

Nonequilibrium Relaxation of an Elastic String in a Random Potential

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We study the nonequilibrium motion of an elastic string in a two dimensional pinning landscape using Langevin dynamics simulations. The relaxation of a line, initially flat, is characterized by a growing length $L(t)$ separating the equilibrated short length scales from the flat long distance geometry that keeps a memory of the initial condition. We show that, in the long time limit, $L(t)$ has a nonalgebraic growth with a universal distribution function. The distribution function of waiting times is also calculated, and related to the previous distribution. The barrier distribution is narrow enough to justify arguments based on scaling of the typical barrier.

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The physics of disordered elastic systems has been the focus of intense activities both on the theoretical and experimental sides. Indeed it is relevant in a large number of experimental situations ranging from periodic systems such as vortex lattices [1], charge density waves [2], and Wigner crystals [3] to domain walls in magnetic [4–7] or ferroelectric [8,9] systems, contact lines [10], and fluid invasion in porous media [11]. Because of the competition between disorder and elasticity, glassy properties arise, and one of the most challenging question is to understand their consequences on the dynamics of the system [12].

Since the system moves by thermal activation over the barriers separating metastable states, the steady state response to a small external force is a way to probe its glassy nature. The glassiness leads to divergent barriers and thus to a slow response known as creep [13,14]. Experiments [4,6,8,9] as well as microscopic calculations of the response [15,16] have confirmed this creep behavior, although questions remain in low dimensions about the value of the creep exponent [17]. Much less is known about the glassy effects in the nonstationary relaxation towards equilibrium. Understanding such nonstationary physics is clearly crucial since it gives complementary information on the barriers and, for experiments, is needed to describe the many systems that are quenched in the glassy state (e.g., by changing rapidly the temperature), and then have to relax. Theoretical attempts to tackle this problem have been made using mean field and renormalization group approaches [18–20]. Direct application of these results to one dimensional domain walls is, however, difficult. Numerical studies, that would give more direct information in low dimension, are also difficult since they have to deal with ultra long time scales dynamics. Simulations have thus been mostly restricted so far to 2-dimensional random Ising models or 2-dimensional periodic elastic systems [21–23]. The relaxation of a directed polymer has been investigated [24–26] by local Monte Carlo dynamics [27], but a precise study of the connection between relaxation and the static glassy properties is still lacking. In this Letter we thus study the slow nonequilibrium relaxation of an

elastic string moving in a two dimensional random media. We prepare the string in a flat configuration and let it relax. Such a protocol is simple, experimentally accessible [4,6,7,9], and exhibits the generic features of the out of equilibrium glassy dynamics. We show that the relaxation is governed by a characteristic growing length, $L(t)$, separating the equilibrated short length scales from the flat long distance ones that keep a memory of the initial condition. In the long time limit, $L(t)$ has a nonalgebraic growth with a universal distribution function. We compute the distribution of waiting times and thus of barriers. This latter distribution is found to be narrow enough to justify the scaling for $L(t)$ based on a typical barrier.

We consider a string described by a single valued function $u(z, t)$, measuring its transverse displacement u from the z axis at time t . The initial condition is flat $u(z, t = 0) = 0$, and we monitor the relaxation towards equilibrium. The string obeys the equation of motion:

$$\gamma \partial_t u(z, t) = c \partial_z^2 u(z, t) + F_p(u, z) + \eta(z, t) \quad (1)$$

where γ is the friction coefficient and c the elastic constant. The pinning force $F_p(u, z) = -\partial_u U(u, z)$ derives from the random bond disorder potential $U(u, z)$ and the thermal noise $\eta(z, t)$ satisfies $\langle \eta(z, t) \rangle = 0$ and $\langle \eta(z, t) \times \eta(z', t') \rangle = 2\gamma T \delta(t - t') \delta(z - z')$ where $\langle \dots \rangle$ is the thermal average. The sample to sample fluctuations of the random potential are given by $\overline{[U(u, z) - U(u', z')]^2} = -2\delta(z - z') R^2(u - u')$ where the overline denotes an average over disorder realizations. In the random bond case the correlator $R(u)$ is short ranged.

To solve (1) numerically we use the method of Ref. [17]. We discretize the string along the z direction, $z \rightarrow j = 0, \dots, L - 1$, keeping $u_j(t)$ as a continuous variable. A second order stochastic Runge-Kutta method is used to integrate the resulting equation. To model a continuous random potential we generate, for each j , a cubic spline $U(u_j, j)$ passing through regularly spaced uncorrelated Gaussian random points [17,28]. To characterize the geometry of the line during the relaxation we introduce the

structure factor $S(q, t) \equiv \overline{s(q, t)} = \frac{1}{L} \overline{\langle u_q^* u_q \rangle}$ where $u_q = \sum_{j=0}^{L-1} u_j(t) e^{-iqj}$ and $q = 2\pi n/L$ with $n = 1, \dots, L-1$.

In the absence of disorder the relaxation of the string can be solved analytically and $S(q, t)$ is

$$S_{\text{pure}}(q, t) = S_{\text{pure}}^{\text{eq}}(q) [1 - \exp(-2cq^2 t/\gamma)] \quad (2)$$

where $S_{\text{pure}}^{\text{eq}}(q) = T/cq^2$ is the structure factor at equilibrium. From (2) we can separate two regimes: (i) At large q the line is equilibrated with the thermal bath and its geometry is described by the equilibrium roughness exponent $\zeta = 1/2$. This behavior can be extracted from the $q^{-(1+2\zeta)}$ power-law decay of the structure factor. (ii) At small q , however, the string still has a memory of the flat initial condition, and the structure factor reaches a plateau: $S_{\text{pure}}(q \rightarrow 0^+, t) = (2T/\gamma)t$. The crossover between these two regimes is driven by a unique growing characteristic length scale $L(t)$ that can be defined from the intersection point of the two limiting behaviors. In the pure case $L_{\text{pure}}(t) = 2\pi\sqrt{2ct/\gamma}$ and its power-law growth defines the dynamical exponent z , as $L(t) \sim t^{1/z}$.

We now discuss our numerical results for the system with disorder. We simulate lines of size $L = 256, 512, 1024$ with $c = \gamma = 1$. We take $R(0) = 1$ and temperatures ranging from $T = 0.1$ to $T = 0.7$. In Fig. 1(a) we show the typical relaxation of a string. Note that in the pure case, for the same parameters, the equilibration of a line of size $L = 256$ occurs after a time $t \sim 10^3$ (see Fig. 2). The presence of barriers in the disordered case makes the dynamics much more slow, and equilibrium is not yet reached at time $t = 10^6$. We show in Fig. 1(b) the evolution of $S(q, t)$. As in the pure case two regimes are observed. At short length scales the line has reached equi-

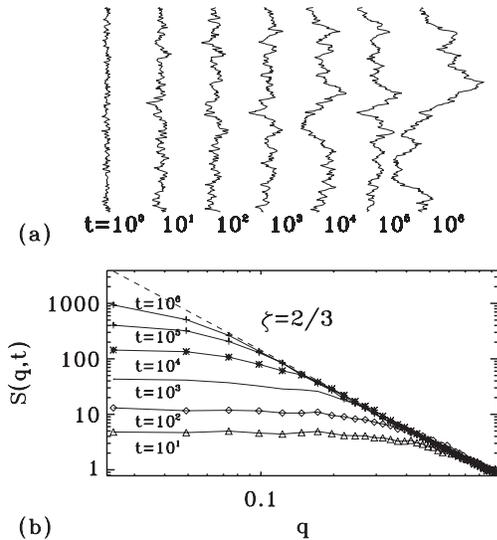


FIG. 1. (a) Typical configurations of the string for different times, at $T = 0.5$. (b) Structure factor for different times at $T = 0.5$, averaged over 1000 disorder realizations. The dashed line corresponds to the thermal equilibrium solution which it is reached at very long times.

librium in the random environment and it is characterized by the well-known roughness exponent $\zeta = 2/3$ [29]. At large length scales a plateau is still present and a crossover growing length $L(t)$ can be defined. Quite generally the scaling form of $S(q, t)$ can be written as

$$S(q, t) = S^{\text{eq}}(q)G(qL(t)) \quad (3)$$

where $G(x \rightarrow 0) \sim x^{1+2\zeta}$ and $G(x \rightarrow \infty) = 1$. The analytical calculation of $L(t)$ is clearly a nontrivial task, but a simple estimate can be done relying on phenomenological scaling arguments, based on creep. At low temperatures the relaxation is dominated by the energy barriers $U(L)$ that must be overcome in order to equilibrate the system up to a length scale L . Using the Arrhenius thermal activation law we can thus express the relaxation time $t(L) \sim \exp[\beta U(L)]$. Even if the exact numerical determination of $U(L)$ is an NP-complete problem it is usually conjectured that the typical barriers of the energy landscape scale, asymptotically with L , the same way as the free energy fluctuations: $U(L) \sim L^\theta$, with $\theta = 1/3$ for a line. Numerical calculations [30] and functional renormalization group calculations [15] seem to confirm this conjecture. Following these arguments we infer that [31]

$$L(t) \approx L_c \left[\frac{T}{U_c} \log\left(\frac{t}{t_0}\right) \right]^{1/\theta} \quad (4)$$

where L_c is the Larkin length [32], U_c the associated energy scale $U_c = U(L_c)$, and t_0 a microscopic time scale.

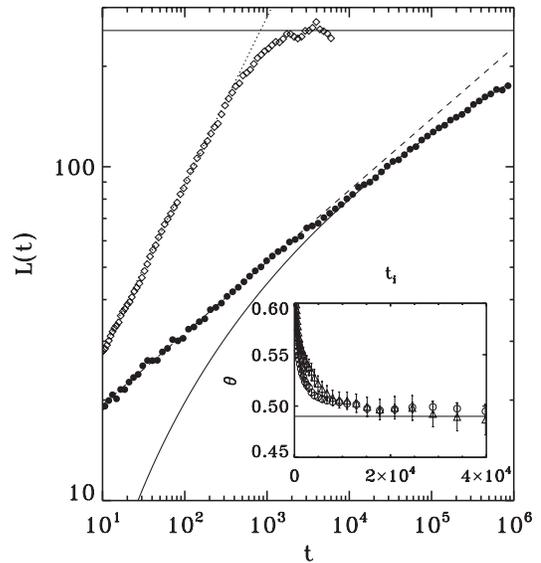


FIG. 2. Growing characteristic length scale $L(t)$ of a string of size $L = 256$. The symbols (\diamond) correspond to the relaxation of the clean system and the dotted line to the analytical result. The symbols (\bullet) correspond to the disordered case. The solid line is a fit to Eq. (4), the dashed line is a fit to the power-law growth at intermediate scales. For $10^4 < t < 10^6$ we get $\theta \approx 0.49$. Inset: exponent θ extracted from the fit to Eq. (4) in the time interval $t_i < t < 10^6$. The symbols (\triangle) correspond to a system size $L = 512$, and the symbols (\circ) to $L = 256$.

An alternative form of $L(t)$ would be the power-law scaling of the clean system, $L(t) \sim t^{1/z}$, but with a new exponent $z > 2$ taking into account the effect of the energy barriers. Note that this proposal corresponds to thermally activated motion over barriers scaling logarithmically with the size L . Such behavior has been observed in various 2-dimensional disordered systems including periodic elastic systems in the so called “marginal glass phase” [20–23]. For this model it is possible to show that the dynamical exponent takes the form $z(T) \propto 1/T$. Moreover the relaxation towards a steady state of an elastic string just above the depinning threshold shows the same power-law behavior with a dynamical exponent $z < 2$ [20].

We now compare our results with the above different scenarios. The growing length scale $L(t)$ can be determined from the average structure factor $S(q, t)$ shown in Fig. 1(b). In practice we define $L(t)$ as the intersection between the plateau $S_t = S(q \rightarrow 0^+, t)$ and the equilibrated structure factor $S^{\text{eq}} \sim q^{-7/3}$. The result is shown in Fig. 2. Note that the whole time dependence of $L(t)$ is described neither by (4) nor by a pure power law. The latter scaling can only approximately fit the short time relaxation: the fitted dynamical exponent z strongly decreases with increasing temperature and ranges from 20 to 4. However, for long times, this power-law scaling can be ruled out due to the observed bending in the log-log scale. To be sure that this bending is not an artifact of the proximity of the finite size equilibration we verified its presence for bigger systems up to a size $L = 1024$ where $L(t) \ll L$ for all considered times. For this reason the logarithmic growth seems to be more adequate for long times. A two-parameters fit to (4) gives an exponent θ which, at long times, becomes size and time independent, as shown in the inset of Fig. 2.

Although the logarithmic growth law describes well our data at long times, we find an exponent $\theta \approx 0.49$, bigger than the expected value $1/3$. If we assume that the dynamics of relaxation is governed by Arrhenius activation, this result indicates either a violation of the expected scaling of barriers or the presence of non-negligible subleading corrections in this scaling at the length scales spanned by $L(t)$ in our simulations. The inset of Fig. 2 shows, for different time-windows, the exponent θ . The saturation of θ excludes strong subleading corrections at least for the largest times reached in the simulations. However, the adequacy of the fit with $\theta \approx 0.49$ in the last three decades is still not enough to exclude the presence of logarithmic corrections in the leading term: $U(L) \sim L^{1/3} \log^\mu(L)$. The latter scenario is consistent with the upper bound scaling found numerically in Ref. [30] for the barriers separating metastable states of a directed polymer in 2-dimensional random media. Such a scaling has been shown [26] to also fit well the Monte Carlo relaxation data for a directed polymer.

The scaling of the barriers $U(L)$ and the subsequent evolution of $L(t)$ refer to *typical* values of U and $L(t)$. On the other hand, for broad enough distributions typical

and mean values can be very different [33]. Therefore, the deviations of the numerical data from the predicted behavior (4) might be produced by a broad distribution of barriers. To check for such a possibility, and to extract the barrier distribution, we study the sample to sample fluctuations of the various observables. A convenient quantity to compute for each evolving sample is the instantaneous value of the structure factor plateau $s = s(q \rightarrow 0^+, t)$ which is directly related to the growing length $l \sim s^{1/(1+2\xi)}$. As raw data directly confirms, this quantity is a stochastic process growing monotonically with the time t . Thus, its sample to sample fluctuations can be directly related to the distribution of relaxation times τ and to the statistics of barriers u by assuming Arrhenius activation, $u \sim \log(\tau)$ [34]. One obtains [35]

$$\Phi_s(u) = 1 - \Phi_u(s) \quad (5)$$

where $u(s)$ is the sample dependent barrier (structure factor plateau) and $\Phi_s(u)$ [$\Phi_u(s)$] its cumulative distribution function for a given value of s (u) [i.e., $\Phi_s(u_0)$ is the probability of finding a barrier u smaller than u_0 , given a fixed value s for the plateau]. Figure 3(a) shows $\Phi_u(s)$ as a function of s for different values of u . For all u the distributions are narrow and, on a logarithmic scale, appear just shifted. This suggest the simple rescaling s/S_t , which collapses all the curves as shown in the inset. Strikingly, we find that this rescaled function for the fully disordered system is indistinguishable, at the resolution of our nu-

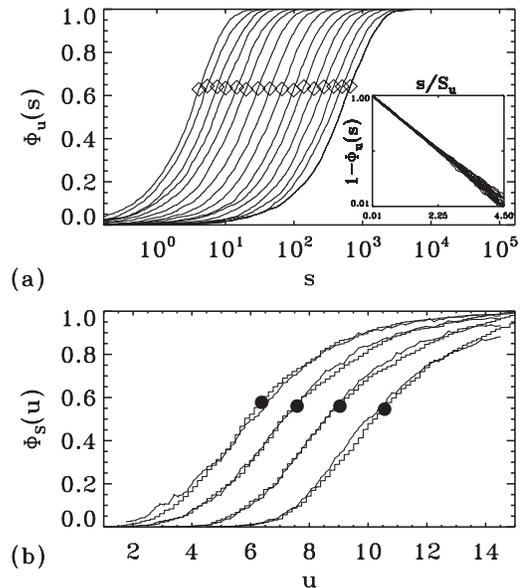


FIG. 3. (a) Cumulative distribution function of the plateau value s for fixed times t ranging from 5 to 10^6 . Symbols (\diamond) indicate the mean value S_t . Inset: collapse of the cumulative distributions in the rescaled variable s/S_t . (b) Cumulative distribution function of the barriers u for $s = 10, 20, 40, 80$. Circles (\bullet) indicate the mean value U_s . Step lines are obtained from raw data, while solid lines are obtained from the rescaled cumulative distribution of s , (5).

merical study, from that $[\Phi_u(x = s/S_t) = 1 - \exp(-x)]$ of the clean system, and from the identical one that one would obtain for the Larkin model [36] of disorder (despite the fact that this model does not have pinning and metastable states). This scaling form implies that the sample to sample fluctuations of the growing length, $l(t)$, are given by $\Phi_u(x = l/L_t) = 1 - \exp[-\alpha x^{1+2\xi}]$, with $\alpha = \Gamma(1 + \frac{1}{1+2\xi})^{1+2\xi}$. The statistics of barriers is obtained from (5) using the evolution S_t vs $u \sim \log(t)$ of Fig. 2. In Fig. 3(b) we show that the cumulative distribution $\Phi_u(s)$ derived using the latter method indeed coincides with the one obtained from a direct analysis of the raw data of s vs u for each sample. As for the sample dependent plateau s (for given values of u), the distributions of u for given values of s are found to be exponentially narrow. Scaling arguments based on typical values are therefore justified, since they can be safely translated directly to the mean values. This indicates that the effect of sample to sample fluctuations cannot explain the deviations of the numerical data with respect to the phenomenological predictions observed in Fig. 2, and that such deviations must come from the scaling of the barriers. Note also that, as visible in Fig. 3(b), the barrier distribution $\Phi_s(u)$, contrary to the distribution of plateaux $\Phi_u(s)$, does *not* scale with u/U_s , where $U_s = \langle u \rangle$ is the mean value. Such a scaling would only work if a *pure* power-law scaling of the barriers with length were perfectly verified. The complex behavior of $\Phi_s(u)$ clearly comes from the existence of two regimes in the scaling of the barriers as a function of time (length) as shown in Fig. 2. However, it remains to be understood why the presence of these two regimes does not affect the perfect collapse for $\Phi_u(s)$ as a function of s/S_t , ranging from the shortest to the longest times. No analytical explanation of this fact, nor of the form of the corresponding scaling function, exists so far.

At long times, approximate power-law scaling for the barriers is recovered, and thus the distribution of barriers would scale with u/U_s . In this case (5) shows that the universal function for all the cumulative distribution functions $\Phi_s(u)$ would be a stretched exponential. This form is different from the one that extremal statistics arguments would suggest [33], prompting for a reexamination of the physical understanding of the barrier distribution in such disordered systems.

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 [34] Strictly speaking u is the logarithm of the *total* waiting time τ to equilibrate a fixed length scale l . However, if, as usually advocated, τ is dominated by the slowest Arrhenius activated processes, then the quantity u we obtain is indeed the barrier at the length scale l , $\tau \sim e^{\beta u(l)}$ [35]. We thus use this denomination, even if it is slightly improper, in what follows.
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