Effects of Ion Motion in Intense Beam-Driven Plasma Wakefield Accelerators

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Recent proposals for using plasma wakefield accelerators (PWFA) as a component of a linear collider have included intense electron beams with densities many times in excess of the plasma density. The beam's electric fields expel the plasma electrons from the beam path to many beam radii in this regime. We analyze here the motion of plasma ions under the beam fields, and find for a proposed PWFA collider scenario that the ions completely collapse inside of the beam. Simulations of ion collapse are presented. Implications of ion motion on the feasibility of the PWFA-based colliders are discussed.

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The plasma wakefield accelerator (PWFA), driven in the blowout regime [1] where the beam is denser than the ambient plasma ($n_b > n_0$), has been the subject of much recent experimental and conceptual investigation [2–5] in the context of its application to a future high-energy linear collider [6,7] (LC). In this regime, the plasma electrons are ejected from the path of the intense driving electron beam, resulting in an electron-rarefied region. This region, containing only ions, possesses linear (in radius r) electrostatic focusing fields that allow high quality propagation of both driving and accelerating beams. In addition, this electron-rarefied region has superimposed upon it electromagnetic fields, which, because the phase velocity of the axisymmetric wake is nearly c, have longitudinal electric fields essentially independent of r. This wake may accelerate a trailing electron beam just as a traveling wave linear accelerator, with strong, linear transverse focusing conveniently supplied by the plasma ions.

The existence of linear focusing in both the PWFA and in the related final-focusing underdense plasma lens [8] is predicated on the assumption that the ions do not move, and thus maintain spatial uniformity. Without uniform ion density n_i, the strong ion-derived focusing fields will not give attractive beam transport characteristics. On the contrary, strong nonlinear (in r) or time-dependent (changing with longitudinal position in the beam, $\zeta = z - ct$) fields give certain degradation of the beam's transverse phase space density, which in the case of a LC must be of unprecedented quality. Thus the issue of ion motion is of critical importance in evaluating the viability of using the PWFA in a collider.

The condition $n_b > n_0$ is inherent in the blowout regime. Indeed, for many scenarios of current interest, the self-consistent driving beam density n_b, as well as that of the accelerating beam, exceeds n_0 by orders of magnitude. Under these circumstances, the beam's electric field is high enough to produce relativistic plasma electrons [9,10], resulting in their ejection to radii large compared to the rms transverse beam size σ_x. If the beam fields are large enough they may also induce the ions to move significantly during the beam passage. We will see below that this is indeed the case for the current proposal, known as an "afterburner," which uses a PWFA as a component in a LC. This proposal, which has been initiated by S. Lee, *et al.* [6], uses a single stage PWFA deployed at the end of a conventional high-energy linear accelerator. In this stage, a portion of the beam charge is used to drive the PWFA, allowing a trailing part of the beam to be doubled (or more) in energy before use in collisions. The afterburner idea was recently explored in the context of present LC complex designs by Raubenheimer [7]. For the parameters in Ref. [7], which are partially based on the afterburner scenario discussed in Ref. [6], our analysis will show violent collapse of the ions. This collapse, which we also illustrate in particle-in-cell (PIC) simulations, has serious implications for the preservation of the accelerating beam emittance, effectively negating the assumed advantage of linear transport in the blowout regime [1].

Most previous analysis of the PWFA has been carried out under the assumption of cylindrical symmetry in the beam, and thus in the plasma response. We begin our discussion on this familiar ground, assuming (consistent with Refs. [6,7]), that the drive beam is indeed axisymmetric. For the purposes of analysis, we invoke some well accepted approximations to give the forms of the electrostatic fields that serve to focus the beams and the ions. The first approximation is that the net transverse force on the beam arises only from the ions' electrostatic fields, which is a defining characteristic of the blowout regime [1]. The second is that the ions move predominately under the influence of the beam electrons' transverse electric field. As the moving ions remain nonrelativistic, they are negligibly affected by the beam-derived magnetic field. These approximations are useful in both 2D and 3D analysis of the beam-plasma-ion interaction, and allow us to estimate the degree to which the ions move due to intense beam fields in the PWFA.

We analyze the cases of the driving and accelerating beam in different ways. The driver, as it is not used directly in the LC experiment, may be taken as axisymmetric, as noted above. On the other hand, the accelerating beam must, because of the demands of the final focus and beam-beam interaction (e.g., beamstrahlung mitigation, crab-crossing), have asymmetric emittances and beam sizes. We make use of the known form of the self-electric field inside such beams [11], and concentrate on the relevant vertical ion motion. For the example PWFA parameters, we assume the values of n_0, the charge and bunch lengths for the driving and accelerating beams to be those quoted in Ref. [7] for the 1 TeV afterburner based on a superconducting LC. These parameters, given in Table I, are similar to those of Ref. [6], which contains analysis and simulations underpinning the physical PWFA model in [7]. While the afterburner design in [7] does not specifically address the emittance of the drive beam, for definiteness we take the transverse normalized emittance of the axisymmetric drive beam to be the geometric mean of the accelerating beam emittances, $\epsilon_n = \sqrt{\epsilon_{n,x}\epsilon_{n,y}}$. We have also assumed that the ionized species is hydrogen, to avoid multiple ionization state and uncontrolled plasma formation inside of the beam [12].

Examining first the axisymmetric beam case, we note the matched β function due to the ion focusing is [13]

$$\beta_{\rm eq} = \sqrt{\gamma/2\pi r_e n_0}. \tag{1}$$

The equilibrium transverse rms beam size σ_x associated with this scenario is $\sigma_{x,\rm eq} = \sqrt{\beta_{\rm eq}\epsilon_n/\gamma}$, where $\gamma \gg 1$ is the beam energy normalized to $m_e c^2$ (the beam is highly relativistic, with velocity $v \simeq c$), and r_e is the classical radius of the electron. In cases relevant to the afterburner, $n_b \gg n_0$, and blowout proceeds very quickly, giving plasma electron rarefaction over nearly the full longitudinal beam extent. We may thus take, in the absence of ion motion, $\sigma_x = \sigma_{x,\rm eq}$ over the whole beam.

Assuming a bi-Gaussian (in r and $\zeta = z - ct$) density distribution of N_b beam electrons, the peak, on-axis beam density is

$$n_{b,0} = \frac{N_b}{(2\pi)^{3/2}\sigma_x^2\sigma_z} = \frac{N_b}{2\pi\epsilon_{n,x}\sigma_z}\sqrt{r_e n_0 \gamma}, \tag{2}$$

where σ_z is the rms beam extent in ζ. With $n_b \gg n_0$, the beam-derived electric field is predominantly radial, and can be evaluated using Gauss' law. In the beam core ($r < \sigma_x$ and $\zeta < \sigma_z$, where $n_b \simeq n_{b,0}$) E_r is nearly linear in r and independent of ζ,

$$E_r \simeq -2\pi e n_{b,0} r = -\frac{eN_b}{\epsilon_{n,x}\sigma_z}\sqrt{r_e n_0 \gamma}\, r. \tag{3}$$

The movement of the nonrelativistic plasma ions is driven mainly by E_r, with approximate equation of motion derived from Eq. (3),

$$\frac{d^2 r}{dt^2} \simeq \frac{ZeE_r}{Am_a} = -\frac{Ze^2 N_b}{Am_a\epsilon_{n,x}\sigma_z}\sqrt{r_e n_0\gamma}\, r. \tag{4}$$

Here Z is the ion charge state, and A the atomic mass in amu. In terms of the distance measured in the beam Galilean frame ζ, Eq. (4) can be recast as

$$\frac{d^2 r}{d\zeta^2} = -\frac{Zr_a N_b}{A\epsilon_{n,x}\sigma_z}\sqrt{r_e n_0 \gamma}\, r = -k_i^2 r, \tag{5}$$

where $r_a = 1.55 \times 10^{-18}$ m is the classical radius of a singly charged ion of mass 1 amu.

Under the stated assumptions, Eq. (5) describes a driven, simple harmonic oscillator with spatial wave number $k_i = \sqrt{Zr_a N_b/A\epsilon_{n,x}\sigma_z}(r_e n_0\gamma)^{1/4}$. For static initial conditions its solution is $r = r_0 \cos k_i\zeta$, with r_0 the initial radial offset of the ion. To account for the variation of $n_{b,0}$ with ζ, we take the beam effective length as $\Delta\zeta = \sqrt{2\pi}\sigma_z$, and the total phase advance of the ion motion in the beam's field is

$$\Delta\phi \simeq k_i\Delta\zeta = \sqrt{\frac{2\pi Z r_a \sigma_z N_b}{A\epsilon_{n,x}}}(r_e n_0 \gamma)^{1/4}. \tag{6}$$

Upon insertion of drive beam parameters from Table I into Eq. (6), we obtain $\Delta\phi \simeq 6.45$. As total collapse of the ions, accompanied by a large near-axis spike in ion density n_i, occurs for $\Delta\phi = \pi/2$, this phase advance is an order of magnitude too large for the assumption of uniform unperturbed ion density to hold. This result could be anticipated by noting that the ratio k_i/k_p is $\sqrt{n_i m_e/2n_0 m_i}$. The existence of a beam with density $n_{b,0} = 1.5 \times 10^5 n_0$, as in our example, indicates that in a beam with $k_p\sigma_z \approx 1$—appropriate for a PWFA driver [1,6]—and $m_i/m_e \approx 1800$, large amounts of ion motion ($k_i\sigma_z \gg 1$) are expected. Possible ways to mitigate this problem are discussed below.

In order to illustrate the severity of ion collapse, as well as aspects of the nonlinearity in the ion motion, we show the results of axisymmetric PIC simulations performed with the code OOPIC [12,14]. The beam parameters in this calculation are the same as in Table I (drive beam). Figure 1 shows n_i in and near the beam region. It can be seen, as expected for the case of such a large $\Delta\phi$, that the ions indeed collapse very quickly. The ion density generally rises as the beam current grows, increasing by a factor of over 200 in the beam core. In this regime ($\Delta\phi \gg 1$) the

TABLE I. Beam and plasma parameters for linear collider afterburner, derived from Ref. [7].

N_b (drive, accelerating)	1.5×10^{10}, $.5 \times 10^{10}$
rms bunch length σ_z	35 μm
γ (drive, accelerating)	$\leq 1 \times 10^6$, $\leq 2 \times 10^6$
Accelerated beam $\varepsilon_{n,(x,y)}$	4×10^{-6}, 9.6×10^{-6} m rad
Drive beam $\varepsilon_{n,x}$	6.2×10^{-7} m rad
Initial ion (electron) density n_0	0.9×10^{16} cm^{-3}
Ion charge state Z	1 (hydrogen)
Matched β function $\beta_{\rm eq}$	3.1 cm
Normalized beam density n_b/n_0	1.5×10^5

ions are released from the beam potential by a combination of its time dependence, nonlinearity, and self-repulsion of the accumulating ions.

With such a large increase in n_i, the self-consistent beam size should be reduced further. This process would take a distance $z > \beta_{\rm eq}$ to establish, which is much longer than our simulation length—the time-step needed in our present calculation is 8 attoseconds. We have analyzed the establishment of new equilibria under the joint evolution of n_b and n_i in Ref. [15]; a computational approach to understanding this problem will be undertaken in the future. It should be noted, however, that there is violent transverse emittance growth associated with this process, due to both the ζ dependence and nonlinear r dependence of the ion-derived fields. Indeed, the growth rate observed in the simulation of Fig. 1 was disastrously high, $d\varepsilon_{n,x}/dz \simeq 6 \times 10^{-4}$ m rad/m, giving 100% growth in only 1 mm of propagation length.

We note that Ref. [6] deals with a case similar to that given in Table I, but with even higher charge. There the beam is also assumed round, with $\sigma_x = 25$ μm, whereas in our self-consistent example we have determined that $\sigma_x \approx 140$ nm. Thus our value of the ion focusing wave number k_i is over 100 times larger than that deduced from the assumed (not derived from a consistent set of beam parameters) case of Ref. [6]. Even with $\Delta\phi \cong 0.1$ deduced for the case of [6], the ion motion is not negligible, and occurs at a level that is also relevant to the accelerating beam. As the ions move further after drive beam passage, the ion perturbation due to the drive beam will be stronger yet inside the trailing beam.

The situation is more constrained for the accelerating beams, which have emittance and charge requirements set by the luminosity of the collider. For beams inside of a cylindrically symmetric ion channel that is preformed by the drive beam, one may assume that the equilibrium β_x and β_y are equal and given by Eq. (1). Thus for the case in Ref. [7], the beam sizes $\sigma_{x,y}$ are a factor of $R = 10$ different. Assuming the beam has elliptical symmetry, the transverse electric fields (E_x and E_y) are equal at the beam edges ($y = \sigma_y$, $x = \sigma_x \equiv R\sigma_y$) [11]. Thus the ion motion component contributing to the ion density perturbation is predominantly vertical.

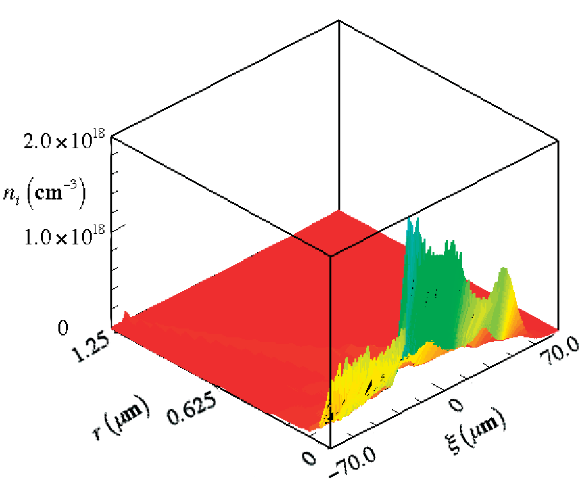

FIG. 1 (color). Surface plot of ion density distribution in (ζ, r), as simulated by OOPIC for drive beam conditions of Table I.

The vertical field inside of the beam core is, in the linear approximation,

$$E_y \approx -\frac{4\pi e n_{b,0}}{(1+R)} y = -\frac{2eN_b\sqrt{r_e n_0 \gamma}}{\varepsilon_{n,y}\sigma_z(1+R)} y \tag{7}$$

$$\approx -\frac{2eN_b}{\sigma_z}\sqrt{\frac{r_e n_0 \gamma}{\varepsilon_{n,y}\varepsilon_{n,x}}} y, \qquad R = \sqrt{\frac{\varepsilon_{n,x}}{\varepsilon_{n,y}}} \gg 1. \tag{8}$$

The linearized (for ions initially inside $y < \sigma_y$, $x < \sigma_x$) vertical equation of motion is

$$y'' = -\frac{2Zr_a N_b}{A\sigma_z}\sqrt{\frac{r_e n_0 \gamma}{\varepsilon_{n,y}\varepsilon_{n,x}}} y = -k_{i,y}^2 y. \tag{9}$$

The focusing strength (k_i^2) is a factor of 2 larger than in the round beam case, if we assume for comparison that the flat beam $\sqrt{\varepsilon_{n,y}\varepsilon_{n,x}}$ is equivalent to $\varepsilon_{n,x}$ in the round beam—this is equivalent to requiring that $n_{b,0}$ is the same in the round and flat beam cases. This factor arises in the flat beam because the field varies strongly only in the vertical dimension. Thus in the flat beam scenario the ion collapse is inherently faster, and even for the smaller N_b and higher energy ($\gamma \leq 2 \times 10^6$) accelerating beam in the Ref. [7], the maximum phase advance is

$$\Delta\phi_y \cong \sqrt{\frac{4\pi Z r_a N_b \sigma_z}{A}\sqrt{\frac{r_e n_0 \gamma}{\varepsilon_{n,y}\varepsilon_{n,x}}}} \approx 6.26. \tag{10}$$

This is again unacceptably large, and should be mitigated by over an order of magnitude in order to preserve the accelerating beam quality.

One may ask if it is possible to choose parameters that ameliorate the ion motion problem, in either the drive or the accelerating beam. The parameters n_0 and σ_z are not actually independent, as $\sqrt{n_0} \propto k_p \propto \sigma_z$; also γ is dictated by the collider design. Thus the only feasible approach would be to use smaller N_b and larger ε_n, as these effects reduce $\Delta\phi$, if only as a square-root. One may not give up N_b in the drive beam, however, without losing acceleration gradient. On the other hand, the drive beam ε_n may be made significantly larger, at the expense of ease in manipulating the beam; for example, if the emittance is too large, one may not easily compress the beam to shorter lengths. One must also then solve the problem of creating the large emittance driver in the presence of a low emittance trailing beam. The constraints of using the beam in the collider interaction point are much more serious for the accelerating beam, however. As one may not arbitrarily choose N_b, $\varepsilon_{n,x}$, or R in the trailing beam, it is not likely that the afterburner case discussed in Ref. [7] can be made feasible.

Thus there seem to be two options that may be pursued. The first is a complete redesign of the LC beam format to accommodate the ion motion problem, in which case the afterburner concept changes from a possible post-design feature to an inherent design constraint. This option is still extremely challenging, however, given the severity of the ion collapse scaling we have deduced here, and the constraints of collider luminosity. A more radical solution would be to eliminate the ions altogether, using a hollow plasma capillary. This has already been proposed in the context of accelerating positrons [6], where the transverse wake is defocusing when plasma is allowed in the beam channel.

Obviously, the loss of ions in the beam path precludes ion focusing of the electrons. It thus also presents an obstacle to implementing another compelling aspect of the afterburner proposal—the use of the plasma lens final focusing [8]. Two scenarios exist for plasma lens final focusing: one in which thin lenses are used, and another in which the beam is adiabatically focused [16] by a steadily increasing n_0 in z, to avert beam size limitations due to synchrotron radiation (the Oide limit [17]). In the first case, if one focuses directly after the PWFA, n_0 must be denser than assumed in the accelerating section. The denser plasma produces stronger focusing than in the upstream equilibrium, and thus yields beam demagnification. However, this scenario does not provide any mitigation of the ion collapse problem. One is forced to consider significant beam expansion before the plasma lens to avoid ion motion and concomitant aberrations.

In the adiabatic focusing case, the rise in plasma density occurs slowly, with n_b increasing as $n_0^{1/2}$. The final beam density thus significantly increases, in a scenario where any increase at all exacerbates an already unacceptable level of ion motion. To illustrate this situation, one may envision achieving minimum β_x and β_y at the collider interaction point of σ_z (limited by the "hour-glass" effect) through adiabatic plasma lensing. The associated beam in this case has $n_b = 1.6 \times 10^{24}$ cm^{-3}; k_i increases by nearly a factor of 10^4 over the accelerating beam case discussed above. This degree of ion motion negates the utility of adiabatic plasma focusing.

In conclusion, we have analyzed ion motion in likely scenarios where the PWFA is used as an afterburner accelerator in a future linear collider, and have found that the assumption of stationary ions which underpins the physics model of the scheme is strongly violated. The subsequent ion motion can produce extremely large perturbations in the ion density, giving rise to transverse fields that disrupt the beam motion. As the ion motion due to the accelerating beam itself is seen to be extremely large for currently conceived collider beam parameters, one must also examine the issue of ion motion in laser-driven plasma accelerators; in accelerator terminology, it is a generalized "transverse wake" problem.

Future work planned includes analysis of the emittance growth caused by nonuniform ion densities. This study requires more in-depth simulations, which we intend to extend to three-dimensions to examine ion motion in flat beams. Further investigations are also planned to search for collider design parameters which may be more compatible with ion motion than current afterburner-inspired schemes. Such parameters are far from those presently under consideration; use of smaller N_b and σ_z bunches seem most promising. It is clear in this regard that more attention should be paid to development of the hollow capillary version of the PWFA, which not only moots the ion motion issue in the electron beam case, but provides for stable acceleration of positrons as well [6]. Finally, we note that experimental tests of ion motion would be desirable. As such, recent PWFA experiments at Stanford [18] employ the parameters which indicate that $\Delta\phi \approx 0.3$ has been achieved.

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