

Signatures of Fractional Statistics in Noise Experiments in Quantum Hall Fluids

Eun-Ah Kim, Michael Lawler, Smitha Vishveshwara, and Eduardo Fradkin

Department of Physics, University of Illinois at Urbana-Champaign, 1110 W. Green St., Urbana Illinois 61801-3080, USA

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The elementary excitations of fractional quantum Hall (FQH) fluids are vortices with fractional statistics. Yet, this fundamental prediction has remained an open experimental challenge. Here we show that the cross-current noise in a three-terminal tunneling experiment of a two dimensional electron gas in the FQH regime can be used to detect directly the statistical angle of the excitations of these topological quantum fluids. We show that the noise also reveals signatures of exclusion statistics and of fractional charge. The vortices of Laughlin states should exhibit a bunching effect, while for higher states in the Jain sequences they should exhibit an “antibunching” effect.

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The classification of fundamental particles in terms of their quantum statistics, Bose-Einstein (bosons) and Fermi-Dirac (fermions), is a fundamental law of Nature enshrined in the principles of quantum mechanics. In the form of the *spin-statistics theorem*, it is also one of the basic *axioms* of quantum field theory (and string theory as well). However, it has long been known that other types of quantum statistics are also possible if the physical system has a reduced dimensionality. Indeed, the possible existence of *anyons* [1,2], particles with *fractional or braid* statistics, interpolating between bosons and fermions, is one of the most startling predictions of quantum mechanics in two space dimensions.

The best experimental candidates for anyons presently known are the vortices [or quasiparticles (qp)] of a strongly interacting 2D electron gas (2DEG) in a strong magnetic field in the FQH regime [3–7]. In this regime, the 2DEG behaves as an incompressible dissipationless topological fluid exhibiting the FQH effect [3,4]. The quasiparticles of FQH fluids have remarkable properties [4–7]: they are finite energy vortices (or solitons) which carry fractional charge and fractional (braid) statistics. Although by now there is strong experimental evidence for fractional charge [8–12], similar evidence is still lacking for fractional statistics (such experimental evidence has been reported very recently [13]).

There are two ways to think about statistics. A first way is through the concept of *braiding (fractional) statistics*, in which the two particle wave function $\Psi(\mathbf{r}_1, \mathbf{r}_2)$ acquires a statistical angle θ upon an adiabatic exchange process,

$$\Psi(\mathbf{r}_1, \mathbf{r}_2) = e^{i\theta}\Psi(\mathbf{r}_2, \mathbf{r}_1). \quad (1)$$

For fractional statistics to occur, the statistical angle θ should take values intermediate between that of bosons ($\theta = 0$, particles commute) and fermions ($\theta = \pi$, particles anticommute) [2,7]. The other way is to consider, in a finite size system, the effect of the presence of particles to the subsequent addition of another particle. The Pauli exclusion principle dictates that each fermion in a system re-

duces the available states to add extra fermions by 1. In contrast, the presence of a boson does not affect the later addition of bosons at all. The natural consequence of the presence of an anyon is an intermediate effect, i.e., a generalized exclusion principle [14,15]. Unfortunately, neither braiding nor exclusion of anyons have been unequivocally observed to date.

The difficulty in detecting fractional statistics lies in the extended nature of these quantum vortices. The adiabatic transport of one vortex around another is a conceptually nice *gedanken* experiment which is difficult to realize in practice. A number of interesting interferometers have been proposed to detect fractional statistics in FQH fluids [16–18], none of which has yet been realized experimentally, except possibly for the recent experiments of Ref. [13]. Theoretical schemes to measure non-Abelian statistics have also been proposed [19,20].

Here we propose an interferometer designed to detect *noise correlations* of the charge currents in a three-terminal experiment (a “*T* junction”) of a 2DEG in the FQH regime. We will show that the structure of the noise in the charge current cross correlations in this *T* junction provides a unique way to detect the fractional statistics of the vortices directly in realistic experimental conditions. This study was inspired by Refs. [21,22] and we will compare our results with these references below.

A zoo of different anyonic qp’s have long been predicted to exist for each filling factor ν (the number of electrons per flux quanta) exhibiting the FQH effect (See Table I for

TABLE I. Fractional charge e^* and statistical angle θ for qp’s of FQH states at filling factor $\nu = p/(2np + 1)$ (n, p are integers) (Ref. [23]) compared with free bosons and free fermions.

	Free boson	$p = 1$	$p > 1$	Free fermion
e^*/e	0, 2, ...	ν	$\frac{\nu}{p}$	1
θ/π	0	$\nu < \frac{1}{2}$	$(1 - \frac{2n}{p}\nu) > \frac{1}{2}$	1

examples). At each allowed ν the 2DEG displays a precise (fractional) quantization of the Hall conductance $\sigma_{xy} = \nu e^2/h$. These quantum states are distinct phases of the 2DEG distinguished by a nontrivial internal or *topological order*, which is robust against arbitrary local perturbations [24,25]. The measurement of fractional statistics will directly prove the existence of topological order. In fully polarized 2DEGs the most prominent states lie in the Jain sequences [26], with filling fractions $\nu = p/(2np + 1)$, including the Laughlin states [4] (for $p = 1$). The statistical angle of vortices of Laughlin states lie in the range $\theta/\pi < 1/2$, and are closer to that of bosons. In contrast, for the higher states in the Jain sequence, with $p > 1$, the statistical angle of their vortices lie in the range $\theta/\pi > 1/2$, which is closer to that of fermions [23,27] (See Table I). We show below that this difference leads to a “bunching” effect for Laughlin states and to an “anti-bunching” effect for non-Laughlin states. In particular, in these sequences there are pairs of FQH states, e.g., $\nu = 1/5$ and $\nu = 2/5$, whose qp’s have the same fractional charge but different fractional statistics. Hence, their qp’s exhibit either bunching or antibunching behavior while having the same charge.

Since the FQH liquids are incompressible, it costs a finite amount of energy (the *gap*) to create a qp excitation in the 2D bulk of the 2DEG. However, these liquids support gapless one-dimensional chiral excitations at the edge [25,28], opening a window for experimental probes. In particular, tunneling between edges allows a practical access to single qp properties, provided the tunneling paths lie inside the FQH liquid (as the qp’s can only exist within the FQH liquid). Single-point contact tunneling has been successfully used for the detection of fractional charge through shot noise measurements [9–11]. However, in order to stage an exchange between two qp’s from multiple independent edges, one needs at least two point contacts. Further, if multiple edges are in point contacts with a single edge each adding or removing qp’s to one edge, the generalized exclusion principle can manifest itself. A T -junction interferometer of the type shown in Fig. 1 is a minimal realization of such a situation.

In this T -junction interferometer, the qp’s are driven to tunnel between one edge and two other edges. Top gates can be used to define and bring together three edges which are attached to external leads by Ohmic contacts. Upon setting the edge 0 at relative voltage V to the two others, qp’s are driven to tunnel between these edge states. The resulting tunneling current operator is

$$I_j(t) = i \frac{e^*}{\hbar} \Gamma_j (e^{i\omega_0 t} \psi_j^\dagger \psi_0 - e^{-i\omega_0 t} \psi_0^\dagger \psi_j), \quad (2)$$

where Γ_j is the tunneling amplitude between edges 0 and $j = 1, 2$, respectively, and ψ_0^\dagger and ψ_j^\dagger are qp creation operators at the respective edges. The Josephson frequency $\omega_0 = e^*V/\hbar$ depends explicitly on the fractional charge.

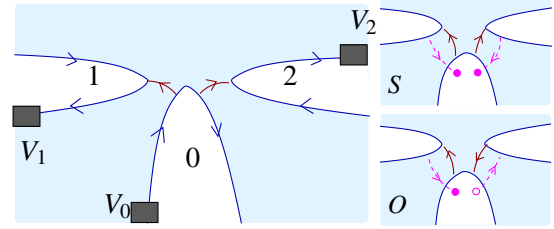


FIG. 1 (color online). The proposed T junction [32] and virtual processes for currents in the same (S) and opposite (O) orientations. Blue lines represent edge states where the edge state 0 is held at potential V relative to edges 1 and 2, i.e., $V_0 - V_1 = V_0 - V_2 = V$. Red solid lines show paths of two quasiparticles tunneling through a FQH liquid from one edge state to two others and magenta dashed lines represent virtual tunneling events.

Each edge state, labeled by $l = 0, 1, 2$, is a 1D chiral Luttinger liquid [25,28], whose equal-position qp propagator, at time t and temperature T , is given by [23]

$$\langle \psi_l(t) \psi_l^\dagger(0) \rangle = \left| \frac{\pi \tau_0 k_B T}{\sinh(\pi k_B T t)} \right|^K e^{-i\theta/2 \text{sgn}(t)}. \quad (3)$$

Here $\psi_l^\dagger(t)$ is the qp creation operator for edge l , τ_0 is a short time cutoff, and k_B is the Boltzmann constant. Equation (3) defines the scaling dimension of the qp operator $\frac{K}{2} = \frac{1}{2p(2np+1)}$. The fractional statistics, defined in Eq. (1) for qp’s of the same edge, is encoded in this propagator, through its explicit dependence on the statistical angle θ . For qp’s of different edges, l and m , the qp operators obey the (interedge) commutation rules

$$\psi_l^\dagger \psi_m^\dagger = e^{-i\alpha_{lm}} \psi_m^\dagger \psi_l^\dagger, \quad (4)$$

with $\alpha_{02} = \alpha_{21} = \alpha_{01} = \theta$ and $\alpha_{lm} = -\alpha_{ml}$. Our goal is to find a measurable quantity which reflects the θ dependence in Eqs. (3) and (4).

The normalized charge current cross-correlation $S(t)$ (noise) ($\Delta I_l = I_l - \langle I_l \rangle$),

$$S(t) \equiv \frac{\langle \Delta I_1(t) \Delta I_2(0) \rangle}{\langle I_1 \rangle \langle I_2 \rangle}, \quad (5)$$

turns out to be the simplest quantity that can exhibit the subtle signatures of fractional statistics [21], and distinguish them from those of fractional charge. We have calculated this nonequilibrium noise, using the Schwinger-Keldysh technique [29,30] to lowest order in the tunneling amplitudes Γ_j . The two virtual tunneling events in Fig. 1 are the lowest order nonvanishing contributions to the correlator $S(t)$ and they take place at all times allowed by causality. Depending on the relative orientations of the currents, the virtual tunneling processes leave two distinct pairs of objects at edge 0, as shown in Fig. 1. When the currents I_1 and I_2 have the same orientation (case S), a qp-qp pair (or a qh-qh pair) is left behind, while when they have opposite orientations (case O), qp-qh pairs are

left behind. Comparing the statistical phase gain upon exchange of qp-qp pairs (or qh-qh pairs) in case S and exchange of the qp-qh pair in case O prescribed in Eqs. (3) and (4), we find that *all* the dependence in fractional statistics is captured by a factor of $\cos\theta$ coming from contributions of case S .

The frequency spectrum of $S(t)$ in Eq. (5) can be written as a sum of two functions, a “direct” and an “exchange” term, with a dependence on θ which is both simple and explicit: [31]

$$\tilde{S}\left(\frac{\omega}{\omega_0}\right) = A\left(\frac{\omega}{\omega_0}, \frac{T}{T_0}, K\right) + \cos\theta B\left(\frac{\omega}{\omega_0}, \frac{T}{T_0}, K\right), \quad (6)$$

where ω is the frequency and $T_0 = \hbar\omega_0/k_B$ [32]. This expression displays a number of remarkable properties: (a) This universal form, which applies for all Jain states, is the lowest order in perturbation theory in the tunneling amplitude. It is accurate provided the tunneling current is small (compared with the Hall current). Nonuniversal terms contribute at higher orders. (b) Fractional charge and statistics play fundamentally distinct roles: fractional charge enters Eq. (6) through the Josephson frequency $\omega_0 = e^*V/\hbar$ and fractional statistics through the $\cos\theta$ factor. (c) Given that for Laughlin states $\theta < \pi/2$ and $\cos\theta > 0$, the exchange term in Eq. (6) provides largely positive (bunching) contributions to the noise whereas for non-Laughlin states with $\theta > \pi/2$, its contribution is largely negative (antibunching). (d) That only case S (Fig. 1) contributes to this factor can be viewed as a manifestation of a generalized exclusion principle [14] since the virtual processes in this case involves adding a qp to edge 0 in the presence of another. This is an intriguing observation given that Eq. (6) was derived using anyonic commutation rules prescribed by *braiding* statistics [7].

We have studied (numerically) the behavior of the functions $\tilde{A}(\omega/\omega_0, T/T_0, K)$ and $\tilde{B}(\omega/\omega_0, T/T_0, K)$, whose details will be reported in a future publication [31]. The salient results of this study, for $\omega \ll \omega_0$ and $\omega \sim \omega_0$, are as follows. For $\omega \ll \omega_0$, both \tilde{A} and \tilde{B} show white noise-

like behavior, and hence we focus on zero frequency values $\tilde{A}(\omega = 0) \equiv \tilde{A}_0$ and $\tilde{B}(\omega = 0) \equiv \tilde{B}_0$ which depend only on the temperature (See Fig. 2). \tilde{A}_0 is negative due to the opposite orientation of the currents in case O of Fig. 1 while \tilde{B}_0 is positive as it only involves case S . Hence, $\tilde{S}(\omega = 0) = -|\tilde{A}_0| + \cos\theta|\tilde{B}_0|$. As shown in Fig. 2, $-|\tilde{A}_0|$ dominates the spectrum at low temperatures ($T \ll T_0$) due to divergent contributions from case O (T^{-K} power law divergence). Nonetheless, $\cos\theta|\tilde{B}_0|$ significantly contributes to the total noise for higher temperatures of $2 \lesssim T/T_0 \lesssim 4$. Thus, the statistical angle θ can be measured directly at a finite temperature.

Near the Josephson frequency $\omega \sim \omega_0$, \tilde{A} and \tilde{B} have qualitatively different sharp features at low temperatures ($T \ll T_0$): $\tilde{A}(\omega/\omega_0)$ changes sign and crosses the frequency axis steeply, while $\tilde{B}(\omega/\omega_0)$ develops a peak of width $2Kk_B T/\hbar\omega_0$. These sharp features provide a direct way to measure the fractional charge [29,30], through the Josephson frequency. They can be used to distinguish between two filling factors with the same fractional charge but different statistics, e.g., $\nu = 1/5$ and $\nu = 2/5$. Since the direct term nearly vanishes close to the Josephson frequency, the exchange term plays a significant role in the total correlation $\tilde{S}(\omega/\omega_0 = 1)$ by shifting it to a positive (negative) value for Laughlin (non-Laughlin) states as shown in Fig. 3(a). The information on statistics and charge contained in the noise near the Josephson frequency is summarized in Fig. 3(b), where we introduced the “effective charge” $\bar{e} \equiv \hbar\bar{\omega}/V$ obtained from the crossing point of \tilde{S} , i.e., $\tilde{S}(\bar{\omega}/\omega_0) = 0$. Comparing \bar{e} ’s for $\nu = 1/5$ and for $\nu = 2/5$ we find them satisfying $\bar{e}_{1/5} < e^* < \bar{e}_{2/5}$ over a broad range of (low) temperatures, however both approaching the fractional charge $e^* = e/5$ as $T \rightarrow 0$. This different asymptotic behavior is a consequence of their different statistical angles: $\theta = \pi/5$ (bunching) for $\nu = 1/5$ and $\theta = 3\pi/5$ (antibunching) for $\nu = 2/5$. This (weak) effect also shows that the vortices of these FQH states have different fractional statistics in spite of having the same fractional charge.

In summary, we have shown that the cross-current noise in a T junction can be used to directly measure the statistical angle θ independently from the fractional charge, and conceptually, from the filling factor ν . The low temperature and low frequency results show that there is a quantitatively significant statistics dependence of this noise. We have shown how to measure both fractional charge and statistics near the Josephson frequency. Our results are independent of nonuniversal short time physics. Safi *et al.* [21] considered a similar setup for Laughlin states at $T = 0$, and obtained an expression for $\tilde{S}(\omega = 0)$ for case O alone as a function of ν (for Laughlin states one cannot distinguish ν from K or e^*/e or θ/π). At *finite temperatures*, case S (whose contribution is negligible next to case O at zero temperature) brings in an explicit statistics dependence which can be distinguished from the effect

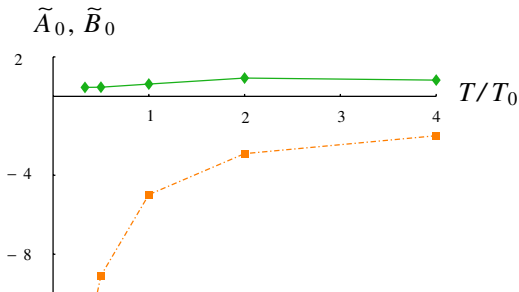


FIG. 2 (color online). The temperature dependence of the direct term \tilde{A}_0 (orange dashed line) and the exchange term \tilde{B}_0 (green solid line) defined in the text, for $\nu = 2/5$ with $T_0 \sim 80$ mK ($V = 40$ μ V). This qualitative behavior is generic to all fractions.

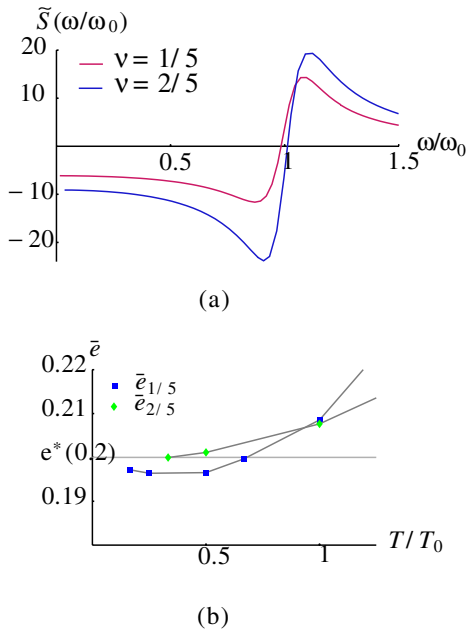


FIG. 3 (color online). (a) $\tilde{S}(\omega/\omega_0)$ for $\nu = 1/5, 2/5$, for $T = 20$ mK, 40 mK, respectively, with $V \sim 40 \mu\text{V}$. $\tilde{S}(\omega/\omega_0 = 1)$ is positive for the Laughlin state $\nu = 1/5$ and negative for the Jain state $\nu = 2/5$. (b) The effective charge $\bar{e}(T)$ in units of e is determined by the crossing points of (a) for $\nu = 1/5$ and $\nu = 2/5$.

of the fractional charge and the scaling dimension for non-Laughlin states. A closely related setup is the four terminal case studied (only for Laughlin states at $T = 0$) by Vishveshwara [22]. Even in this case, an actual measurement of the cross-correlation functions will involve the quantity $S(t)$ we calculated here.

The current state-of-the-art noise experiments [11,33] operate at bias voltages of 10–150 μV (corresponding to a Josephson frequency $\omega_0 \sim 1$ GHz), and temperatures of 10–100 mK. These parameters access the ratios of T/T_0 in the range shown here, making our proposal quite feasible. To remain in the perturbative regime the voltage V has to be large compared to a crossover scale determined by the tunneling amplitudes, so that the current is a small fraction (~ 0.1) of the Hall current at the same voltage, but still with a measurable noise. Some recent experiments are consistent with fractional statistics. A somewhat large qp charge, extracted in recent shot noise experiments, suggests bunching behavior [33]. Aharonov-Bohm oscillations in an antidot with a superperiod have been interpreted as evidence for fractional statistics [13].

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