Δ Resonance Contribution to Two-Photon Exchange in Electron-Proton Scattering

S. Kondratyuk and P. G. Blunden

Department of Physics and Astronomy, University of Manitoba, Winnipeg, Manitoba, Canada R3T 2N2

W. Melnitchouk

Jefferson Lab, 12000 Jefferson Avenue, Newport News, Virginia 23606, USA

J. A. Tjon

Jefferson Lab, 12000 Jefferson Avenue, Newport News, Virginia 23606, USA
Department of Physics, University of Maryland, College Park, Maryland 20742-4111, USA

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We calculate the effects on the elastic electron-proton scattering cross section of the two-photon exchange contribution with an intermediate Δ resonance. The Δ two-photon exchange contribution is found to be smaller in magnitude than the previously evaluated nucleon contribution, with an opposite sign at backward scattering angles. The sum of the nucleon and Δ two-photon exchange corrections has an angular dependence compatible with both the polarization-transfer and the Rosenbluth methods of measuring the nucleon electromagnetic form factors.

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The electromagnetic form factors reflect the essentially nonlocal nature of the nucleon in its interactions with photons. As the basic observables parametrizing nucleon compositeness, the form factors have long been studied both experimentally and theoretically. This interest has been renewed recently due to the increased precision of electron-proton scattering experiments and the availability of two alternative methods of extracting the form factors from the data: the Rosenbluth method—also known as the longitudinal-transverse (LT) separation technique [1,2]—and the polarization-transfer (PT) technique [3]. If one uses the traditional one-photon exchange calculation to extract the form factors, the two methods lead to apparently incompatible results: while the PT method yields a ratio of the electric to magnetic form factors which falls off linearly with the square of the momentum transfer Q^2, the LT separation experiments give an approximately constant ratio [3–5]. Finding an explanation of this discrepancy is important for the use of electron-proton scattering as a precise and reliable tool in hadronic physics.

Several theoretical studies [6,7] have suggested that the problem could be at least partially resolved by including higher-order two-photon exchange corrections in the analysis of electron-proton scattering data, in addition to the lowest-order one-photon exchange (Born) approximation. The recent explicit calculation [6] has shown that with the two-photon exchange taken into account in the analysis of electron-proton scattering, the ratio of the form factors extracted from the LT separation measurements becomes more compatible with the ratio from the PT experiments. However, the two-photon exchange diagrams calculated in Ref. [6] contained only nucleons in the intermediate state; the contribution of other hadrons has not been included until now. In view of the prominent role of the Δ resonance (unlike other excited states) in many hadronic reactions, it is essential to evaluate its contribution to the two-photon exchange in electron-proton scattering. Without an explicit calculation the results with only the nucleon intermediate state can be viewed only as suggestive in resolving the discrepancy. Some aspects of the Δ contribution were addressed before [8], using various approximate approaches.

This Letter presents a quantum field theoretical calculation of the two-photon exchange "box" and "crossed-box" diagrams with a Δ resonance in the intermediate state. We will show that the Δ and nucleon contributions tend to partially cancel each other, their sum nevertheless yielding a predominantly negative two-photon exchange correction. The modified cross section has an angular dependence consistent with both the LT separation and PT measurements of the form factors.

We consider scattering of electrons (mass $m_e \approx 0.511 \times 10^{-3}$ GeV) off protons (mass $M_N \approx 0.938$ GeV) with the four-momenta assigned as $e(p_1) + p(p_2) \to e(p_3) + p(p_4)$. The differential cross section for this process is written in the form $d\sigma = d\sigma_B(1 + \delta_N + \delta_\Delta)$ where $d\sigma_B$ is the lowest-order Born contribution and δ_N (δ_Δ) is the higher-order correction obtained from two-photon exchange diagrams containing nucleons (Δ's) in the intermediate state. (Other higher-order effects—such as the vacuum polarization and the electron-photon vertex corrections—are known [9] to be irrelevant to the differences between the PT and LT analyses; we therefore focus here on the two-photon exchange effects only.) It is convenient to divide $d\sigma$ by the well-known factor describing the scattering from a structureless "proton" and thus use the reduced cross section

$$d\sigma_R = \left[G_M^2(Q^2) + \frac{\epsilon}{\tau}G_E^2(Q^2)\right](1 + \delta_N + \delta_\Delta). \qquad (1)$$

Here the Born contribution is written in terms of the electric and magnetic form factors of the proton, $G_E(Q^2)$ and $G_M(Q^2)$, which are functions of the momentum transfer squared $Q^2 \equiv -q^2 \equiv 4\tau M_N^2 = -(p_1 - p_3)^2$. The kinematic variable ϵ is related to the scattering angle θ through $\epsilon = [1 + 2(1+\tau)\tan^2(\theta/2)]^{-1}$, which is equal to the photon polarization in the Born approximation.

We denote the Born scattering amplitude as $\mathcal{M}_B$ and the two-photon exchange amplitudes with the nucleon and Δ intermediate states as $\mathcal{M}_N^{\gamma\gamma}$ and $\mathcal{M}_\Delta^{\gamma\gamma}$, respectively. From the equation $d\sigma = d\sigma_B(1 + \delta_N + \delta_\Delta) = |\mathcal{M}_B + \mathcal{M}_N^{\gamma\gamma} + \mathcal{M}_\Delta^{\gamma\gamma}|^2$, where $d\sigma_B = |\mathcal{M}_B|^2$, we derive to first order in the electromagnetic coupling $e^2/(4\pi) \approx 1/137$:

$$\delta_{N,\Delta} = 2\frac{\mathrm{Re}(\mathcal{M}_B^\dagger \mathcal{M}_{N,\Delta}^{\gamma\gamma})}{|\mathcal{M}_B|^2}. \tag{2}$$

The nucleon part δ_N of the two-photon exchange was analyzed in Ref. [6]. Below we evaluate the Δ two-photon exchange contribution δ_Δ. The scattering amplitude $\mathcal{M}_\Delta^{\gamma\gamma}$ is given by the sum of the box and crossed-box loop diagrams depicted in Fig. 1.

We use the $\gamma N\Delta$ vertex of the following form [10]:

$$\begin{aligned}\Gamma^{\nu\alpha}_{\gamma\Delta\to N}(p,q) &\equiv iV^{\nu\alpha}_{\Delta\mathrm{in}}(p,q)\\ &= i\frac{eF_\Delta(q^2)}{2M_\Delta^2}\{g_1[g^{\nu\alpha}\not{p}\not{q} - p^\nu\gamma^\alpha\not{q} - \gamma^\nu\gamma^\alpha p\cdot q\\ &\quad + \gamma^\nu\not{p}q^\alpha] + g_2[p^\nu q^\alpha - g^{\nu\alpha}p\cdot q]\\ &\quad + (g_3/M_\Delta)[q^2(p^\nu\gamma^\alpha - g^{\nu\alpha}\not{p})\\ &\quad + q^\nu(q^\alpha\not{p} - \gamma^\alpha p\cdot q)]\}\gamma_5 T_3,\end{aligned} \tag{3}$$

where $M_\Delta \approx 1.232$ GeV is the Δ mass, p_α and q_ν are the four-momenta of the incoming Δ and photon, respectively, T_3 is the isospin transition operator, and g_1, g_2, and g_3 are the coupling constants. An analysis of Eq. (3) in the Δ rest frame suggests that g_1, $g_2 - g_1$, and g_3 may be interpreted as magnetic, electric, and Coulomb components, respectively, of the $\gamma N\Delta$ vertex. The form factor in Eq. (3) is necessary for ultraviolet regularization of the loop integrals evaluated below; we use the simple dipole form $F_\Delta(q^2) = \Lambda_\Delta^4/(\Lambda_\Delta^2 - q^2)^2$, where Λ_Δ is the cutoff. The form factor entails some model dependence of our results, which is unavoidable in any dynamical hadronic calculation. The vertex with an outgoing Δ is given by $\Gamma^{\alpha\nu}_{\gamma N\to\Delta}(p,q) \equiv iV^{\alpha\nu}_{\Delta\mathrm{out}}(p,q) = \gamma_0[\Gamma^{\nu\alpha}_{\gamma\Delta\to N}(p,q)]^\dagger\gamma_0$, with p_α and q_ν the four-momenta of the outgoing Δ and incoming photon, respectively. The $\gamma N\Delta$ vertex is orthogonal to the four-momenta of both the photon and the Δ: $q_\nu\Gamma^{\nu\alpha}_{\gamma\Delta\to N}(p,q) = 0$ and $p_\alpha\Gamma^{\nu\alpha}_{\gamma\Delta\to N}(p,q) = 0$. The first of these equations ensures the usual electromagnetic gauge invariance of the calculation while the second allows us to use only the physical spin 3/2 component,

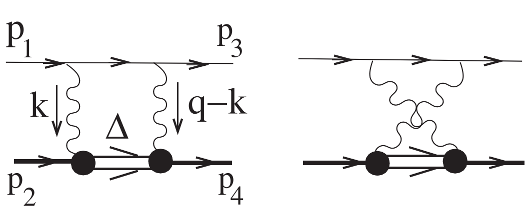

FIG. 1. Two-photon exchange box and crossed-box graphs for electron-proton scattering with a Δ intermediate state.

$$\begin{aligned}S^\Delta_{\alpha\beta}(p) &= \frac{-i}{\not{p} - M_\Delta + i0}\mathcal{P}^{3/2}_{\alpha\beta}(p),\\ \mathcal{P}^{3/2}_{\alpha\beta}(p) &= g_{\alpha\beta} - \frac{1}{3}\gamma_\alpha\gamma_\beta - \frac{1}{3p^2}(\not{p}\gamma_\alpha p_\beta + p_\alpha\gamma_\beta\not{p}),\end{aligned} \tag{4}$$

of the Rarita-Schwinger propagator, the background spin 1/2 component vanishing when contracted with the adjacent $\gamma N\Delta$ vertices [11]. At present we do not include a width in the Δ propagator as its influence on the unpolarized cross section should be small.

The loop integrals corresponding to the box and crossed-box diagrams in Fig. 1 can be written as

$$\mathcal{M}^{\gamma\gamma}_\Delta = -e^4\int\frac{d^4k}{(2\pi)^4}\frac{N^\Delta_{\mathrm{box}}(k)}{D^\Delta_{\mathrm{box}}(k)} - e^4\int\frac{d^4k}{(2\pi)^4}\frac{N^\Delta_{x-\mathrm{box}}(k)}{D^\Delta_{x-\mathrm{box}}(k)}, \tag{5}$$

with the numerators and denominators given by

$$\begin{aligned}N^\Delta_{\mathrm{box}}(k) &= \bar{U}(p_4)V^{\mu\alpha}_{\Delta\mathrm{in}}(p_2+k, q-k)[\not{p}_2 + \not{k} + M_\Delta]\\ &\quad\times\mathcal{P}^{3/2}_{\alpha\beta}(p_2+k)V^{\beta\nu}_{\Delta\mathrm{out}}(p_2+k,k)U(p_2)\bar{u}(p_3)\gamma_\mu\\ &\quad\times[\not{p}_1 - \not{k} + m_e]\gamma_\nu u(p_1),\end{aligned} \tag{6}$$

$$\begin{aligned}N^\Delta_{x-\mathrm{box}}(k) &= \bar{U}(p_4)V^{\mu\alpha}_{\Delta\mathrm{in}}(p_2+k, q-k)[\not{p}_2 + \not{k}\\ &\quad + M_\Delta]\mathcal{P}^{3/2}_{\alpha\beta}(p_2+k)V^{\beta\nu}_{\Delta\mathrm{out}}(p_2\\ &\quad + k, k)U(p_2)\bar{u}(p_3)\gamma_\nu[\not{p}_3 + \not{k}\\ &\quad + m_e]\gamma_\mu u(p_1),\end{aligned} \tag{7}$$

$D^\Delta_{\mathrm{box}}(k) = [k^2][(k-q)^2][(p_1-k)^2 - m_e^2][(p_2+k)^2 - M_\Delta^2]$, $D^\Delta_{x-\mathrm{box}}(k) = D^\Delta_{\mathrm{box}}(k)|_{p_1-k\to p_3+k}$, where U and u denote the proton and electron four-spinor wave functions, respectively. Compared to the case of the nucleon [6], the presence of a Δ in the intermediate state entails a more complicated structure of the numerator. Also the loop integrals with a Δ are not infrared divergent, in contrast with the nucleon contribution where the infrared part is very important [9]. The evaluation of Eq. (5) involves preliminary algebraic manipulations to effect cancellations between terms in the numerators and denominators and subsequent integration of the thus simplified expressions. The result is obtained analytically in terms of the standard Passarino-Veltman dilogarithm functions [12]. In the calculation we used the computer package FEYNCALC [13].

The first and second loop integrals in Eq. (5) must be mutually related by crossing symmetry, which can be formulated in terms of the numerator of Eq. (2) using the Mandelstam variables $s = (p_1 + p_2)^2$, $t = (p_1 - p_3)^2$, and $u = (p_2 - p_3)^2 = 2M_N^2 + 2m_e^2 - t - s$. Denoting $f^{\gamma\gamma}(s, t) \equiv \mathcal{M}_B^\dagger \mathcal{M}_\Delta^{\gamma\gamma}$ and writing it as the sum $f^{\gamma\gamma}(s, t) = f_{\rm box}^{\gamma\gamma}(s, t) + f_{x\text{-box}}^{\gamma\gamma}(s, t)$, where the first (second) term is calculated using only the first (second) integral in Eq. (5), the crossing symmetry requires that $f_{x\text{-box}}^{\gamma\gamma}(s, t) = -f_{\rm box}^{\gamma\gamma}(u, t)|_{u=2M_N^2+2m_e^2-t-s}$. We calculated the integrals in Eq. (5) explicitly and checked that our results obey this constraint.

The Δ two-photon exchange correction to the differential cross section can be expressed as a quadratic form in the $\gamma N\Delta$ coupling constants $g_M = g_1$, $g_E = g_2 - g_1$, and $g_C = g_3$: $\delta_\Delta = C_M g_M^2 + C_{ME} g_M g_E + C_E g_E^2 + C_C g_C^2 + C_{EC} g_E g_C + C_{MC} g_M g_C$ with the coefficients depending on the kinematical variables. The relative contributions of the coupling constants g_M, g_E, and g_C to δ_Δ can be assessed from Table I, where the C_M, C_{ME}, etc., are given as functions of ϵ for a fixed Q^2. Here we used the dipole $\gamma N\Delta$ form factor with the cutoff $\Lambda_\Delta = 0.84$ GeV, which describes a Δ resonance whose mean-square radius is comparable to that of the nucleon. This choice is consistent with various parametrizations from pion electroproduction [14].

In the following we discuss the results obtained with the fixed coupling constants $g_M = 7$ and $g_E = 2$. These couplings were used in the dressed K-matrix model [10] (adjusted for a different normalization of the vertex used in the present calculation), yielding a good coupled-channel description of pion-nucleon scattering, pion photoproduction, and Compton scattering at low and intermediate energies. In particular, the $E2/M1$ ratio obtained in Ref. [10] is $R_{EM} \approx -3\%$, in agreement with the PDG [15] value: $-(2.5 \pm 0.5)\%$. Recent analyses [14] of pion electroproduction suggest that the Coulomb coupling constant g_C is small and negative. In our calculation we vary g_C in the range $[-2, 0]$. With these values of g_M, g_E, and g_C one can see from Table I that the magnetic coupling dominates the Δ two-photon exchange correction, whereas the electric coupling has a much smaller effect. Since the contribution of the Coulomb component is strongly suppressed (not exceeding 0.2%), we omit it from further discussion, setting $g_C = 0$ in the rest of the Letter.

The ϵ dependence of the sum of the Δ and nucleon two-photon exchange corrections is shown in Fig. 2, for two fixed values of Q^2. The dependence on the $\gamma N\Delta$ form factor can be seen by comparing the results obtained with the cutoffs $\Lambda_\Delta = 0.84$ and 0.68 GeV (the latter choice corresponds to a Δ that is spatially "bigger" than the nucleon). The purely nucleon contribution, shown for comparison, was calculated as in Ref. [6] using the γNN form factors extracted from the PT experiments [3,4]. The Δ correction is more prominent at higher momentum transfers. The Δ tends to reduce the effect of the nucleon two-photon exchange, making the modulus of the negative nucleon correction somewhat smaller at backward angles (i.e., at low ϵ). The combined effect of the nucleon and Δ two-photon exchanges produces a negative correction to the cross section at small ϵ, decreasing in magnitude as ϵ increases. (The diminishing of the two-photon exchange correction at forward angles is consistent with the analysis of electron-proton and positron-proton scattering data [16].) The main features of the Δ contribution—its smallness and its tendency to attenuate the nucleon contribution at backward angles—are insensitive to the $\gamma N\Delta$ form factor, being to that extent model independent. The detailed interplay between the Δ and the nucleon contributions is more complicated at forward angles, as can be seen from Fig. 2.

The calculated differential cross section is shown by the solid lines in Fig. 3, including the Born term and the sum of the two-photon exchange corrections $\delta_N + \delta_\Delta$ with the nucleon and the Δ intermediate states. The reduced cross section Eq. (1), scaled for convenience by the square of the standard dipole form factor $G_D(Q^2) = 1/(1 + Q^2/0.84^2)^2$,

TABLE I. The ϵ dependence of the coefficients $C_{M,ME,E,C}$ at $Q^2 = 3$ GeV^2 ($C_{EC,MC} < 10^{-10}$ for any kinematics considered).

ϵ	$C_M \times 10^4$	$C_{ME} \times 10^4$	$C_E \times 10^4$	$C_C \times 10^4$
0.1	2.92	1.49	−1.64	−1.09
0.2	2.53	0.94	−1.61	−1.00
0.3	2.17	0.50	−1.57	−0.88
0.4	1.83	0.14	−1.52	−0.72
0.5	1.54	−0.11	−1.45	−0.50
0.6	1.23	−0.32	−1.37	−0.21
0.7	0.95	−0.46	−1.27	0.18
0.8	0.65	−0.55	−1.15	0.79
0.9	0.31	−0.57	−0.98	1.98

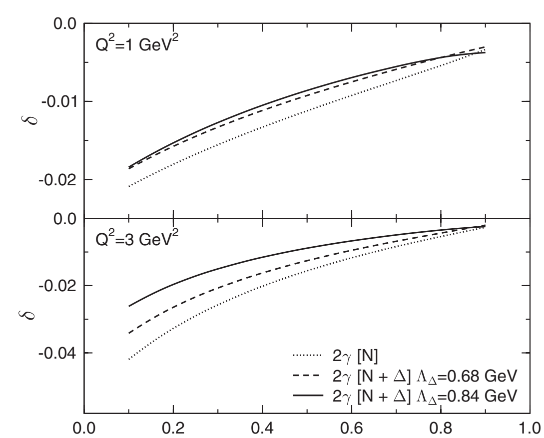

FIG. 2. Sum of the nucleon (N) and Δ contributions to the two-photon exchange correction to the electron-proton scattering cross section, using two values of the cutoff Λ_Δ.

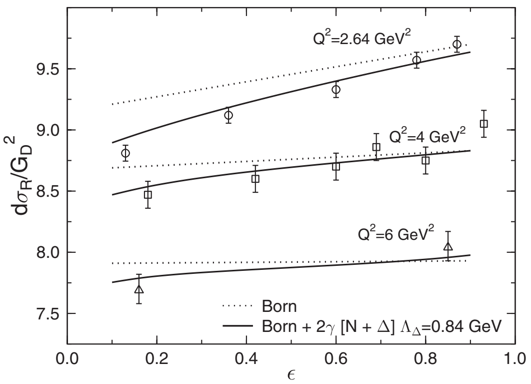

FIG. 3. Effect of adding the two-photon exchange to the Born cross section, the latter evaluated with the nucleon form factors from the PT experiment [3,4]. The reduced cross section is scaled as described in the text. The data points are taken from Refs. [1,2].

is compared in Fig. 3 with the LT separation measurements from SLAC [1] (at $Q^2 = 4$ and 6 GeV^2) and JLab [2] (at $Q^2 = 2.64\ \text{GeV}^2$). The dotted lines show the Born contribution alone, using the nucleon form factors $G_{E,M}(Q^2)$ taken from the analysis of the JLab PT experiment [3,4]. One can see that including only the Born term is inadequate in the analysis of the data. The addition of the two-photon exchange correction increases the slope of the cross section, also exhibiting some nonlinearity in ϵ. Thus the results of the PT and LT separation experiments become essentially compatible by including the nucleon and Δ two-photon exchange corrections.

To summarize, we calculated the correction to the electron-proton scattering cross section due to the two-photon exchange with a Δ intermediate state, treated on the same footing as the intermediate nucleon contribution. For realistic choices of the $\gamma N \Delta$ vertex we found that the Δ contribution alters the cross section by an amount from -1% to $+2\%$, and is largest at backward scattering angles. For the cross section obtained using the LT separation technique, the Δ two-photon exchange contribution slightly reduces the magnitude of the (negative) nucleon correction. Generally, the cross section including the nucleon and Δ two-photon exchange corrections has the angular dependence that can accommodate the results of both the LT separation and PT methods of measuring the nucleon form factors. This calculation therefore provides explicit and compelling evidence that the two-photon exchange contribution (with the lowest mass, N and Δ intermediate states) can resolve the form factor discrepancy. To reconcile these two methods completely, theoretical analyses of the data might need additional ingredients. For example, one may take into account the dependence of the γNN and $\gamma N \Delta$ vertices on the hadronic off-shell momenta (as was suggested in [17]). Heavier hadron resonances or quark degrees of freedom should also become important at higher momentum transfers (see, e.g., [18]).

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[1] R. C. Walker *et al.*, Phys. Rev. D **49**, 5671 (1994); L. Andivahis *et al.*, Phys. Rev. D **50**, 5491 (1994).
[2] I. A. Qattan *et al.*, Phys. Rev. Lett. **94**, 142301 (2005).
[3] M. K. Jones *et al.*, Phys. Rev. Lett. **84**, 1398 (2000); O. Gayou *et al.*, Phys. Rev. Lett. **88**, 092301 (2002).
[4] E. J. Brash, A. Kozlov, Sh. Li, and G. M. Huber, Phys. Rev. C **65**, 051001(R) (2002).
[5] J. Arrington, Phys. Rev. C **68**, 034325 (2003).
[6] P. G. Blunden, W. Melnitchouk, and J. A. Tjon, Phys. Rev. Lett. **91**, 142304 (2003); nucl-th/0506039 [Phys. Rev. C (to be published)].
[7] P. A. M. Guichon and M. Vanderhaegen, Phys. Rev. Lett. **91**, 142303 (2003); Y. C. Chen, A. Afanasev, S. J. Brodsky, C. E. Carlson, and M. Vanderhaegen, Phys. Rev. Lett. **93**, 122301 (2004).
[8] S. D. Drell and M. Ruderman, Phys. Rev. **106**, 561 (1957); G. K. Greenhut, Phys. Rev. **184**, 1860 (1969); J. A. Campbell, Phys. Rev. **180**, 1541 (1969).
[9] L. C. Maximon and J. A. Tjon, Phys. Rev. C **62**, 054320 (2000).
[10] S. Kondratyuk and O. Scholten, Phys. Rev. C **64**, 024005 (2001).
[11] V. Pascalutsa and R. G. E. Timmermans, Phys. Rev. C **60**, 042201 (1999).
[12] G. 't Hooft and M. Veltman, Nucl. Phys. **B153**, 365 (1979).
[13] R. Mertig, M. Böhm, and A. Denner, Comput. Phys. Commun. **64**, 345 (1991) (available at http://www.feyncalc.org/).
[14] T. Sato and T.-S. H. Lee, Phys. Rev. C **63**, 055201 (2001); G. L. Caia, V. Pascalutsa, J. A. Tjon, and L. E. Wright, Phys. Rev. C **70**, 032201 (2004).
[15] S. Eidelman *et al.* (Particle Data Group), Phys. Lett. B **592**, 1 (2004).
[16] J. Arrington, Phys. Rev. C **69**, 032201 (2004).
[17] S. Kondratyuk, K. Kubodera, and F. Myhrer, Phys. Rev. C **71**, 028201 (2005).
[18] A. V. Afanasev, S. J. Brodsky, C. E. Carlson, Y.-C. Chen, and M. Vanderhaegen, Phys. Rev. D **72**, 013008 (2005).