fully polarized fields. The black curves are obtained from decomposing, via Eq. (3), the correlation matrix resulting from the superposition of the three independent beams in known states of polarization, while the red curves are obtained following similar calculations based on $\bar{W}$ measured by using the independent probe configurations. The orthogonal field components have been represented as polarization ellipses with appropriate magnitudes. The scale on the graphs is arbitrary, but the same for the entire figure, allowing for a direct comparison. The first row is an elliptically polarized field. The second row is a two-dimensional, unpolarized field comprised of the incoherent superposition of the first field with a linearly polarized field of equal intensity normal to the polarization ellipse of the first. The third state is three-dimensional unpolarized light resulting from the incoherent superposition of three perpendicularly oriented, linearly polarized fields of equal magnitude. This is an interesting case, as there is no unique equivalent ensemble of three states (in fact, any three orthogonal fields with equal intensities results in this distribution). The last row represents an interesting partially polarized, three-dimensional field. It is the result of the superposition of three right circular states with mutually perpendicular directions of propagation. Reconstruction of the states of polarization is good, with experimental errors being mainly present in the intensities.

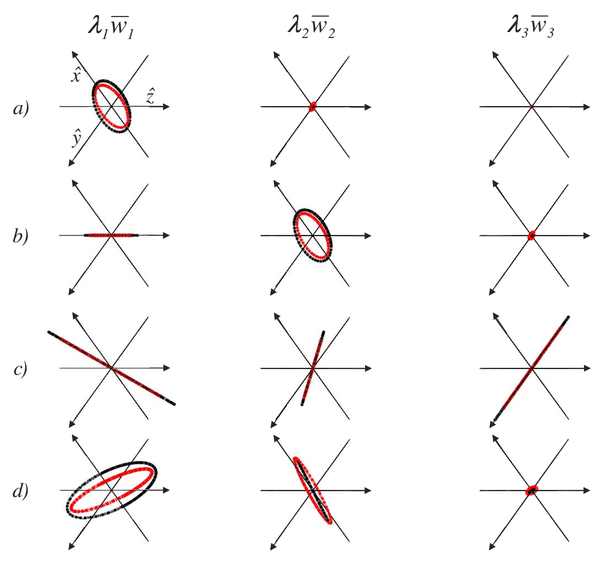


FIG. 2 (color online). Field components corresponding to four different 3D field configurations: (a) elliptically polarized state, (b) 2D unpolarized state, (c) 3D unpolarized state, (d) partially polarized state. The red dots represent the measured values, while the black curves denote the expected components calculated from the known polarization states of the three overlapped beams as shown in Fig. 1.

The measured $\bar{W}_{ij}(\mathbf{r}, \mathbf{r}, \omega)$ matrix caries all information pertaining to second-order field correlations at a particular point. Generally, it is expressed in terms of the auto and cross correlations of mutually perpendicular states of polarization corresponding to the measurement reference frame. While these nine parameters fully specify the polarization information contained in the field at that point, they may not be the most convenient descriptor of that information. In fact, there are a number of physically significant interpretations of the information present in these nine quantities [5,8].

These interpretations are given in terms of calculated parameters and decompositions of the cross-spectral density matrix. The representation of the field correlations in terms of the natural basis of Eq. (3) is one. Essentially, we can represent any fluctuating field as the superposition of three independent, fully polarized fields in mutually orthogonal states of polarization. This interpretation is useful for representing the state of polarization of the field at a point as shown in Fig. 2. It also allows for the unique identification of the polarized portion of the field.

A polarized field is one for which the electric field vector moves on an ellipse with increasing time. It has been shown [7] that this is equivalent to a field for which the orthogonal field components are fully correlated, i.e., there exists a fixed phase and amplitude relationship between all field components. This implies that each element of the correlation matrix factorizes, and that there is only one element in the decomposition of Eq. (3) with a nonzero intensity. The polarized portion of a general, fluctuating field is then that unique part that can be represented by a correlation matrix which fully factorizes, and is given by the portion of the maximal field component which is in excess of the other two [5]. Specifically, the correlation matrix of any three-dimensional field can be uniquely decomposed into a portion that represents a polarized field and a portion for which there is no unique polarized component (this second piece can be considered the unpolarized portion, as there is no polarized component) as

$$\bar{W} = (\lambda_1 - \lambda_2)\bar{w}_1 + (2\lambda_2 + \lambda_3)\bar{W}_U, \tag{4}$$

where $\lambda_i$ are the eigenvalues (in decreasing order of their magnitudes) of the $\bar{W}$ matrix, $w_1$ is the correlation matrix of a unit trace, fully polarized corresponding to the maximal eigenvalue of $\bar{W}$, and $\bar{W}_U$ is the correlation matrix of a unit trace, unpolarized field. This also allows for a physically meaningful definition of the degree of polarization in terms of the ratio of energy densities of the polarized and total field [6]:

$$P = \frac{\lambda_1 - \lambda_2}{\lambda_1 + \lambda_2 + \lambda_3}. \tag{5}$$

We have demonstrated that all the nine elements of the field-field correlation matrix of three-dimensional, fluctuating fields can be measured in practice. By utilizing independent probe configurations, it is possible to extract the second-order correlations of mutually orthogonal electric field components at a point. As the field correlations of any three-dimensional, fluctuating field can be regarded as the incoherent superposition of three fully polarized orthogonal fields, it is possible to represent the information contained in the cross-spectral density matrix of a field at a point in terms of three polarization ellipses and their relative intensities. We have shown that there is a good agreement between the generated field and the experimental measurements. Also, it is important to note that this procedure allows for the unique determination of the polarized component of a random field. The results presented here constitute the practical demonstration of what could be called three-dimensional optical polarimetry and its applications may find use in near-field optics, in connection with fluorescence, high energy lasers, and imaging.

The authors thank Emil Wolf for insightful discussion on this subject.

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