Superweakly Interacting Massive Particle Solutions to Small Scale Structure Problems

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Collisionless, cold dark matter in the form of weakly interacting massive particles (WIMPs) is well motivated in particle physics, naturally yields the observed relic density, and successfully explains structure formation on large scales. On small scales, however, it predicts too much power, leading to cuspy halos, dense cores, and large numbers of subhalos, in apparent conflict with observations. We consider super-WIMP dark matter, produced with large velocity in late decays at times 10^5–10^8 s. As analyzed by Kaplinghat in a more general setting, we find that super-WIMPs have sufficiently large free-streaming lengths and low phase space densities to help resolve small scale structure problems while preserving all of the above-mentioned WIMP virtues.

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The microscopic identity of dark matter (DM) is one of the major puzzles in basic science today. In the current standard cosmological picture, the Universe contains nonbaryonic dark matter with abundance $\Omega_{\rm DM}h^2 = 0.095$–$0.129$ [1], where $\Omega_{\rm DM}$ is the energy density in units of the critical density, and $h \simeq 0.71$ is the reduced Hubble parameter. This component is typically assumed to be cold, collisionless, and non-self-interacting dark matter, which we refer to as CDM throughout this work. CDM is remarkably successful in explaining the observed large scale structure down to length scales of $\sim$1 Mpc.

Among the most well motivated CDM candidates are weakly interacting massive particles (WIMPs), with masses of the order of the weak scale $M_W \sim 100$ GeV and interaction cross sections $\sigma \sim g^2 M_W^{-2}$. WIMPs emerge naturally from several well motivated particle physics frameworks and include the lightest supersymmetric particle (LSP) in R-parity conserving supersymmetry models [2], the lightest Kaluza-Klein state in models with universal extra dimensions [3], and branons in brane-world models [4]. In addition, WIMPs naturally have thermal relic densities of the desired order of magnitude.

Despite its considerable successes, however, CDM appears to face difficulty in explaining the observed structure on length scales $\lesssim 1$ Mpc. Numerical simulations assuming CDM predict overdense cores in galactic halos [5], too many dwarf galaxies in the local group [6], and have trouble producing enough disk galaxies without angular momentum loss [5,7]. Although there is not currently consensus that the small scale problems of CDM are insurmountable [8], the number and variety of problems put considerable pressure on CDM and have motivated many alternative dark matter candidates. These include self-interacting dark matter [9], collisional dark matter [10], thermal warm dark matter (WDM) [11], annihilating dark matter [12], nonthermal WIMP production [13], and other proposals, such as the possibility of a broken scale invariance in the power spectrum [14].

A common feature of these new hypotheses is that they preserve the successes of standard CDM on large scales, but reduce power on small scales. Unfortunately, this virtue is achieved at a cost: in contrast to WIMPs, these candidates are generally not well motivated independently by particle physics, and their relic abundance is also not naturally in the correct range. For example, to explain the observed small scale structure, thermal WDM particles must have a mass greater than about 500 eV [15,16]. On the other hand, the observed relic density is naturally achieved for masses $\sim$10 eV. To resolve this discrepancy requires either an unreasonably large number ($\sim 10^3$) of light degrees of freedom at the time of decoupling or, alternatively, a nonstandard cosmology with a large injection of entropy at late times.

Here we consider superweakly interacting massive particle (super-WIMP) dark matter. In super-WIMP scenarios, a WIMP freezes out as usual, but then decays to a stable DM particle that interacts *superweakly* [17,18]. Examples of super-WIMPs include nonthermally produced weak-scale gravitinos [17–21], axinos [22], and quintessinos [23] in supersymmetry, and Kaluza-Klein graviton and axion states in models with universal extra dimensions [24]. Super-WIMPs preserve the virtues of WIMPs: they exist in the same well motivated frameworks and naturally have the right relic density, since they inherit it from late-decaying WIMPs. This latter property and the effect on small scale structure discussed here are absent for thermally produced gravitinos.

In contrast to WIMPs, super-WIMPs are produced with large velocities at late times. For example, gravitino or Kaluza-Klein graviton super-WIMPs are naturally produced at $\tau_X \sim M_{\rm Pl}^2/M_W^3 \sim 10^5$ s–10^8 s, where the reduced Planck mass $M_{\rm Pl} \equiv (8\pi G_N)^{-1/2} \simeq 2.4 \times 10^{18}$ GeV enters because these super-WIMPs interact only gravitationally. This has two effects. First, the velocity dispersion reduces the phase space density, smoothing out cusps in DM halos. Second, such particles damp the linear power spectrum, reducing power on small scales and improving consistency with structure formation. As we will show, these effects are sufficiently strong that super-WIMPs may provide a natural resolution to small scale structure problems. Similar conclusions have been reached in the more general setting explored in Ref. [25].

We first consider effects coming from the velocity dispersion. Such effects may be characterized by $Q \equiv \rho/\langle v^2\rangle^{3/2}$, the dark matter mass per unit volume of 6-dimensional phase space, where ρ is the mass density and $\langle v^2\rangle$ is the velocity dispersion [26]. Q has a number of important properties. The fine-grained value of Q remains constant for collisionless, dissipationless gases. It may be determined analytically in scenarios with decaying dark matter. In addition, the coarse-grained value of Q can only decrease, a property that follows from the relation of Q to thermodynamic entropy.

The coarse-grained value of Q can be estimated from rotation curves, gas emission, and gravitational lensing [26,27]. Galaxies with coarse-grained Q near $Q_0 \equiv 1.0 \times 10^{-27}\ \mathrm{GeV}^4 \simeq 1.2 \times 10^{-4}(M_\odot/\mathrm{pc}^3)/(\mathrm{km/s})^3$ have been observed [27]. Given the properties of Q noted above, this imposes a lower limit on fine-grained Q of $Q > Q_0$. At the same time, it has been argued [26,27] that values of Q close to Q_0 with, for example, $Q_0 \lesssim Q \lesssim 4Q_0$, are preferred, as they reduce the maximal cuspiness of galactic halos to that actually observed.

We now determine the fine-grained value of Q in super-WIMP scenarios. Throughout this work, we assume that super-WIMPs are produced between 10^3 and 10^{12} s so that the Universe is radiation dominated and the number of effective relativistic degrees of freedom is constant. We define a relativistic version of Q, $\tilde{Q} \equiv \rho/\langle u^2\rangle^{3/2}$, where $u = p/m$ is the three-momentum normalized by the particle's mass. Because u redshifts, $\tilde{Q}$ has the advantage that it is invariant given expansion in the Universe for both relativistic and nonrelativistic matter, but reduces to Q at late times when the matter becomes nonrelativistic.

We may estimate the value of Q at structure formation by determining the value of $\tilde{Q}$ when super-WIMPs are produced. For simplicity, we assume that all super-WIMPs are produced at the decay lifetime τ_X. The exponential distribution of production times gives $\mathcal{O}(1)$ corrections to the results described here and will be discussed in detail elsewhere [25,28]. We find

$$Q = \tilde{Q}(\tau_X) \simeq Q_0 u_X^{-3}\left[\frac{10^6\ \mathrm{s}}{\tau_X}\right]^{3/2}\left[\frac{\Omega_{\mathrm{SWIMP}}h^2}{0.11}\right], \quad (1)$$

where $u_X \equiv u(\tau_X)$. Note that for $u_X \sim 1$ and $\tau_X \sim 10^6$ s, natural values in the cases of gravitino and Kaluza-Klein graviton super-WIMPs, the phase space density is in the preferred range to eliminate cuspy halos.

We now turn to the effect on the power spectrum. Initially, the matter density has small inhomogeneities. These inhomogeneities evolve linearly at first, but eventually evolve nonlinearly to form the structure observed today. In the present universe, the scale of nonlinearity is expected to be around 30 Mpc. After this point, N-body simulations are necessary to analyze the evolution of the power spectrum. These analyses show that CDM predicts an excess of power on scales under 1 Mpc.

One way to reduce power on small scales and ameliorate this problem is through the free-streaming of DM. From its production time τ_X until matter-radiation equality at $t_{\mathrm{EQ}} \simeq 2.2 \times 10^{12}$ s, super-WIMPs can stream out of overdense regions into underdense regions, smoothing out inhomogeneities. A free-streaming scale much larger than 1 Mpc is excluded by observations of Lyman alpha clouds [29]. (For instance, for WDM, the constraints $m_{\mathrm{WDM}} \gtrsim 550$ eV [15] and $m_{\mathrm{WDM}} \gtrsim 750$ eV [16] correspond, given our definition of free-streaming scale, to $\lambda_{\mathrm{FS}} \lesssim 1.4$ Mpc and $\lambda_{\mathrm{FS}} \lesssim 1.0$ Mpc, respectively.) However, values close to this (roughly, $1.0\ \mathrm{Mpc} \gtrsim \lambda_{\mathrm{FS}} \gtrsim 0.4$ Mpc) could resolve the present disagreements.

The free-streaming scale for super-WIMP dark matter can be estimated to be

$$\lambda_{\mathrm{FS}} = \int_{\tau_X}^{t_{\mathrm{EQ}}} \frac{v(t)dt}{a(t)} = \lambda(t_{\mathrm{EQ}}) - \lambda(\tau_X), \quad (2)$$

where $a(t)$ is the cosmic scale factor, $v(t)$ is the super-WIMP velocity, and

$$\lambda(t) = 2R_{\mathrm{EQ}}u_{\mathrm{EQ}}\ln\left[\frac{1}{u(t)} + \sqrt{1 + \frac{1}{u^2(t)}}\right], \quad (3)$$

where $R_{\mathrm{EQ}} \equiv ct_{\mathrm{EQ}}/a(t_{\mathrm{EQ}}) \simeq 93$ Mpc and $u_{\mathrm{EQ}} \equiv u(t_{\mathrm{EQ}})$ [13,23]. In the common case that super-WIMPs are relativistic when produced ($u_X \gtrsim 1$) but nonrelativistic at t_{EQ} ($u_{\mathrm{EQ}} \ll 1$), Eq. (2) simplifies to

$$\lambda_{\mathrm{FS}} \approx \lambda(t_{\mathrm{EQ}}) \approx 1.0\ \mathrm{Mpc}\ u_X\left[\frac{\tau_X}{10^6\ \mathrm{s}}\right]^{1/2}\left\{1 + 0.14\ln\left[\left(\frac{10^6\ \mathrm{s}}{\tau_X}\right)^{1/2}\frac{1}{u_X}\right]\right\}, \quad (4)$$

demonstrating that production times of $\sim 10^6$ s naturally also provide the preferred order of magnitude for the free-streaming scale. Note that in this case, Q and λ_{FS} are both functions of $u_X\tau_X^{1/2}$, and so are correlated, as they are for WDM. However, in the super-WIMP scenario with $u_X \lesssim 1$, $\lambda_{\mathrm{FS}} < \lambda(t_{\mathrm{EQ}})$ and this degeneracy is broken, opening up new possibilities [28].

We now consider the particular case of gravitino super-WIMPs. The next-to-lightest supersymmetric particle (NLSP) may be either a scalar (e.g., a sneutrino or a slepton) or a fermion (a neutralino). If the NLSP is a sneutrino, gravitinos are produced at time [18]

$$\tau_{\tilde{\nu}} = 48\pi M_{\mathrm{Pl}}^2 \frac{m_{\tilde{G}}^2}{m_{\tilde{\nu}}^5}\left[1 - \frac{m_{\tilde{G}}^2}{m_{\tilde{\nu}}^2}\right]^{-4} \quad (5)$$

with $u_X = (m_{\tilde{\nu}}^2 - m_{\tilde{G}}^2)/(2m_{\tilde{\nu}}m_{\tilde{G}})$. These then determine Q and λ_{FS} through Eqs. (1)–(3); the results are shown in Fig. 1. Remarkably, preferred values of Q and λ_{FS} are simultaneously realized in super-WIMP scenarios with

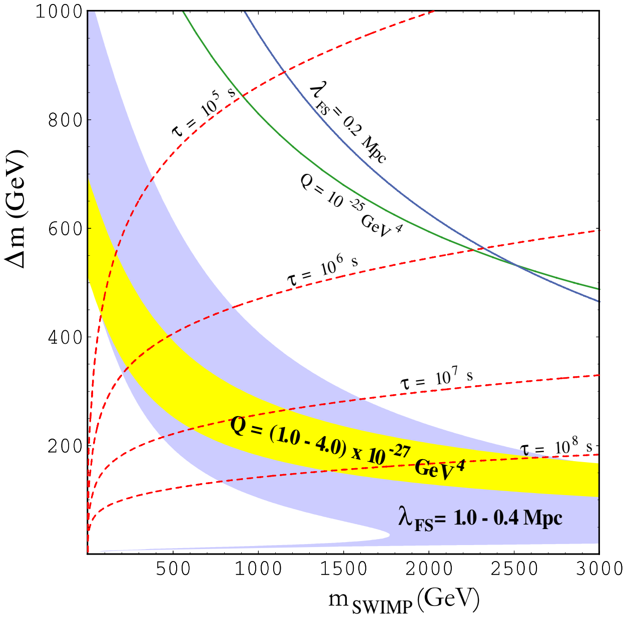

FIG. 1 (color online). Preferred regions (shaded) of phase space density Q and free-streaming length $\lambda_{\rm FS}$ in the $(m_{\rm SWIMP}, \Delta m)$ plane, where $\Delta m \equiv m_{\rm NLSP} - m_{\rm SWIMP}$, for gravitino super-WIMP with a sneutrino NLSP. The regions under both bands are disfavored. In the regions above both bands, super-WIMP dark matter becomes similar to CDM; representative values of Q and $\lambda_{\rm FS}$ are shown. Contours of typical lifetimes $\tau_{\tilde{\nu}}$ are also shown. We have assumed $\Omega_{\rm SWIMP} h^2 = 0.11$.

natural, weak-scale masses with $m_{\rm NLSP} \gtrsim 500$ GeV. The super-WIMP scenario is also constrained by big bang nucleosynthesis and the Planckian spectrum of the cosmic microwave background. These constraints [30] have been evaluated in several studies [17–21,31]. For a sneutrino NLSP, the main constraint comes from hadronic and electromagnetic energy produced in the three-body decays $\tilde{\nu} \to l W \tilde{G}, \nu Z \tilde{G}$. These modes have small branching ratios of the order of 10^{-3} and so the parameter space is not strongly constrained for $\tau_X \gtrsim 10^6$ s [20].

For a charged slepton NLSP, the lifetime is identical to that given in Eq. (5). However, charged sleptons are not collisionless and are electromagnetically coupled to the baryon-photon plasma. The opposite tendencies of pressure repulsion and gravitational attraction generate acoustic waves with density perturbation oscillations of photons, baryons, and sleptons. After gravitino production, however, the photon-baryon fluid is coupled only gravitationally to the neutral super-WIMP. Power is therefore reduced on scales that enter the horizon before this decoupling, and this effect can be more important than the free-streaming damping discussed above [28,32].

If the decaying WIMP is a neutralino, the super-WIMP production time is different. In general, the neutralino is a mixture of neutral B-ino, W-ino, and Higgsino states. For the specific case of a photino, the lifetime is [18]

$$\tau_{\tilde{\gamma}} = 48\pi M_{\rm Pl}^2 \frac{m_{\tilde{G}}^2}{m_{\tilde{\gamma}}^5}\left[1 - \frac{m_{\tilde{G}}^2}{m_{\tilde{\gamma}}^2}\right]^{-3}\left[1 + 3\frac{m_{\tilde{G}}^2}{m_{\tilde{\gamma}}^2}\right]^{-1}. \quad (6)$$

The resulting Q- and $\lambda_{\rm FS}$-preferred regions are shown in Fig. 2. As in the sneutrino NLSP case, preferred values for both quantities are obtained for natural weak-scale masses; if anything, the preferred super-WIMP and NLSP masses are lighter and more natural. For typical neutralinos, hadrons are produced in two-body decays, leading to extremely severe constraints from Big Bang nucleosynthesis. For photinos, however, the constraints are relaxed, since hadrons are produced only in three-body decays, and the super-WIMP resolution to small scale structure problems may be realized.

In conclusion, we have examined the implications of super-WIMP dark matter for small scale structure. Because super-WIMP dark matter is produced with large velocity in late decays, its phase space density is decreased, smoothing out halo cusps. At the same time, super-WIMPs damp the linear power spectrum, which lowers the concentration of dark matter in the cores of galactic halos. This effect may bring numerical simulations into agreement with observations of dwarf galaxies and reduce the excess of dwarf galaxies relative to CDM predictions.

The effects on small scale structure may also be achieved by WDM or dark matter with exotic interactions. In contrast to those possibilities, however, super-WIMP are automatically present in particle physics models with supersymmetry or extra dimensions and are naturally produced with the correct relic density. Dark matter produced in late decays will necessarily be warmer than CDM. It is remarkable, however, that for super-WIMP gravitinos,

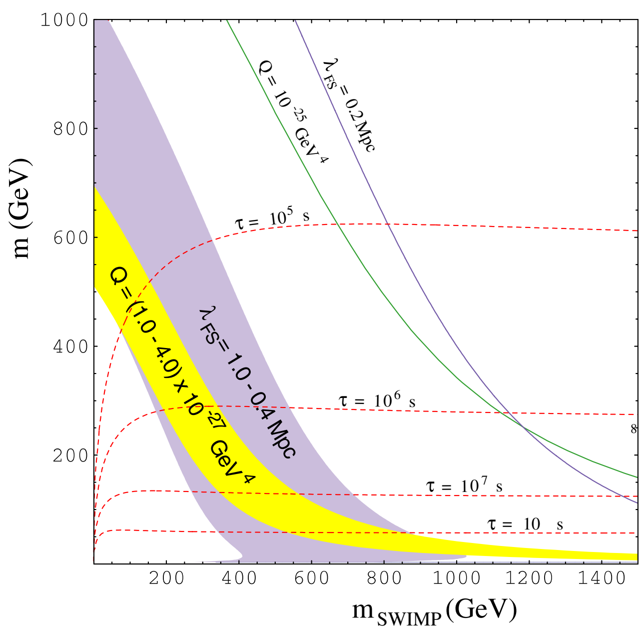

FIG. 2 (color online). Same as in Fig. 1, but for gravitino super-WIMP with a photino NLSP.

where the production times and velocities are determined by the fixed energy scales $M_{\rm EW}$ and $M_{\rm Pl}$, the predicted values of both Q and $\lambda_{\rm FS}$ are in favorable ranges to resolve outstanding problems without violating other constraints from cosmology and particle physics. Super-WIMP therefore appears to combine the most appealing features of both CDM and WDM. This explanation will be probed by future observations, especially those constraining the epoch of reionization [25,33]: reionization by redshift 6 implies $Q \gtrsim 0.1 \times Q_0$, compatible with the preferred values analyzed in this work, but confirmation of indications from WMAP of earlier reionization could restrict parameter space greatly. At the same time, given the virtues described here, it would be especially interesting to see if the promise of super-WIMP is realized by N-body simulations of structure formation and semianalytic analyses.

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