Two-Dimensional Bright Solitons in Dipolar Bose-Einstein Condensates

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We analyze the physics of bright solitons in 2D dipolar Bose-Einstein condensates. These solitons, which are not possible in short-range interacting gases, constitute the first realistic proposal of fully mobile stable 2D solitons in ultracold gases. In particular, we discuss the necessary conditions for the existence of stable 2D bright solitary waves by means of a 3D analysis of the lowest-lying excitations. We show that the anisotropy of the dipolar potential is crucial, since sufficiently large dipolar interactions can destabilize the 2D soliton. Additionally, we study the scattering of solitary waves, which, contrary to the contact-interacting case, is inelastic and could lead to fusion of the waves. Finally, the experimental possibilities for observability are discussed.

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During the past years, the physics of ultracold atomic and molecular gases has attracted considerable interest. Although these gases are very dilute, their properties are crucially determined by the interparticle interactions [1]. Up to now, only short-range van der Waals interactions have played a significant role in typical experiments. However, very recent developments are paving the way towards a new fascinating research area, namely, that of degenerate dipolar gases. A major breakthrough has been very recently performed at Stuttgart University, where a Bose-Einstein condensate (BEC) of ^{52}Cr atoms has been realized [2]. ^{52}Cr atoms are particularly interesting, since they present a large magnetic dipole moment, $\mu = 6\mu_B$. Hence, ^{52}Cr BEC constitutes the first realization of a degenerate dipolar gas. On the other hand, recent developments on cooling and trapping of molecules [3], on photoassociation [4], and on Feshbach resonances of binary mixtures [5] open exciting perspectives towards a degenerate gas of polar molecules [6].

In addition to the usual short-range forces, dipolar particles oriented by an external field interact via dipole-dipole interaction, which is long-range and anisotropic, being partially attractive. New exciting physics is, therefore, expected in these systems. Recent theoretical analyses have shown that the stability and excitations of dipolar gases are crucially determined by the trap geometry [7–9]. Ultracold dipolar particles are also attractive in the context of strongly correlated atomic gases [10,11], as physical implementation of quantum computation [12], and for the study of ultracold chemistry [13].

The nonlinearity of the BEC physics is one of the major consequences of the interparticle interactions. In this sense, resemblances between BEC physics and nonlinear physics (in particular, nonlinear optics) have been analyzed in detail. Several remarkable experiments have been reported in this context, including four-wave mixing [14], BEC collapse [15], and the creation of dark, bright, and gap solitons [16]. The physics of BEC solitons has indeed aroused a large interest. For short-range interactions and at sufficiently low temperature, the BEC physics is provided by a nonlinear Schrödinger equation (NLSE) with cubic nonlinearity (Gross-Pitaevskii equation) [1]. The 1D NLSE admits solitonic solutions for bright (dark) solitons [17] for attractive (repulsive) interatomic interactions, the equivalent of self-focusing (self-defocusing) nonlinearity in Kerr media. For larger dimensions, the NLSE admits no stable solitons, although discrete solitons may be found in the presence of periodic potentials [18]. These solutions may be created in cold atoms in optical lattices [19] but possess a very limited mobility. The use of optical lattices in one or two directions has been recently proposed to allow for mobile 2D and 3D BEC solitons along a free direction [20]. We stress, however, that these structures move freely only along the free direction, clearly differing from the solutions discussed in this Letter.

A dipolar BEC is, as we discuss below, described by a NLSE with nonlocal nonlinearity, induced by the dipole-dipole interaction. Nonlocal nonlinearity has been a subject of active investigation in disparate physical systems during the past years. Nonlocality is known to be important, e.g., in the physics of plasmas [21], where the nonlocal response is induced by heating and ionization, and in the physics of nematic liquid crystals, where it is the result of long-range molecular interactions [22]. Nonlocality plays a crucial role in the physics of solitons and modulational instability [23–26]. In particular, any symmetric nonlocal nonlinear response with positive definite Fourier spectrum has been mathematically shown to arrest collapse in arbitrary dimensions [24]. However, as discussed below, these conditions are violated by the anisotropy of the dipole-dipole interaction, which leads to an even richer physics. Multidimensional solitons have been experimentally observed in nematic liquid crystals [22] and in photorefractive screening solitons [27]. Multidimensional solitons have been also discussed in BEC with short-range interactions, by considering the collapse arrest induced by the first nonlocal correction to the local pseudopotential [23,26]. However, in typical BEC experiments this collapse arrest occurs for an extremely small condensate size [26], which, except for the case of a very small particle number, leads to extremely large densities, at which three-body losses destroy the BEC. In this sense, this Letter constitutes the first realistic proposal to obtain multidimensional fully mobile BEC solitons.

In this Letter, we are interested in the physics of solitons in a 2D dipolar BEC. Since, as discussed below, the scattering of these solutions is inelastic, we refer henceforth to them as 2D solitary waves (SWs). Whereas, as discussed above, for short-range interacting BEC a 2D stable SW is not possible, we show that the dipole-dipole interactions may stabilize a 2D SW. We discuss the appropriate conditions at which this is possible, which involve the tuning of the dipole-dipole interactions as discussed in Ref. [28]. We study the stability of the solitary waves, by means of a fully 3D analysis of the lowest-lying excitations, showing that, contrary to what is expected in the presence of general long-range interactions [24], the anisotropy of the dipole-dipole interactions is indeed a crucial feature, since it induces destabilization for sufficiently large nonlocal potentials. In the final part of this Letter, the scattering of 2D SWs is analyzed. We show that this scattering is inelastic, and it can lead under appropriate conditions to the fusion of SWs. Finally, we conclude with a discussion about the experimental feasibility and generation of the SWs.

In the following, we consider a BEC of N particles with electric dipole d (the results are equally valid for magnetic dipoles) oriented in the z direction by a sufficiently large external field and that, hence, interact via a dipole-dipole potential: $V_d(\vec{r}) = g_d(1 - 3\cos^2\theta)/r^3$, where $g_d = \alpha N d^2/4\pi\epsilon_0$, with ϵ_0 the vacuum permittivity, θ the angle formed by the vector joining the interacting particles and the dipole direction, and $-1/2 \leq \alpha \leq 1$ a tunable parameter by means of rotating orienting fields [28]. This tunability becomes crucial in our discussion.

A dipolar BEC at sufficiently low temperatures is described by a NLSE with nonlocal nonlinearity [7]:

$$i\hbar\frac{\partial}{\partial t}\Psi(\vec{r},t) = \left[-\frac{\hbar^2}{2m}\nabla^2 + U(\vec{r}) + g|\Psi(\vec{r},t)|^2 + \int d\vec{r}' V_d(\vec{r}-\vec{r}')|\Psi(\vec{r}',t)|^2\right]\Psi(\vec{r},t), \quad (1)$$

where $\int |\psi(\vec{r},t)|^2 d\vec{r} = 1$, and $g = 4\pi\hbar^2 aN/m$ is the coupling constant which characterizes the contact interaction, with a the s-wave scattering length. In the following, we consider $a > 0$, i.e., repulsive short-range interactions. We assume an external trapping potential $U(\vec{r}) = m\omega_z^2 z^2/2$, with no trapping in the xy plane.

In the following, we are interested in the possibility to achieve a 2D SW in dipolar BECs. In order to have a good insight on this issue, we introduce a Gaussian ansatz for the wave function:

$$\Psi_0(\vec{r}) = \frac{1}{\pi^{3/4} l_z^{3/2} L_\rho L_z^{1/2}} \exp\left(-\frac{x^2+y^2}{2l_z^2 L_\rho^2} - \frac{z^2}{2l_z^2 L_z^2}\right), \quad (2)$$

where $l_z = \sqrt{\hbar/m\omega_z}$, and L_ρ and L_z are dimensionless variational parameters related with the widths in the xy plane and the z direction, respectively. We have numerically checked that this ansatz is indeed a very good approximation of the exact solutions of Eq. (1) for the situations under consideration. Using this ansatz, the energy of the system reads:

$$\frac{2E}{\hbar\omega_z} = \frac{1}{L_\rho^2} + \frac{1}{2L_z^2} + \frac{L_z^2}{2} + \frac{1}{\sqrt{2\pi}L_\rho^2 L_z}\left[\frac{\tilde{g}}{4\pi} + \frac{\tilde{g}_d}{3} f\left(\frac{L_\rho}{L_z}\right)\right], \quad (3)$$

where $\tilde{g} = 2g/\hbar\omega_z l_z^3 = 8\pi Na/l_z$ and $\tilde{g}_d = 2g_d/\hbar\omega_z l_z^3$, and $f(\kappa) = (\kappa^2-1)^{-1}[2\kappa^2 + 1 - 3\kappa^2 H(\kappa)]$, with $H(\kappa) = \arctan(\sqrt{\kappa^2-1})/\sqrt{\kappa^2-1})$, and $\kappa = L_\rho/L_z$.

The minimization of E leads to the equations:

$$1 + \frac{\tilde{g}}{2(2\pi)^{3/2}L_z}\left[1 - \frac{2\pi}{3}\beta F\left(\frac{L_\rho}{L_z}\right)\right] = 0, \quad (4)$$

$$L_z = \frac{1}{L_z^3} + \frac{\tilde{g}}{2(2\pi)^{3/2}L_\rho^2 L_z^2}\left[1 - \frac{4\pi}{3}\beta G\left(\frac{L_\rho}{L_z}\right)\right], \quad (5)$$

where $F(\kappa) = (1-\kappa^2)^{-2}[-4\kappa^4 - 7\kappa^2 + 2 + 9\kappa^4 H(\kappa)]$, $G(\kappa) = (1-\kappa^2)^{-2}[-2\kappa^4 - +10\kappa^2 + 1 - 9\kappa^2 H(\kappa)]$, and $\beta = \tilde{g}_d/\tilde{g}$. Equations (4) and (5) admit a solution, and, hence, a localized wave, only under certain conditions. A simplified picture may be achieved by considering the fully 2D case in which the confinement in z is strong enough to guarantee $L_z = 1$. In that case, both kinetic and interaction energy scale as $1/L_\rho^2$. In the absence of dipole-dipole interactions ($\tilde{g}_d = 0$), and irrespective of the value of L_ρ, $E(L_\rho)$ is, depending on the value of g, monotonic either growing with L_ρ (collapse instability) or decreasing with L_ρ (expansion instability). This reflects the well-known fact that 2D solitons are not stable in NLSE with contact interactions. In the case of a dipolar BEC, the situation is remarkably different, since the function f depends explicitly on L_ρ. This allows for the appearance of a minimum in $E(L_\rho)$ (inset in Fig. 1), which from the asymptotic values of f [$f(0) = -1$ and $f(\kappa \to \infty) = 2$] should occur if:

$$\frac{\tilde{g}_d}{3\sqrt{2\pi}} < 1 + \frac{\tilde{g}}{2(2\pi)^{3/2}} < \frac{-2\tilde{g}_d}{3\sqrt{2\pi}}. \quad (6)$$

A simple inspection shows that this condition can be fulfilled only if $\tilde{g}_d < 0$, i.e., only if the dipole is tuned as previously discussed with $\alpha < 0$ (this is true also for $L_z \neq 1$). In that case, the tuning of the dipole-dipole interaction may allow for the observation of a stable SW, characterized by an internal energy $E_S < 0$ (inset in Fig. 1). Note that, if $Na/l_z \gg 1$, then we arrive to the condition $|\beta| > 3/8\pi \simeq 0.12$. A direct resolution of Eq. (9) (below) for large $\tilde{g}$ shows stable SWs for $|\beta| > 0.12$, in excellent agreement with the Gaussian ansatz.

In order to gain more understanding on the stability of the 2D SWs, we have analyzed the lowest-lying modes of a SW, namely, breathing and quadrupolar modes. To this aim, we employ a Gaussian ansatz of the form [29,30]:

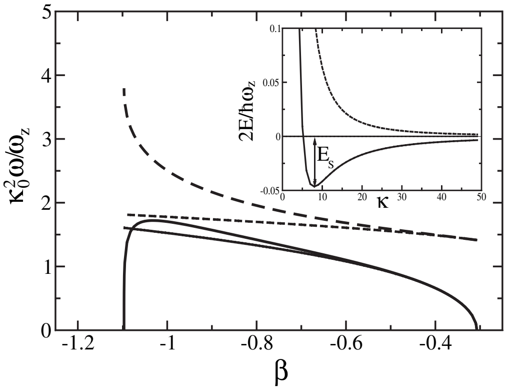

FIG. 1. Breathing (bold solid line) and $m = \pm 2$ quadrupole (bold dashed line) mode for $\tilde{g} = 20$, with $\kappa_0 = L\rho/L_z$, as a function of β. 2D results are shown in thin lines. Inset: $E(\kappa)$ for $\tilde{g} = 500$, and $\beta = -0.10$ (dashed line) and $\beta = -0.20$ (solid line).

$$\Psi(\vec{r};t) = \Psi_0\left(\frac{x}{b_x}, \frac{y}{b_y}, \frac{z}{b_z}\right) e^{i\beta_x x^2 + i\beta_y y^2 + i\beta_z z^2}, \tag{7}$$

where $\{b_i(t), \beta_i(t)\}$, $i = x, y, z$, are time-dependent parameters, and insert this ansatz into the corresponding Lagrangian density

$$\mathcal{L} = \frac{i\hbar}{2}(\Psi\dot{\Psi}^* - \dot{\Psi}\Psi^*) + \frac{\hbar^2}{2m}|\nabla\Psi|^2 + \frac{1}{2}m\omega_z^2 z^2 |\Psi(\vec{r},t)|^2 + \frac{g_c}{2}|\Psi(\vec{r},t)|^4 + \frac{1}{2}|\Psi(\vec{r},t)|^2 \int d\vec{r}' V_d(\vec{r}-\vec{r}')|\Psi(\vec{r}',t)|^2. \tag{8}$$

After integrating $L = \int d\vec{r}\mathcal{L}$, we obtain the corresponding Euler-Lagrange equations for $\{b_i(t), \beta_i(t)\}$. Linearizing these equations around the stationary solution obtained from Eqs. (4) and (5), we obtain the expressions for the frequencies of the lowest-lying modes [30]. A typical behavior of the lowest modes as a function of β is depicted in Fig. 1. The lowest-lying mode has for any value of β a breathing character. For sufficiently small values of $|\beta|$, the frequency of the breathing mode tends to zero, and eventually the system becomes unstable against expansion. This corresponds to the disappearance of the minimum in the inset in Fig. 1. In this regime, the 2D picture provides a good description of the physics of the problem, as shown in Fig. 1. For sufficiently large values of $|\beta|$, the 3D character of the system becomes crucial, leading to a different sort of instability, in this case against 3D collapse. This is reflected in the decrease of the frequency of the breathing mode. Hence, as expected from general arguments for nonlocal nonlinearity [24], the dipolar interaction can stabilize the 2D SW. However, a new crucial feature is introduced by the anisotropic character of the dipolar interaction, since too large dipolar interactions can destabilize the SWs.

As shown in Fig. 1, a 2D calculation offers a good description of the problem for sufficiently small values of $|\beta|$. In the 2D case, the system can be considered as "frozen" into the ground state $\varphi_0(z)$ of the harmonic oscillator in the z direction, and, hence, the BEC wave function factorizes as $\Psi(\vec{r}) = \psi(\vec{\rho})\varphi_0(z)$. Employing this factorization, the convolution theorem, the Fourier transform of the dipole potential, $\tilde{V}_d(k) = (4\pi/3)(3k_z^2/k^2 - 1)$, and integrating over the z direction, we arrive at the 2D NLSE:

$$i\hbar\frac{\partial}{\partial t}\psi(\vec{\rho},t) = \left[-\frac{\hbar^2}{2m}\nabla_\rho^2 + \frac{g}{\sqrt{2\pi}l_z}|\psi(\vec{\rho},t)|^2 + \frac{4\sqrt{\pi}g_d}{3\sqrt{2}l_z} \times \int \frac{d\vec{k}_\rho}{(2\pi)^2} e^{i\vec{k}_\rho\cdot\vec{\rho}}\tilde{n}(\vec{k}_\rho) h_{2D}\left(\frac{k_\rho l_z}{\sqrt{2}}\right)\right]\psi(\vec{\rho},t), \tag{9}$$

where $\tilde{n}$ is the Fourier transform of $n(\vec{\rho}) = |\psi(\vec{\rho})|^2$, and $h_{2D}(k) = 2\text{–}3\sqrt{\pi}ke^{k^2}\,\mathrm{erfc}(k)$, with erfc$(x)$ the complementary error function. Below, we employ Eq. (9) to analyze numerically the dynamics of 2D SWs.

Up to now, we have analyzed a single localized wave, showing that a stable SW may exist under proper conditions. In order to deepen our understanding of the 2D SWs, and their comparison with the solitons in a 1D NLSE, we have analyzed the scattering of two SWs for different values of their initial center-of-mass kinetic energy $E_{\rm kin}$. Direct numerical simulations of the 2D nonlocal NLSE show that the scattering of dipolar 2D SWs is inelastic. In particular, as shown in Fig. 2, for sufficiently slow localized waves (for the case considered in Fig. 2, $E_{\rm kin} \leq 2.9|E_S|$), two SWs merge when colliding. As observed in Fig. 2, the solitary waves, when approaching, transfer their center-of-mass kinetic energy into internal energy, transforming into a single localized structure. This structure, although localized, is in an excited state, and clear oscillations may be observed. For larger initial kinetic energies, the waves move apart from each other after the collision, but the transfer of kinetic energy into internal energy is enough to unbind the SW, and the SWs are destroyed. This

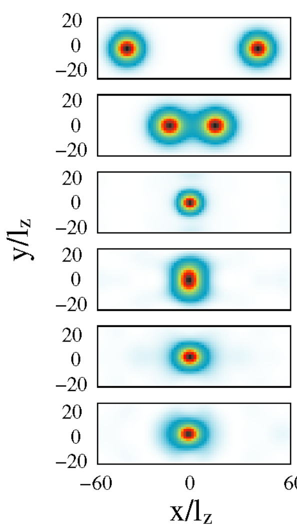

FIG. 2 (color online). Density plot of the fusion of two dipolar 2D SWs for $\tilde{g} = 20$, $\beta = -0.5$, and $\tilde{k} = 0.01$. From top to bottom: $\omega_z t/2 = 0, 1000, 2000, 3000, 4000, 5000$.

inelastic character of the scattering of dipolar 2D SWs clearly distinguishes these solutions from solitons in a 1D NLSE [17] and will be the subject of further investigation.

In the final part of this Letter, we discuss some issues concerning the experimental realization of stable 2D SWs in ultracold dipolar gases. As we have previously mentioned, the experimental generation of these structures demands the tuning of the dipole-dipole interaction and, in particular, the inversion of its sign. In addition to this condition, the dipole-dipole interaction must be sufficiently large, $|g_d|/g > 0.12$. For ^{52}Cr, $|g_d|/g \simeq 0.03$ for $\alpha = -1/2$, and, hence, the tuning of the dipolar interaction must be combined with a reduction of the contact interactions via Feshbach resonances. This combination can be problematic, since the absolute value of the magnetic field must be kept constant while tuning the dipolar strength [31]. However, optical Feshbach resonances could be applied instead [32]. Additionally, as commented in the introduction of this Letter, current developments open optimistic perspectives for the achievement of a BEC of polar molecules in the near future. Since appropriate molecules in the lowest vibronic state can present very large electric dipole moments [6], and the dipole-dipole interaction may be tuned in these gases by means of rotating electric fields, a broad regime of values of β may be available, and, hence, a stable 2D SW should be easily observable.

In summary, dipolar BEC offers a new highly controllable physical scenario for the analysis of nonlocal nonlinearity. We have shown that, contrary to the case of short-range interacting BECs, stable 2D SWs can be generated in dipolar BEC by means of tuning techniques, for a sufficiently large ratio between dipole-dipole and short-range interaction. However, our fully 3D stability analysis shows that, contrary to general long-range interactions, the anisotropy of the dipolar interaction is crucial, since it induces destabilization of the SWs for sufficiently large dipolar interactions. We have also shown that these SWs scatter inelastically and may undergo fusion for sufficiently low scattering energies. Our Letter constitutes the first realistic proposal for the observation of fully mobile 2D SWs in BEC. These 2D SWs can be employed for the transport of cold atoms in 2D which can be easily guided by means of Raman laser pulses, without complicated wave guiding arrangements

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