

## A Universal Criterion for Plastic Yielding of Metallic Glasses with a $(T/T_g)^{2/3}$ Temperature Dependence

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Room temperature ( $T_R$ ) elastic constants and compressive yield strengths of  $\sim 30$  metallic glasses reveal an average shear limit  $\gamma_C = 0.0267 \pm 0.0020$ , where  $\tau_Y = \gamma_C G$  is the maximum resolved shear stress at yielding, and  $G$  the shear modulus. The  $\gamma_C$  values for individual glasses are correlated with  $t = T_R/T_g$ , and  $\gamma_C$  for a single glass follows the same correlation (vs  $t = T/T_g$ ). A cooperative shear model, inspired by Frenkel's analysis of the shear strength of solids, is proposed. Using a scaling analysis leads to a universal law  $\tau_{CT}/G = \gamma_{C0} - \gamma_{C1}(t)^{2/3}$  for the flow stress at finite  $T$  where  $\gamma_{C0} = (0.036 \pm 0.002)$  and  $\gamma_{C1} = (0.016 \pm 0.002)$ .

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For a dislocation free crystal, Frenkel [1] calculated the theoretical shear strength by assuming cooperative shearing obtaining  $\tau_Y \cong G/5$ . The yield strength of metallic glasses is thought to be determined by the cooperative shear motion of atomic clusters termed shear transformation zones (STZ's) [2–5]. Compressive strengths of  $\sigma_Y \sim 0.02Y$  are observed with a weak dependence on normal stress or pressure [6,7]. Here, we report elastic constants and compressive yield stresses for  $\sim 30$  metallic glasses. Yielding at  $T_R$  can be described by a critical shear strain  $\gamma_C = 0.0267 \pm 0.0020$ ; a better description of  $\gamma_C$  includes a dependence on the dimensionless temperature  $t = T/T_g$ . A cooperative shear model (CSM) is introduced that predicts a temperature dependent  $\tau_C$  (or  $\gamma_C$ ) having a “ $T^{2/3}$ ” form. The CSM is based on the concept of inherent states (IS) and potential energy landscapes (PEL) developed by Stillinger *et al.* [8,9], Wales *et al.* [10,11], and Milandro and Lacks [12].

Table I shows measured density, ambient  $T$  elastic constants  $Y$ ,  $G$ ,  $B$ ,  $\nu$  (Poisson's ratio), yield strength in compression,  $\sigma_Y$ , elastic strain limit ( $\sigma_Y/Y$ ), and glass transition temperature,  $T_g$ , for  $\sim 30$  metallic glasses [13–29]. Note that  $\sigma_Y/Y$  varies over the range  $0.014 < \varepsilon_Y < 0.022$ . Ignoring the small normal stress dependence of the shear yield strength [6,7,13,14], one can plot  $\tau_Y$  vs  $G$  ( $\tau_Y = \sigma_Y/2$ ) to find the corresponding elastic shear strain limit as shown in Fig. 1. We obtain linear correlation with a best fit of  $\gamma_C = \tau_Y/G = 0.0267 \pm 0.0020$ , but there remains significant scatter in  $\gamma_C$ . Examination shows that glasses with low  $T_g$  tend to exhibit smaller  $\gamma_C$  than those with high  $T_g$ . Consider the reduced temperature,  $t = T_R/T_g$ . We plot  $\gamma_C$  for each individual alloy vs  $t$  (open circles) in Fig. 2. The plot includes data (filled circles) for the temperature dependent  $\tau_Y$  of Vitreloy 1 (fixed  $T_g$  and varying  $T$ ) by Lu *et al.* [13], low temperature data for bulk  $\text{La}_{55}\text{Al}_{25}\text{Cu}_{20}$  [30] (squares), melt spun ribbons of  $\text{Pd}_{85.5}\text{Si}_{14.5}$  [31] (stars), and  $\text{Fe}_{40}\text{Ni}_{40}\text{P}_{14}\text{B}_6$  [31] (horizontal triangles), and bulk  $\text{Pd}_{77.5}\text{Cu}_6\text{Si}_{16.5}$  [2(b),24] (vertical triangles). The data for ribbons were “normalized” to obtain

agreement with other data at  $T = 0$  K ( $G$  was not known for the ribbons). The “peak flow stress” of Vitreloy 1 vs  $T$  (from  $T_R$  to above  $T_g$ ) was taken as  $\tau_Y$ . Figure 2 shows that  $\gamma_C$  is a systematic function of  $t$ .

Following Frenkel, the elastic energy of an STZ is here described by a periodic elastic energy density vs strain:

$$\phi(\gamma) = \phi_0/2[1 - \cos(\pi\gamma/2\gamma_C)] = \phi_0 \sin^2(\pi\gamma/4\gamma_C) \quad (1)$$

with a minima at  $\gamma = 0$ , a barrier at  $2\gamma_C$  ( $\gamma_C$  is the yield strain), and a total barrier energy density  $\phi_0$ . The critical yield stress is  $\phi'|_{\max} = \tau_c = \pi\phi_0/4\gamma_C$ . For the unstressed solid,  $G = \phi''|_{\gamma=0}$  giving  $\phi_0 = (8/\pi^2)G\gamma_C^2$ . The “free enthalpy” density of the stressed STZ is  $h(\gamma) = \phi(\gamma) - \tau\gamma$ . In an unstressed solid, the total potential energy barrier for an STZ is  $W = \phi_0\zeta\Omega = (8/\pi^2)G\gamma_C^2\zeta\Omega$ , where  $\Omega$  is the actual volume of the STZ defined by the plastic “core,” and  $\zeta$  is a correction factor arising from matrix confinement of a “dressed” STZ [2,32]. For a Gaussian shaped strain fluctuation with core diameter  $\sigma$ , one can estimate  $\zeta \sim 2-4$  and  $W \sim 3\Omega\phi_0$ . The details of  $\zeta$  depend on the shape and size of the fluctuation and the elastic constants  $G$  and  $\nu$  for the material.

For an infinite crystal of indistinguishable atoms, the periodic minima of  $\phi(\gamma)$  are equivalent; i.e., there is no configurational entropy. For a glass, there are  $\Gamma$  stable atomic configurations or inherent states (IS's) [8,9,33] with  $\Gamma = \exp(N\Delta s_C)$ , where  $N$  is the number of atoms in the STZ, and  $\Delta s_C$  the configurational entropy per atom of the IS's or “basin denumeration function” [9,33]. While  $\Delta s_C$  is well defined in the thermodynamic limit  $N \rightarrow \infty$ , it decreases [9,10,33] for small  $N$ . The characteristic strain ( $\gamma_C$ ) or “configurational displacement” separating neighboring configurations will increase for  $N \sim 100$  or less. On the other hand, the total barrier  $W$  also scales with STZ volume  $\Omega$  (or  $N$ ). Therefore  $W \sim \gamma_C^2\Omega$  is expected have a minimum for some intermediate  $N^*$ . We estimate that  $N^*$  is likely of order  $\sim 100$  atoms. Yielding occurs when the applied stress causes a critical density of “minimum” barrier STZ's to become unstable.

TABLE I. Summary of data on alloy compositions and properties used in this Letter.

Alloy	Property								
	$\rho$ (g/cc)	$Y$ (GPa)	$G$ (GPa)	$B$ (GPa)	$\nu$	$\sigma_y$ (GPa)	$T_g$ (K)	$\sigma_y/Y$	Ref.
1. Zr <sub>41.2</sub> Ti <sub>13.8</sub> Ni <sub>10</sub> Cu <sub>12.5</sub> Be <sub>22.5</sub>	5.9	95	34.1	114.1	0.352	1.86	618	0.0196	[13–15]
		97.2	35.9	111.2	0.354	1.85	613	0.0190	
2. Zr <sub>48</sub> Nb <sub>8</sub> Ni <sub>12</sub> Cu <sub>14</sub> Be <sub>18</sub>	6.7	93.9	34.3	118	0.367	1.95	620	0.0208	[15]
3. Zr <sub>55</sub> Ti <sub>5</sub> Cu <sub>20</sub> Ni <sub>10</sub> Al <sub>10</sub>	6.62	85	31	118	0.375	1.63	625	0.0192	[15]
4. Zr <sub>57.5</sub> Nb <sub>5</sub> Cu <sub>15.4</sub> Ni <sub>12</sub> Al <sub>10</sub>	6.5	84.7	30.8	117.6	0.379	1.58	663	0.0187	[15]
5. Zr <sub>55</sub> Al <sub>19</sub> Co <sub>19</sub> Cu <sub>7</sub>	6.2	101.7	37.6	114.9	0.352	2.2	733	0.0216	[16]
6. Pd <sub>40</sub> Cu <sub>30</sub> Ni <sub>10</sub> P <sub>20</sub>	9.28	92	34.5	151.8	0.399	1.72	593	0.0187	[17]
7. Pd <sub>40</sub> Cu <sub>30</sub> Ni <sub>10</sub> P <sub>20</sub>	9.28	92	33	146	0.394	1.72	593	0.0187	[18]
8. Pd <sub>40</sub> Cu <sub>30</sub> Ni <sub>10</sub> P <sub>20</sub>	9.30	92	35.8	144.7	0.394	1.75	595	0.0190	[17]
9. Pd <sub>60</sub> Cu <sub>20</sub> P <sub>20</sub>	9.78	91	32.3	167	0.409	1.70	604	0.0187	[15]
10. Pd <sub>40</sub> Cu <sub>40</sub> P <sub>20</sub>	9.30	93	33.2	158	0.402	1.75	548	0.0188	[15]
11. Ni <sub>45</sub> Ti <sub>20</sub> Zr <sub>25</sub> Al <sub>10</sub>	6.4	109.3	40.2	129.6	0.359	2.37	791	0.0217	[19]
12. Ni <sub>40</sub> Ti <sub>17</sub> Zr <sub>28</sub> Al <sub>10</sub> Cu <sub>5</sub>	6.48	127.6	47.3	140.7	0.349	2.59	862	0.0203	[19]
13. Ni <sub>60</sub> Nb <sub>35</sub> Sn <sub>5</sub>	8.64	183.7	66.32	267	0.385	3.85	885	0.0210	[20]
14. Ni <sub>60</sub> Sn <sub>6</sub> (Nb <sub>0.8</sub> Ta <sub>0.2</sub> ) <sub>34</sub>	9.24	161.3	59.41	189	0.357	3.50	875	0.0217	[16]
15. Ni <sub>60</sub> Sn <sub>6</sub> (Nb <sub>0.6</sub> Ta <sub>0.4</sub> ) <sub>34</sub>	9.80	163.7	60.1	197.6	0.361	3.58	882	0.0219	[16]
16. Cu <sub>64</sub> Zr <sub>36</sub>	8.07	92	34	104.3	0.352	2.0	787	0.0217	[21]
17. Cu <sub>46</sub> Zr <sub>54</sub>	7.62	83.5	30.0	128.5	0.391	1.40	696	0.0168	[22]
18. Cu <sub>46</sub> Zr <sub>42</sub> Al <sub>7</sub> Y <sub>5</sub>	7.23	84.6	31	104.1	0.364	1.60	713	0.0189	[23]
19. Pd <sub>77.5</sub> Cu <sub>6</sub> Si <sub>16.5</sub>	10.4	89.7	31.8	166	0.409	1.5	550	0.0167	[24]
20. Pt <sub>60</sub> Ni <sub>15</sub> P <sub>25</sub>	15.7	96.1	33.8	202	0.420	1.4	488	0.0146	[25]
21. Pt <sub>57.5</sub> Cu <sub>14.7</sub> Ni <sub>5</sub> P <sub>22.8</sub>	15.2	95.7	33.4	243.2	0.434	1.45	490	0.0151	[26]
22. Pd <sub>64</sub> Ni <sub>16</sub> P <sub>20</sub>	10.1	91.9	32.7	166	0.405	1.55	452	0.0169	[24]
23. MgGd <sub>10</sub> Cu <sub>25</sub>	4.04	49.1	18.6	46.3	0.32	0.98	428	0.020	[16]
24. La <sub>55</sub> Al <sub>25</sub> Cu <sub>10</sub> Ni <sub>5</sub> Co <sub>5</sub>	6.0	41.9	15.6	44.2	0.342	0.85	430	0.0203	[15]
25. Ce <sub>70</sub> Al <sub>10</sub> Ni <sub>10</sub> Cu <sub>10</sub>	6.67	30.3	11.5	27	0.313	0.65	359	0.0215	[27]
26. Cu <sub>50</sub> Hf <sub>43</sub> Al <sub>7</sub>	11.0	113	42	132.8	0.358	2.2	774	0.0195	[16]
27. Cu <sub>57.5</sub> Hf <sub>27.5</sub> Ti <sub>15</sub>	9.91	103	37.3	117.5	0.356	1.94	729	0.0188	[16]
28. Fe <sub>61</sub> Mn <sub>10</sub> Cr <sub>4</sub> Mo <sub>6</sub> Er <sub>1</sub> C <sub>15</sub> B <sub>6</sub>	6.89	193	75	146	0.280	4.16	870	0.0216	[28]
29. Fe <sub>53</sub> Cr <sub>15</sub> Mo <sub>14</sub> Er <sub>1</sub> C <sub>15</sub> B <sub>6</sub>	6.92	195	75	180	0.32	4.2	860	0.0215	[28]
30. Au <sub>49.5</sub> Ag <sub>5.5</sub> Pd <sub>2.3</sub> Cu <sub>26.9</sub> Si <sub>16.3</sub>	11.6	74.4	26.5	132.3	0.406	1.20	405	0.0141	[29]
31. Au <sub>55</sub> Cu <sub>25</sub> Si <sub>20</sub>	12.2	69.8	24.6	139.8	0.417	1.00	348	0.0143	[29]

The barrier at finite  $\tau$ ,  $W_\tau$ , approaches zero as  $\tau \rightarrow \tau_C$ . It is easily shown that  $\phi_{0\tau}$  (barrier energy density at finite  $\tau \rightarrow \tau_C$ ) decreases as  $\phi_{0\tau} \sim (\tau_C - \tau)^{3/2}$  while the shear modulus (at finite  $\tau$ )  $G_\tau \sim (\tau_C - \tau)^{1/2}$ . The strain difference between the energy minimum and barrier configuration (saddle point) scales as  $\delta\gamma_\tau \sim (\tau_C - \tau)^{1/2}$  as  $\tau \rightarrow \tau_C$ . Mechanical instability of the STZ at  $\tau_C$  takes the form of a “fold catastrophe” [11,34]. For  $\tau \rightarrow \tau_C$ , the parameters  $W_\tau = \phi_{0\tau}\zeta\Omega$ ,  $\delta\gamma_\tau$ , and  $G_\tau$ , are related by the scaling law

$$\phi_{0\tau}/(G_\tau(\delta\gamma_\tau)^2) = R = 1/4, \quad (2)$$

so that  $W_\tau = G_\tau\gamma_{C\tau}^2\zeta\Omega$ ,

where  $R$  is the “fold ratio.” Wales *et al.* [11,34] have shown that, for binary Lennard Jones (LJ) glasses (256 atoms) and liquid salt clusters (71 atoms), this scaling relation holds *on average*, even far from  $\tau_C$ . For the Frenkel landscape of Eq. (1),  $R$  actually varies from 1/4

to  $\pi^2/32$  as  $\tau$  varies from  $\tau_C$  down to 0. Analysis of simulation results [12] for shear induced destabilization of individual IS’s of a 500 atom LJ glass shows that Eq. (2) is obeyed (within 10%) over  $0 < \tau < \tau_C$ . Assuming scaling holds on average:

$$W_\tau = W_0(T)[(\tau_C - \tau)/\tau_C]^{3/2} = \phi_0[(\tau_C - \tau)/\tau_C]^{3/2}\zeta\Omega = 4RG_{0T}\gamma_C^2[(\tau_C - \tau)/\tau_C]^{3/2}\zeta\Omega, \quad (3)$$

where  $G_{0T}$  is the shear modulus of the *unstressed glass* which includes a *weak* dependence on  $T$  (Debye-Grüneisen thermal expansion) for a fixed glass configuration. The scaling law holds for any function  $\phi(\gamma)$  for which  $d^2\phi(\gamma)/d\gamma^2$  is analytic around the inflection point. At finite  $T$  and applied  $\tau$ , thermal strain fluctuations will carry the system over the barrier  $W_\tau$ . For plastic flow to occur on a given time scale (or strain rate  $\dot{\gamma}$ ), the rate of barrier crossing must reach a critical value comparable to  $\dot{\gamma}$ . Using

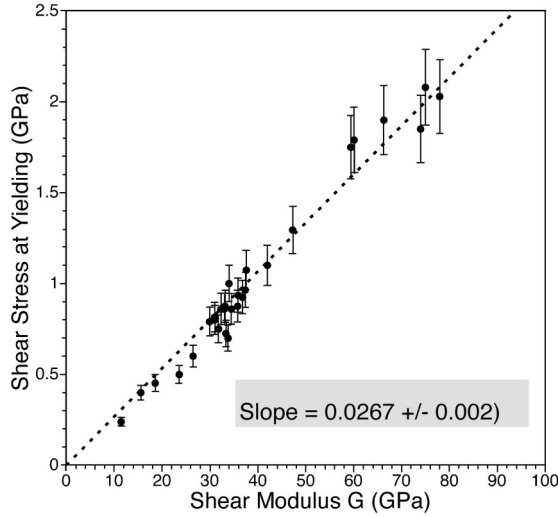


FIG. 1. Experimental shear stress at yielding,  $\tau_Y = \sigma_Y/2$  vs shear modulus  $G$  at room temperature for 30 bulk metallic glasses.

an attempt frequency  $\omega_0$  requires

$$\text{yielding rate} = \omega_0 \exp(-W_\tau/kT) = C\dot{\gamma}, \quad (4)$$

with  $C$  a dimensionless constant of order unity, and thus

$$\begin{aligned} W_\tau/kT &= -\ln(C\dot{\gamma}/\omega_0) \\ &= \{4RG_{0T}\gamma_C^2[(\tau_{C0} - \tau_{CT})/\tau_C]^{3/2}\zeta\Omega\}/kT. \end{aligned} \quad (5)$$

Here,  $\tau_{C0}$  is the yield stress at  $T = 0$ , while  $\tau_{CT}$  is at finite  $T$ . One obtains

$$\tau_{CT} = \tau_{C0} - \tau_{C0}[kT \ln(\omega_0/C\dot{\gamma}) / (4RG_{0T}\gamma_C^2\zeta\Omega)]^{2/3}. \quad (6)$$

At  $T = T_g$  and  $\tau = 0$ , the barrier is  $W_0(T_g) = 4RG_{0T_g}\gamma_C^2\zeta\Omega$ . If  $T_g$  itself is defined by a critical barrier crossing rate due to fluctuations, one obtains  $W_0(T_g) = \beta T_g$ , with  $\beta$  constant. This yields

$$W_{\tau,T_g} = \beta T_g [(\tau_C - \tau)/\tau_C]^{3/2}. \quad (7)$$

Equation (6) for the yield stress becomes

$$\tau_{CT} = \tau_{C0} - \tau_{C0}[(k/\beta) \ln(\omega_0/C\dot{\gamma})(G_{0T}/G_{0T_g})]^{2/3} t^{2/3}, \quad (8)$$

where  $t = T/T_g$ . The factor  $(G_{0T}/G_{0T_g})$  incorporates the weak dependence of  $G$  on the thermal expansion of a fixed glass configuration.  $G_{0T}$  has been experimentally determined [15,18] to be a linear function of  $T$ . For Vitreloy 1 [15], one finds  $dG_{0T}/dT \sim 4 \times 10^{-3}$  (GPa/K) with  $G_0 \sim 37$  GPa at 0 K. Since the thermal expansion coefficient will drop at very low  $T$ , one obtains an upper bound for the fractional change in  $(G_{0T}/G_{0T_g})$  from 0 K to  $T_g$  as  $\Delta_{\max} = (T_g/G_0)(dG_{0T}/dT) \sim 0.07$ . Similarly, we estimate  $\Delta_{\max} \sim 0.11$  for Pd<sub>40</sub>Ni<sub>40</sub>P<sub>20</sub> [18]. The  $t$  dependence of  $(G_{0T}/G_{0T_g})$  gives a maximum correction to the second term in Eq. (8)

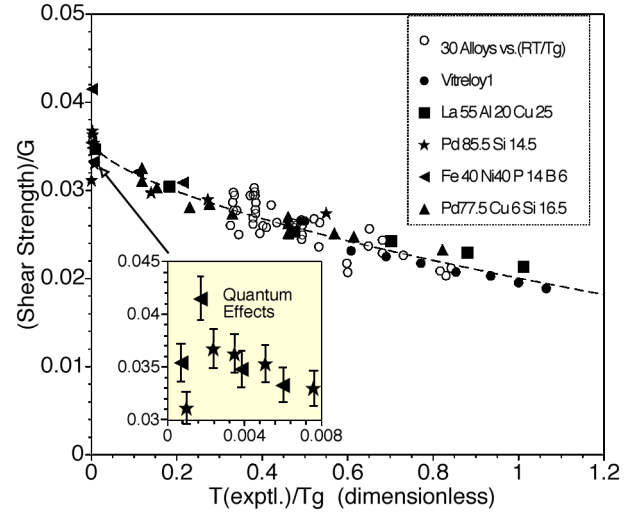


FIG. 2 (color online). Experimental shear strain at yielding ( $\tau_Y/G$ ) vs  $t = T/T_g$ . Small open circles show results at room temperature on 30 alloys of varying  $T_g$ . Solid symbols show the  $t$  dependence of  $\tau_Y/G$  for various individual alloys as indicated. The reader is referred to the text for references and details.

at  $t \sim 1$  (near  $T_g$ ) of order 5%–7%. The logarithmic term in Eq. (8) involves  $\omega_0$  and is estimated to be the frequency of shear phonon of nm wavelength ( $\sim 10^{13}$  Hz). With typical strain rates (in yielding experiments) of  $10^{-2}$ – $10^{-4}$  s<sup>-1</sup>, we have  $\ln(\omega_0/C\dot{\gamma}) \sim 30$ . An order of magnitude change in either  $\omega_0$  or  $\dot{\gamma}$  changes the logarithmic term by  $\sim 5\%$ . The dependence of  $\tau_{CT}$  on  $T$  is thus dominated by the  $t^{2/3}$  term.

The dotted curve in Fig. 2 was obtained using Eq. (8) (square bracket taken as a constant) to “fit” the dependence of  $\tau_{CT}$  at  $T_R$  (fixed  $T$ ) and varying  $T_g$  for the 30 metallic glasses (open circles) and the  $t$  dependence of individual alloys (filled symbols). This fit gives  $\tau_Y/G = \gamma_{C0} + \gamma_{C1}t^m$  where  $\gamma_{C0} = 0.036 \pm 0.002$ ,  $\gamma_{C1} = 0.016 \pm 0.002$ , and  $m = 0.62 \pm 0.2$ . Equation (8) explains both the  $T$  dependence of  $\tau_Y$  for all individual amorphous alloys and the “ $T_g$  dependence” for 30 glasses at fixed  $T_R$  with an exponent “ $m$ ” consistent (within error) with the predicted value “ $2/3$ .” The coefficients  $\gamma_{C0}$  and  $\gamma_{C1}$  are approximately universal constants. The present classical model is expected to break down at very low  $T$ . Low  $T$  data shown in the inset of Fig. 2 suggest “quantum effects” on yielding when shear phonon modes “freeze out” at very low  $T$ .

For the CSM, the elastic response of an STZ is nonlinear as  $\tau$  increases from 0 to  $\tau_C$ . The actual critical strain at  $\tau_C$  is not  $\tau_{CT}/G$  (as in experiments), but rather  $\pi\tau_{CT}/2G$  with the factor  $\pi/2$  arising from nonlinear elasticity. The role of nonlinear elasticity and nonaffine atomic displacements in the shear response of an STZ has been recently discussed [35,36]. In both the Frenkel model and simulations [35], the compliance at finite stress,  $G_\tau^{-1}$ , diverges at  $\tau_C$ . Experimentally, one measures yield stress, not strain,

so the experimental  $\gamma_C$  at yield underestimates the actual strain of the STZ. Further, the macroscopic material comprises a statistical distribution of STZ's with distributed values of  $G$  and  $\tau_C$  varying with location and orientation. Yielding is expected when a critical fraction of unstable STZ's results in global instability.

In conclusion, plastic yielding of metallic glasses at  $T_R$  is roughly described by an average elastic shear limit criterion,  $\tau_Y = \gamma_C G$ , where  $G$  is the shear modulus of the unstressed glass, and  $\gamma_C = 0.0267 \pm 0.0020$ . Closer analysis reveals that  $\gamma_C$  depends on  $t = T/T_g$ . A CSM inspired by Frenkel's work and recent molecular dynamics simulations is developed to explain these empirical findings. Yielding is treated as a fold catastrophe obeying a scaling law  $W(\tau)/[G_\tau(\delta\gamma)^2] = R$ . Applied on average, this scaling law leads to a " $t^{2/3}$  law,"  $\tau_{CT}/G = \gamma_{C0} - \gamma_{C1}(t)^{2/3}$ , for flow stress of metallic glasses where  $\gamma_{C0}$  and  $\gamma_{C1}$  are weakly material dependent. A fit to all experimental data yields  $\gamma_{C0} = (0.036 \pm 0.002)$ ,  $\gamma_{C1} = (0.016 \pm 0.002)$ , and an exponent  $m = 0.62 \pm 0.2$ . A similar derivation of the " $T^{2/3}$ " law has appeared in the literature on yielding in crystals [37]. Gaunt also derived a similar law for thermally activated domain wall motion in disordered magnets [38]. To the extent that plastic yielding in nonmetallic glasses (oxides, molecular glasses, etc.) is a fold catastrophe, one might expect Eq. (8) to be valid more generally although  $\gamma_{C0}$  may vary. The present CSM model may establish a basis for a broader understanding of glass physics.

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