Astrophysical Neutrinos: Flavor Ratios Depend on Energy

Tamar Kashti and Eli Waxman

Physics Faculty, Weizmann Institute of Science, Rehovot 76100, Israel (Received 27 July 2005; published 26 October 2005)

Electromagnetic (and adiabatic) energy losses of π 's and μ 's modify the flavor ratio (measured at Earth) of neutrinos produced by π decay in astrophysical sources, $\Phi_{\nu_e}:\Phi_{\nu_{\mu}}:\Phi_{\nu_{\tau}}$, from 1:1:1 at low energy to 1:1.8:1.8 at high energy. The transition occurs over 1–2 decades of ν energy, and is correlated with a modification of the neutrino spectrum. For γ -ray bursts, e.g., the transition is expected at ~100 TeV and may be detected by km-scale ν telescopes. Measurements of the transition energy and energy width will provide unique probes of the physics of the sources. π and μ energy losses also affect the ratio of $\bar{\nu}_e$ flux to total ν flux, which may be measured at the W resonance (6.3 PeV): It is modified from 1/6 (1/15) at low energy to 1/9 (practically 0) at high energy for neutrinos produced in pp ($p\gamma$) interactions.

DOI: 10.1103/PhysRevLett.95.181101

PACS numbers: 96.40.Tv, 14.60.Pq, 98.70.Rz, 98.70.Sa

The existence of extra-Galactic high energy neutrino sources (for reviews see [1]) is implied by observations of ultrahigh energy cosmic rays with energies of $\epsilon >$ 10¹⁹ eV. The acceleration of particles in extra-Galactic sources is expected to produce a cosmic-ray spectrum extending over many decades of energy (although the observed spectrums is likely dominated by Galactic sources below $\sim 10^{19}$ eV), leading to the production of a wide energy spectrum of extra-Galactic high energy neutrinos. Possible sources of high energy neutrinos include γ -ray bursts (GRBs) and active galactic nuclei (AGNs). High energy (> 0.1 TeV) neutrino telescopes are being constructed in order to detect extra-Galactic neutrinos (for review see [2]). Their detection will allow one to identify the high energy cosmic-ray sources, and to probe their physics. It may also provide information on fundamental neutrino properties.

High energy neutrinos are believed to be produced in astrophysical sources mainly through the decay of charged pions, $\pi^+ \rightarrow \mu^+ + \nu_{\mu} \rightarrow e^+ + \nu_{\mu} + \nu_e + \bar{\nu}_{\mu}$ or $\pi^- \rightarrow \mu^- + \bar{\nu}_{\mu} \rightarrow e^- + \nu_{\mu} + \bar{\nu}_e + \bar{\nu}_{\mu}$, produced in interactions of high energy protons with "target" photons $(p\gamma)$ or nucleons (pp, pn) [3]. The ratio of the fluxes of neutrinos of different flavors is therefore expected to be, at the source, $\Phi^0_{\nu_e}: \Phi^0_{\nu_{\mu}}: \Phi^0_{\nu_{\tau}} = 1:2:0$ (Φ_{ν_l} stands for the combined flux of ν_l and $\bar{\nu}_l$). Neutrino oscillations then lead to an observed flux ratio on Earth of $\Phi_{\nu_e}: \Phi_{\nu_{\mu}}: \Phi_{\nu_{\tau}} = 1:1:1$ [5]. For neutrinos produced in pp (pn) collisions, where both π^+ 's and π^- 's are produced, the ratio of $\bar{\nu}_e$ flux to total ν flux is 1/6, while for neutrinos produced in $p\gamma$ collisions, where only π^+ 's are produced, the ratio is 1/15 [6,7].

Km-scale optical Cerenkov neutrino telescopes, such as IceCube, are expected to be capable of discriminating between neutrino flavors [8]. Moreover, although these detectors cannot distinguish between neutrinos and antineutrinos at most energies, they may allow one to determine the ratio of ν_e and $\bar{\nu}_e$ fluxes by detecting $\bar{\nu}_e$'s at the W resonance ($\bar{\nu}_e e^- \rightarrow W^- \rightarrow$ anything) around 6.3 PeV (see, e.g., [6] for discussion of resonant $\bar{\nu}_e$ detection in IceCube). It has been pointed out [5,9] that measurements of the flavor composition of astrophysical high energy neutrinos may enable one to probe new physics, by searching for deviations from the standard flavor ratio 1:1:1. Measurements of the ν_e to $\bar{\nu}_e$ flux ratio may allow one to probe the physics of the sources [6], by discriminating between the two primary modes of pion production, $p\gamma$ and pp collisions.

Although a 1:1:1 flavor ratio appears to be a robust prediction of models where neutrinos are produced by pion decay, we point out in this Letter that energy dependence of the flavor ratio is a generic feature of models of high energy astrophysical neutrino sources. Pions are typically produced in environments where they may suffer significant energy losses prior to decay, due to interaction with radiation and magnetic fields [4,10]. Since the pion lifetime is shorter than the muon's, at sufficiently high energy the probability for pion decay prior to significant energy loss is higher than the corresponding probability for muon decay. This leads to suppression at high energy of the relative contribution of muon decay to the neutrino flux. The flavor ratio is modified to 0:1:0 at the source, similar to that of atmospheric neutrinos at high energies where the muons do not decay (see, e.g., [11]), implying a 1:1.8:1.8 ratio on Earth [8].

The energy dependence of the neutrino flavor content provides a unique probe of the sources. On the other hand, it complicates attempts to study new physics based on measuring deviations from Φ_{ν_e} : $\Phi_{\nu_{\mu}} = 1:1$. Furthermore, if muon energy losses become important at ≤ 1 PeV, it would affect the flux of $\bar{\nu}_e$ near the W resonance, rendering the suggestion to probe neutrino mixing angles with neutrinos around the W resonance [12] impractical, and making the discrimination between $p\gamma$ and pp collisions more difficult (due to reduction of $\Phi_{\bar{\nu}_e}$: $\Phi_{\nu}^{\text{total}}$).

We first derive below approximate analytic expressions describing the energy dependence of the neutrino flux,

flavor ratio, and antiparticle content, for sources that produce pions with a power-law energy distribution [a differential number flux $\Phi_{\pi}(\epsilon_{\pi}) \propto \epsilon_{\pi}^{-k}$], and assuming the charged particle energy loss rate to be proportional to a power of the particle energy, $\dot{\epsilon} \propto -\epsilon^n$. Such energy dependence is expected for synchrotron and inverse-Compton emission (below the Klein-Nishina regime), in which case $\dot{\epsilon} \propto -\epsilon^2$, and for "adiabatic" energy loss (energy loss due to expansion of the plasma in which the particles are confined), in which case $\dot{\epsilon} \propto -\epsilon^1$. We then discuss several specific models of high energy neutrino sources, and the general implications of our results.

The energy dependence of the flavor ratio.—We consider neutrinos from astrophysical sources produced by the decay of charged pions. The decay of the pion and the subsequent decay of the muon lead to a flavor ratio of $\Phi^0_{\nu_e}: \Phi^0_{\nu_u}: \Phi^0_{\nu_\tau} = 1:2:0$. We note that in the production of a charged pion, the high energy proton may be converted to a high energy neutron (e.g., $p + \gamma \rightarrow n + \pi^+$). The neutron may escape the source and decay, producing an additional $\bar{\nu}_e$. However, in this decay the neutrino carries only a small fraction, $\sim 10^{-3}$, of the original proton energy, comparable to $(m_n - m_p)/m_n$, much lower than the energy of neutrinos produced by the pion decay. Since in most astrophysical sources the number of emitted neutrinos drops rapidly with energy, the contribution of neutron decay to the neutrino flux would generally be small and is therefore neglected here.

The flavor ratio $\Phi_{\nu_e}^0: \Phi_{\nu_{\mu}}^0: \Phi_{\nu_{\tau}}^0 = 1:2:0$ is modified at high energy, where the lifetime of the muons becomes comparable to, or smaller than, the time for significant electromagnetic (or adiabatic) energy loss. The particle lifetime, $\tau_{x,\text{decay}}$ where *x* stands for π^{\pm} or μ^{\pm} , is proportional to ϵ_x . Thus, the ratio of cooling time, $\tau_{x,\text{cool}} \equiv \epsilon_x/|\dot{\epsilon}_x|$, to lifetime is rapidly decreasing with energy, $\tau_{x,\text{cool}}/\tau_{x,\text{decay}} \propto \epsilon_x^{-n}$. We denote the energy at which $\tau_{x,\text{cool}} = \tau_{x,\text{decay}}$ by $\epsilon_{0,x}$. The $1/\epsilon_x^n$ dependence of the cooling and decay time ratio implies that $\epsilon_{0,\pi}/\epsilon_{0,\mu}$ is approximately given by $(\tau_{0,\pi}/\tau_{0,\mu})^{-1/n} \sim 10^{2/n}$, where $\tau_{0,\pi} = 2.6 \times 10^{-8}$ s and $\tau_{0,\mu} = 2.2 \times 10^{-6}$ s are the pion and muon rest-frame lifetimes.

We consider below neutrinos produced by the decay of π^+ 's. The results for neutrinos from π^- decay are simply given by replacing each particle with its antiparticle. For pion decay, each of the four final leptons carry approximately 1/4 of the pion energy. The (differential number) flux of ν_{μ} 's of energy ϵ_{ν} produced by π^+ decay is therefore approximately given by

$$\Phi^{0}_{\nu_{\mu}}(\boldsymbol{\epsilon}_{\nu}) = -\partial_{\boldsymbol{\epsilon}_{\nu}} \int_{4\boldsymbol{\epsilon}_{\nu}}^{\infty} d\boldsymbol{\epsilon}_{i} \Phi_{\pi}(\boldsymbol{\epsilon}_{i}) P_{\pi}(\boldsymbol{\epsilon}_{i}, 4\boldsymbol{\epsilon}_{\nu}).$$
(1)

Here, $\Phi_{\pi}(\epsilon_i)$ is number flux (per unit energy) of pions produced by the source with energy ϵ_i , and $P_{\pi}(\epsilon_i, 4\epsilon_{\nu})$ is the probability that a pion produced with energy ϵ_i would decay before its energy drops below $4\epsilon_{\nu}$.

For $\dot{\epsilon} \propto \epsilon^n$ with n > 0, $P(\epsilon_i, \epsilon_f) = 1 - \exp[-\epsilon_0^n(\epsilon_f^{-n} - \epsilon_i^{-n})/n]$. Assuming $\Phi_{\pi}(\epsilon_i) \propto \epsilon_i^{-k}$ with k > 1, we find that the neutrino flux is suppressed due to energy loss by a factor

$$\frac{\Phi_{\nu_{\mu}}^{0}(\epsilon_{\nu})}{\Phi_{0}^{(\text{no loss})}} = s(-s)^{1-k/n} e^{-s} \gamma \left(\frac{k-1}{n}, -s\right).$$
(2)

Here $s \equiv (\epsilon_{0,\pi}/4\epsilon_{\nu})^n/n$, $\gamma(a, z)$ is the lower incomplete gamma function, and $\Phi_0^{(no \log s)}$ is the flux that would have been obtained had energy losses been negligible.

Similarly, for ν 's produced by the decay of μ^+ we have

$$\Phi^{0}_{\bar{\nu}_{\mu}(\nu_{e})}(\boldsymbol{\epsilon}_{\nu}) = \partial_{\boldsymbol{\epsilon}_{\nu}} \int_{3\boldsymbol{\epsilon}_{\nu}}^{\infty} d\boldsymbol{\epsilon}_{\mu} \\ \times \int_{\frac{4}{3}\boldsymbol{\epsilon}_{\mu}}^{\infty} d\boldsymbol{\epsilon}_{i} \Phi_{\pi} P_{\mu}(\boldsymbol{\epsilon}_{\mu}, 3\boldsymbol{\epsilon}_{\nu}) \partial_{\boldsymbol{\epsilon}_{\mu}} P_{\pi}(\boldsymbol{\epsilon}_{i}, 4\boldsymbol{\epsilon}_{\mu}/3),$$
(3)

and

$$\frac{\Phi_{\bar{p}_{\mu}(\nu_{e})}^{0}(\boldsymbol{\epsilon}_{\nu})}{\Phi_{0}^{(\text{no loss})}} = \frac{q^{n}}{1-q^{n}}(-s)^{1-k+n/n} \left[e^{-s} \gamma \left(\frac{k-1}{n}, -s\right) - e^{-q^{n}s} q^{1-k} \gamma \left(\frac{k-1}{n}, -q^{n}s\right) \right],$$
(4)

where $q \equiv 4\epsilon_{0,\mu}/3\epsilon_{0,\pi}$.

The flavor content given in Eqs. (2) and (4) is modified as the ν 's propagate from the source to Earth. For propagation over cosmological distances d, $d \gg \hbar c \epsilon / \Delta m^2 c^4$, and the observed fluxes $\Phi_{\nu_{\alpha}}$ ($\alpha = e, \mu, \tau$) are related to the production fluxes, $\Phi_{\nu_{\alpha}}^0$, by (see, e.g., [11])

$$\Phi_{\nu_{\alpha}} = \sum_{\beta} P_{\alpha\beta} \Phi^{0}_{\nu_{\beta}} = \sum_{\beta} \sum_{i} |U_{\alpha i}|^{2} |U_{\beta i}|^{2} \Phi^{0}_{\nu_{\beta}}.$$
 (5)

Here, $U_{\alpha i}$ is the neutrino mixing matrix, $|\nu_{\alpha}\rangle = \sum_{i=1}^{3} U_{\alpha i}^{*} |\nu_{i}\rangle$ where ν_{i} (i = 1, 2, 3) are the mass eigenstates.

 $U_{\alpha i}$ can be written as a function of three mixing angles $\theta_{12}, \theta_{23}, \theta_{13}$ and a Dirac phase δ . The best fit for $\delta = 0$ [13] is $\theta_{12} = 34^{\circ} \pm 2.5^{\circ}, \theta_{23} = 45^{\circ} \pm 6^{\circ}, \theta_{13} = 0^{\circ} \pm 8^{\circ}$ (90% confidence level). The observed neutrino flux ratio is $\Phi_{\nu_e}: \Phi_{\nu_\mu}: \Phi_{\nu_\tau} = 1:1.00 \pm 0.15:1.00 \pm 0.15$ for initial flux ratio $\Phi_{\nu_e}^0: \Phi_{\nu_\mu}^0: \Phi_{\nu_\tau}^0 = 1:2:0$, and $\Phi_{\nu_e}: \Phi_{\nu_\mu}: \Phi_{\nu_\tau} = 1:1.8^{\pm}0.70:1.8 \pm 0.47$ for $\Phi_{\nu_e}^0: \Phi_{\nu_\mu}^0: \Phi_{\nu_\tau}^0 = 0:1:0$ (90% confidence level). Here, we have obtained an approximate estimate of the error bars by assuming the errors in different mixing angles are not correlated, i.e., $\Delta(\Phi_{\nu_\alpha}/\Phi_{\nu_e}) = \{\sum_{ij} \Delta \theta_{ij}^2 [\partial_{\theta_{ij}}(\Phi_{\nu_\alpha}/\Phi_{\nu_e})]^2\}^{1/2}$.

The results of applying Eq. (5) to the initial flux ratios given by Eqs. (2) and (4), are shown in Figs. 1(a)–1(c), for the case of $\Phi_{\pi} \propto 1/\epsilon_{\pi}^2$ and $\dot{\epsilon}_x \propto \epsilon_x^2$. As expected, the muon



FIG. 1 (color online). Flavor and antiparticle content of the flux of astrophysical neutrinos produced by pion decay, for $\Phi_{\pi} \propto 1/\epsilon_{\pi}^2$ and $\dot{\epsilon}_x \propto \epsilon_x^2$. (a),(b) The energy fluxes in different flavors, $\epsilon_{\nu_l}^2 \Phi_{\nu_l}$ (normalized to $\epsilon_{\nu_e}^2 \Phi_{\nu_e}$ at low energy). Φ_{ν_l} stands for the combined flux of ν_l and $\bar{\nu}_l$, and these plots are therefore valid for neutrinos produced by any combination of π^+ and π^- decay. (c) The ratio between $\Phi_{\nu_{\mu}(\nu_{\tau})}$ and Φ_{ν_e} (solid line), with 90% C.L. lines of ν_{μ} (dashed lines) and ν_{τ} (dotted lines) fluxes. (d) The ratio of $\bar{\nu}_e$ to total ν flux on Earth, solid (dashed) line for neutrinos produced by $p\gamma$ (*pp*) interactions.

(pion) decay neutrino flux is suppressed above $\epsilon_{\nu} \sim \epsilon_{0,\mu}/3$ ($\epsilon_{\nu} \sim \epsilon_{0,\pi}/4$) by a factor $\propto \epsilon_{\nu}^{-n} = \epsilon_{\nu}^{-2}$, reflecting the fact that at high energy the probability for decay prior to significant energy loss is $\propto \epsilon^{-n} = \epsilon_{\nu}^{-2}$, and the flavor ratio transition takes place over an energy range of $\sim \epsilon_{0,\pi}/\epsilon_{0,\mu} \sim (\tau_{0,\pi}/\tau_{0,\mu})^{-1/n} \sim 10^{2/n} \sim 10$.

The energy at which a flavor ratio transition takes place (from 1:1:1 to 1:1.8:1.8), and the width of the transition, provide unique handles on the properties of the source. $\epsilon_{0,\mu}$ may constrain, e.g., the (radiation and magnetic field) energy density at the source, and the width of the transition may discriminate between adiabatic and electromagnetic energy losses. It should be kept in mind, however, that while the flavor ratio transition is expected to be a generic feature of high energy neutrinos produced in astrophysical sources, the behavior in the transition region may be more complicated that described by Eqs. (2) and (4), which were obtained under idealized assumptions. For example, neutrinos may be emitted from different regions (in a single source) with different values of $\epsilon_{0,\mu}$ (thus "smearing" the transition), and suppression of the inverse-Compton scattering cross section in the Klein-Nishina regime may lead to deviations from a simple power-law energy dependence of the energy loss rate.

Neutrinos from specific sources.—GRBs are possible sources of high energy neutrinos. The γ rays are believed to be produced by the dissipation of the kinetic energy of a highly relativistic wind. Neutrinos that are expected to be produced in the same region where the γ rays are produced have characteristic energies ≥ 100 TeV [4]. The pions and muons cool by synchrotron radiation, $\dot{\epsilon}_x \propto \epsilon_x^2$, and the muon cooling energy in the γ -ray production region is [10,14]

$$\epsilon_{0,\mu} \approx 10^3 \frac{\Gamma_{2.5}^4 \Delta t_{-3}}{L_{53}^{1/2}} \text{ TeV.}$$
 (6)

Here $\Gamma = 10^{2.5}\Gamma_{2.5}$ is the wind Lorentz factor, $L = 10^{53}L_{53}$ erg/s is the kinetic energy luminosity of the wind (assuming spherically symmetric wind), and $\Delta t = 10^{-3}\Delta t_{-3}$ s is the observed variability time scale of the γ -ray signal.

If GRBs are associated, as commonly believed, with collapses of massive stars, neutrinos of lower energy, $\epsilon_{\nu} \ge 1$ TeV, may be emitted in the early stage of GRB evolu-

tion, when the relativistic wind penetrates the progenitor star [15]. The high energy density of radiation implies, in this scenario, $\epsilon_{0,\mu} < 1$ TeV due to inverse-Compton losses, and the Klein-Nishina suppression of inverse-Compton losses at high energy implies $\epsilon_{0,\pi} \gg 1$ TeV. In this case, an observed flavor ratio of 1:1.8:1.8 would therefore be expected at all energies (>1 TeV).

Measurements of the energy dependence of the neutrino flavor ratio would therefore provide constraints on the physical parameters of the source, and may allow one to discriminate between different scenarios for neutrino production in GRBs.

Equation (6) holds also for neutrinos produced in blazar AGN jets [10]. For the parameters characterizing these objects, $\Gamma \sim 10$, $L \sim 10^{47}$ erg/s, and $\Delta t \sim 10^4$ s, we have $\epsilon_{0,\mu} \sim 4 \times 10^6$ TeV, implying a flavor transition at $\sim 10^6$ TeV. When the dominant cooling process is adiabatic cooling, a similar cooling energy is obtained. However, at these energies, the number of neutrinos detected from AGNs may be too small to allow detection of the transition.

Implications for the flux of $\bar{\nu}_e$.—The fraction of $\bar{\nu}_e$ flux out of the total flux decreases at high energy due to muon cooling. For neutrinos produced in pp interactions, where π^+ 's and π^- 's are produced at roughly equal numbers, a flavor ratio of 1:1:1 is obtained at low energy for both ν_{α} and $\bar{\nu}_{\alpha}$, implying $\Phi_{\bar{\nu}_e}$: $\Phi_{\nu}^{\text{total}} = 1:6$. At high energy, the flavor ratio changes to 1:1.8:1.8, implying $\Phi_{\bar{\nu}_e}$: $\Phi_{\nu}^{\text{total}} =$ 1:9. For neutrinos produced in $p\gamma$ interactions, where only π^+ 's are created, $\Phi_{\bar{\nu}_e}$: $\Phi_{\nu}^{\text{total}} = 1:15$ at low energy. At high energy, the muon energy losses suppress the production of antineutrinos, resulting in a strong suppression of $\Phi_{\bar{\nu}_e}/\Phi_{\nu}^{\text{total}}$. This is illustrated in Fig. 1(d), where the energy dependence of the $\bar{\nu}_e$ to ν_e flux ratio is plotted for $\Phi_{\pi} \propto 1/\epsilon_{\pi}^2$ and $\dot{\epsilon}_x \propto \epsilon_x^2$.

If the transition is below 1 PeV, as expected for GRBs, it will reduce the flux of $\bar{\nu}_e$ near the W resonance, making the detection of $\bar{\nu}_e$, and hence the discrimination between $p\gamma$ and pp collisions, more difficult. It should be pointed out in this context, that significant production of π^- 's may occur in sources where pions are produced only through interactions of nucleons with photons, if the photoproduction optical depth is large enough, so that the neutron produced in a $p\gamma \rightarrow n + \pi^+$ interaction is likely to interact with a photon before escaping the source $(n\gamma \rightarrow p + \pi^-)$. Thus, a discrimination between photoproduction and inelastic nuclear collision sources based on the observed $\Phi_{\bar{\nu}_e}/\Phi_{\nu}^{total}$ ratio may not be straightforward.

Conclusions.—We have shown that a flavor ratio transition, from $\Phi_{\nu_e}: \Phi_{\nu_{\mu}}: \Phi_{\nu_{\tau}} = 1:1:1$ at low energy to 1:1.8:1.8 at high energy, is expected to be a generic feature of high energy neutrinos produced in astrophysical

sources. The location and energy width of the transition provide unique handles on the properties of the source, and may allow one to discriminate between different scenarios for neutrino production. The modified flavor ratio affects the experimental upper limits on the total neutrino flux, which are commonly obtained assuming a 1:1:1 ratio (e.g., [7]). It also changes the ratio of $\bar{\nu}_e$ to total ν flux from 1/6 (1/15) at low energy to 1/9 (practically 0) at high energy for neutrinos produced in pp ($p\gamma$) interactions.

We have neglected in the current analysis the possibility of matter oscillations, which are typically unimportant for the high energy sources under consideration (e.g., [4]). However, it should be kept in mind that matter oscillations may, in general, play a role in modifying the predicted flavor ratio. For example, the flavor ratio of neutrinos produced by a GRB jet buried in a blue supergiant would be modified by Mikheyev-Smirnov-Wolfenstein resonance to 1:1.5:1.5 outside the source and 1.2:1.4:1.4 on Earth.

We thank C. Peña-Garay, Y. Nir, and B. Katz for helpful discussions. This research was supported in part by ISF and Minerva grants.

- T. K. Gaisser, Nucl. Phys. B, Proc. Suppl. 117, 318 (2003);
 E. Waxman, in *Proceedings of the Nobel Symposium 129: Neutrino Physics* (World Scientific, Enköping, Sweden, 2004).
- [2] F. Halzen, in *Proceedings of the Nobel Symposium 129: Neutrino Physics* (Ref. [1]).
- [3] ν_{τ} 's may also be produced, by production of charmed mesons, but the higher energy threshold and lower cross section for charmed meson production typically imply a negligible ν_{τ} fraction, e.g., [4].
- [4] E. Waxman and J. N. Bahcall, Phys. Rev. Lett. 78, 2292 (1997).
- [5] J. G. Learned and S. Pakvasa, Astropart. Phys. 3, 267 (1995).
- [6] L.A. Anchordoqui, H. Goldberg, F. Halzen, and T.J. Weiler, Phys. Lett. B 621, 18 (2005).
- [7] M. Ackermann et al., Astropart. Phys. 22, 127 (2004).
- [8] J. F. Beacom, N. F. Bell, D. Hooper, S. Pakvasa, and T. J. Weiler, Phys. Rev. D 68, 093005 (2003).
- [9] H. Athar, M. Jezabek, and O. Yasuda, Phys. Rev. D 62, 103007 (2000).
- [10] J. P. Rachen and P. Meszaros, Phys. Rev. D 58, 123005 (1998).
- [11] M. C. Gonzalez-Garcia and Y. Nir, Rev. Mod. Phys. 75, 345 (2003).
- [12] P. Bhattacharjee and N. Gupta, hep-ph/0501191.
- [13] A. Strumia and F. Vissani, Nucl. Phys. B726, 294 (2005).
- [14] E. Waxman and J. N. Bahcall, Phys. Rev. D 59, 023002 (1999).
- [15] P. Meszaros and E. Waxman, Phys. Rev. Lett. 87, 171102 (2001).