Surprises in Threshold Antikaon-Nucleon Physics

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(Received 6 August 2005; published 19 October 2005)

Low energy $\bar{K}N$ interactions are studied within unitary chiral perturbation theory at next-to-leading order with ten coupled channels. We pay special attention to the recent precise determination of the strong shift and width of the kaonic hydrogen $1s$ state by the DEAR Collaboration that has challenged our theoretical understanding of this sector of strong interactions. We typically find two classes of solutions, both of them reproducing previous data, that either can or cannot accommodate the DEAR measurements. The former class has not been previously discussed.

DOI: 10.1103/PhysRevLett.95.172502 PACS numbers: 36.10.Gv, 11.80.−m, 12.39.Fe, 13.75.Jz

Low energy antikaon-nucleon interactions have been the object of extensive study almost for the last 50 years. Based on early data on $K^- p$ scattering, Dalitz and Tuan predicted [1] in 1959 the existence of a subthreshold $\bar{K}N$ resonance, the $\Lambda(1405)$, first seen experimentally three years later [2]. Despite this success, $K^- p$ scattering is still challenging our understanding of strong interactions. First, this resonance, being too light, appears puzzling for quark model [3] and lattice QCD [4] communities. This fact can be interpreted as one more evidence that the $\Lambda(1405)$ is a dynamically generated resonance as claimed in Refs. [1,5–7]. Second, there has been disagreement between the sign of the $K^- p$ scattering lengths extracted either from scattering or from the $1s$ $K^- p$ atomic level shift until 1998 when it was settled down by the KpX experiment at KEK [8]. Now, the around factor of 2 more precise DEAR measurement [9] brings in a further disagreement with all previous theoretical results from SU(3) chiral dynamics, which are, however, compatible with the KEK measurement [7,10,11]. Third, the physical $\Lambda(1405)$ has not yet been considered up to very recently [7,12] as the admixture of two nearby poles, so that different reactions weighting more one pole or the other produce different resonant shapes peaking at different energies. For experimental evidences on this issue, see [13]. Fourth, the recently discovered strange tribaryons $S^0(3115)$ and $S^1(3140)$ [14] have most likely shown that deeply bound states of $\bar{K}$, as predicted in [15] and even deeper, do exist. If so, the $\bar{K}$-nucleus potential is therefore definitely strongly attractive in contrast with the up to now prevailing beliefs and claims of a shallow potential. This is of foremost importance as it is a way to obtain very dense nuclear matter [15], $(3 \sim 4) \times \rho_0$, as well as to get kaon condensation in nuclear matter (e.g., neutron stars) [16], or strangeness clusters in heavy ion collisions. A sounder theoretical explanation of such strongly attractive $\bar{K}$-nucleus potential is called for. Fifth, there is an exhaustive search of the strangeness content of the proton with positive results in several experiments worldwide [17]. Furthermore, the recent evaluation [18] of the pion-nucleon sigma term $\sigma_{\pi N}$ points toward a rather large strangeness content of the proton, with a contribution to the nucleon mass between 110 to 220 MeV. One can address this issue by calculating the proton scalar form factor, $\langle p|\bar{s}s|p\rangle$, which by unitarity is related with the $I = 0$ S-wave $\bar{K}N$ amplitudes [19], the subject of this Letter. All these issues concern our basic knowledge of strong interactions and require as a necessary first step a precise understanding and calculation of the $\bar{K}N$ strong amplitudes, especially at low and subthreshold energies.

In the limit of massless u, d, and s quarks, the QCD Lagrangian is symmetric under the chiral group $SU(3)_L \times SU(3)_R$. Once this symmetry is spontaneously broken to the diagonal $L + R$ subgroup there appear eight Goldstone bosons which acquire mass proportionally to the nonvanishing quark masses—pions, kaons, and etas. Their low energy interactions are therefore fixed and can be cast in a Taylor expansion in powers of momenta and quark masses modulated by unknown coefficients. This is known as chiral perturbation theory (CHPT) [20]. However, in a system like $\bar{K}N$, where the $\Lambda(1405)$ resonance is so close to threshold, CHPT needs to be supplied with a nonperturbative resummation scheme. We follow here the unitary CHPT (UCHPT) [7] pioneered in [21]. This setup, that does not qualify as an effective quantum field theory while CHPT does, was used in [11] to study $\bar{K}N$ scattering as well. There, the authors were not able to reproduce simultaneously previous $\bar{K}N$ scattering data and the new precise DEAR measurement and called for a possible inconsistency, first pointed out in Ref. [22], between the latter and former data. We will show below that this is not the case.

Meson-baryon interactions are described to lowest order in the CHPT expansion, i.e., at $\mathcal{O}(p)$, by the Lagrangian

$$\mathcal{L}_1 = \langle i\bar{B}\gamma^\mu[D_\mu, B]\rangle - m_0\langle\bar{B}B\rangle + \frac{D}{2}\langle\bar{B}\gamma^\mu\gamma_5\{u_\mu, B\}\rangle + \frac{F}{2}\langle\bar{B}\gamma^\mu\gamma_5[u_\mu, B]\rangle, \tag{1}$$

where m_0 stands for the octet baryon mass in the SU(3) chiral limit. The trace $\langle\cdots\rangle$ runs over flavor indices and axial-vector couplings are constrained by $F + D = g_A = 1.26$. We use $D = 0.80$ and $F = 0.46$ extracted from hyperon decays [23]. Furthermore, $u_\mu = iu^\dagger(\partial_\mu U)u^\dagger$, $U(\Phi) = u(\Phi)^2 = \exp(i\sqrt{2}\Phi/f)$, with f the pion decay constant in the chiral limit, and the covariant derivative $D_\mu = \partial_\mu + \Gamma_\mu$ with $\Gamma_\mu = [u^\dagger, \partial_\mu u]/2$. The 3×3 flavor matrices Φ and B collect the lightest octets of pseudoscalar mesons (π, K, η) and baryons (N, Σ, Λ, Ξ), respectively. At next-to-leading order (NLO) in CHPT, i.e., $\mathcal{O}(p^2)$, the meson-baryon interactions are described by the Lagrangian

$$\mathcal{L}_2 = b_0\langle\bar{B}B\rangle\langle\chi_+\rangle + b_D\langle\bar{B}\{\chi_+, B\}\rangle + b_F\langle\bar{B}[\chi_+, B]\rangle + b_1\langle\bar{B}[u_\mu,[u^\mu, B]]\rangle + b_2\langle\bar{B}\{u_\mu,\{u^\mu, B\}\}\rangle + b_3\langle\bar{B}\{u_\mu,[u^\mu, B]\}\rangle + b_4\langle\bar{B}B\rangle\langle u_\mu u^\mu\rangle + \cdots. \quad (2)$$

Here ellipses denote terms that do not produce new contributions to S-wave scattering at $\mathcal{O}(p^2)$. In addition, $\chi_+ = u^\dagger\chi u^\dagger + u\chi^\dagger u$, $\chi = 2B_0\mathcal{M}_q$, $\mathcal{M}_q$ is the diagonal quark mass matrix (m_u, m_d, m_s) and $B_0 f^2 \equiv -\langle 0|\bar{q}q|0\rangle$ the quark condensate in the SU(3) chiral limit. The b_i are fitted to data.

We evaluate within CHPT at $\mathcal{O}(p^2)$ all two-body scattering amplitudes with strangeness $S = -1$ corresponding to the ten coupled channels: $\pi^0\Lambda$, $\pi^0\Sigma^0$, $\pi^-\Sigma^+$, $\pi^+\Sigma^-$, K^-p, $\bar{K}^0 n$, $\eta\Lambda$, $\eta\Sigma^0$, $K^0\Xi^0$, and $K^+\Xi^-$, in increasing threshold energy order. Each channel is labeled by its position (1 to 10) in the previous list. We denote the CHPT amplitudes at $\mathcal{O}(p)$ by $T^{(1)}_{\chi ij}$ and at $\mathcal{O}(p^2)$ by $T^{(2)}_{\chi ij}$, with subindices ij indicating the scattering process $i \to j$. We employ these perturbative amplitudes, given explicitly in [24], as input to UCHPT at NLO. Two-body partial wave amplitudes can be written in matrix notation as [7]:

$$T(W) = [I + \mathcal{T}(W)g(s)]^{-1}\mathcal{T}(W), \quad (3)$$

with W the total energy in the center of mass (c.m.) frame and $s = W^2$. This equation was derived in [7] employing a coupled channel dispersion relation for the inverse of a partial wave T_{ij}. The unitarity or right-hand cut is taken into account easily by the discontinuity of $T^{-1}(W)$ for W above the $i_{\rm th}$ threshold, given by the phase space factor $\delta_{ij}q_i/8\pi W$, with q_i the c.m. three momentum. This is included in the diagonal matrix $g(s)$ where $g(s)_i$ is the $i_{\rm th}$ channel unitarity bubble. The dispersion relation above is once subtracted so that we introduce a subtraction constant $\tilde{a}_i$ for each channel in the $g(s)_i$ function. In our problem, isospin symmetry reduces the number of subtraction constants from 10 to 6 [12], $\tilde{a}_1, \tilde{a}_2 = \tilde{a}_3 = \tilde{a}_4, \tilde{a}_5 = \tilde{a}_6, \tilde{a}_7, \tilde{a}_8$, and $\tilde{a}_9 = \tilde{a}_{10}$. Our $\tilde{a}_i$ satisfy $\tilde{a}_i \equiv a_i(\mu) - 2\log\mu + 1$, with $a_i(\mu)$ the subtraction constants in [7]. On the other hand, we keep the physical masses of mesons and baryons in the calculation of $g(s)_i$ which produces pronounced cusp effects. This is the only source of isospin breaking in our scattering amplitudes. The interacting kernel $\mathcal{T}(W)$ in (3) is a 10×10 symmetric matrix incorporating local and pole terms as well as crossed channel dynamics contributions in the dispersion relation for T^{-1}. The matrix $\mathcal{T}$ ($\mathcal{T} = \mathcal{T}_1 + \mathcal{T}_2 + \cdots$, where subindices indicate the chiral order) is fixed by matching (3) with the baryon CHPT amplitudes T_χ order by order [7]. At leading order, $\mathcal{O}(p)$, $\mathcal{T}_1 = T^{(1)}_\chi$ [7] while at NLO, $\mathcal{O}(p^2)$, $\mathcal{T}_2 = T^{(2)}_\chi$. The matching can be done to any arbitrary order and for $\mathcal{O}(p^3)$ or higher $\mathcal{T}_n \neq T^{(n)}_\chi$.

The data we include in our fits are the $\sigma(K^-p \to K^-p)$ elastic cross section, the charge exchange one, $\sigma(K^-p \to \bar{K}^0 n)$, and several hyperon production reactions, $\sigma(K^-p \to \pi^+\Sigma^-)$, $\sigma(K^-p \to \pi^-\Sigma^+)$, $\sigma(K^-p \to \pi^0\Sigma^0)$, and $\sigma(K^-p \to \pi^0\Lambda)$. In addition, we also fit the precisely measured ratios at the K^-p threshold:

$$\gamma = \frac{\sigma(K^-p \to \pi^+\Sigma^-)}{\sigma(K^-p \to \pi^-\Sigma^+)} = 2.36 \pm 0.04,$$
$$R_c = \frac{\sigma(K^-p \to \text{charged particles})}{\sigma(K^-p \to \text{all})} = 0.664 \pm 0.011, \quad (4)$$
$$R_n = \frac{\sigma(K^-p \to \pi^0\Lambda)}{\sigma(K^-p \to \text{all neutral states})} = 0.189 \pm 0.015;$$

see [7] for references. The first two ratios, being Coulomb corrected, are measured with 1.7% precision, i.e., of the same order as the expected isospin violation which we neither fully consider here nor was in [11]. Indeed, all the other observables we fit have uncertainties larger than 5%. Since we just include S-wave amplitudes and P waves start to contribute at higher momenta [24], we only include in the fit the K^-p cross sections low energy data points, namely, with laboratory frame K^- three-momentum $p_L \leq 0.2$ GeV. This also enhances the sensitivity to the lowest energy region in which we are particularly interested and where UCHPT is more suitable. We also include in the fits the $\pi^\pm\Sigma^\mp$ event distributions from [25] in average—this largely eliminates the $I = 1$ contribution. To calculate them we follow [7]. The number of data points included in each fit without the DEAR data is 94. Unless the opposite is stated, we also include in the fits the DEAR [9] measurement of the shift and width of the $1s$ kaonic hydrogen level

$$\Delta E = 193 \pm 37(\text{stat}) \pm 6(\text{syst})\ \text{eV}, \quad \Gamma = 249 \pm 111(\text{stat}) \pm 39(\text{syst})\ \text{eV}, \quad (5)$$

which is around a factor two more precise than the KEK [8] measurement, $\Delta E = 323 \pm 63 \pm 11$ eV and $\Gamma = 407 \pm 208 \pm 100$ eV. To calculate the shift and width of the $1s$ kaonic hydrogen state we use the results in [22] incorporating isospin breaking corrections. We compare them with the ones from the Deser formula [26]. We further constrain our fits by computing several πN observables calculated in baryon SU(3) CHPT at $\mathcal{O}(p^2)$ with the values of the low energy constants determined in the fit. Unitarity corrections in the πN sector are not as large as in the $S = -1$ sector and hence a calculation within pure SU(3) baryon

CHPT is more reliable. Thus, we calculate a^+_{0+}, the isospin-even S-wave scattering length, the pion-nucleon σ term $\sigma_{\pi N}$, and m_0 (from the value of the proton mass) at $\mathcal{O}(p^2)$. We do not consider the isospin-odd πN scattering length a^-_{0+} since at this order it is just given by g_A, in good agreement with experiment [27]. The $\sigma_{\pi N}$ term receives sizable higher order corrections from the mesonic cloud which are expected to be positive and around 10 MeV [28]. Since we evaluate it just at $\mathcal{O}(p^2)$, we enforce $\sigma_{\pi N} = 20$, 30, or 40 MeV in the fits ($\sigma_{\pi N} = 45 \pm 8$ MeV [29]). For the same reason, we enforce $m_0 = 0.7$ or 0.8 GeV ($m_0 = 0.77 \pm 0.11$ GeV from the second entry of Ref. [28] or $1.07 \gtrsim m_0 \gtrsim 0.71$ GeV [30]). We also include the value $a^+_{0+} = -(1 \pm 1) \times 10^{-2} m_\pi^{-1}$ in the fit procedure. This value results after considering its experimental one $a^+_{0+} = -(0.25 \pm 0.49) \times 10^{-2} m_\pi^{-1}$ [31] and the theoretical expectation of positive $\mathcal{O}(p^3)$ corrections around $+1 \times 10^{-2} m_\pi^{-1}$ from unitarity [27]. We stress that for all the fits we minimize strictly the χ^2, that is, each data point is weighted according to its experimental error. We do not include the data from [32] in the $\sigma(K^- p \to \pi^- \Sigma^+)$ cross section since they are incompatible with all the other data.

We typically find two classes of fits, namely, class A, which give rise to $1s$ kaonic hydrogen ΔE and Γ around the DEAR measurement, and class B fits, which are at variance with the DEAR measurement but close to the results derived from Martin's scattering lengths [10].

In Fig. 1, we show the shift and width of the $1s$ kaonic hydrogen state in the first panel and the cross sections and event distribution data in the rest of the panels. The solid and dashed lines correspond to the fits with $\sigma_{\pi N} = 40$ MeV and $m_0 = 0.8$ GeV, called A_4^+ and B_4^+, respectively—we discuss all the other fits in [24]. Since the fit B_4 strongly disagrees with the DEAR measurement, we include in this fit the KEK measurement and not the DEAR one. In the first panel of Fig. 1, the solid circle on the left is for A_4^+ while the solid one on the right is for B_4^+. The empty circle is obtained using the Deser formula [26] with the $K^- p$ scattering length from A_4^+. We observe a gentle correction to the Deser formula result when using the expression including the isospin breaking corrections from [22]. The downward triangle is the result of using Martin's scattering lengths [10] in [22]. The squares correspond to the fits with $\sigma_{\pi N} = 30$ MeV and $m_0 = 0.8$ GeV; for details see [24]. The isospin even πN scattering length results always around $-1 \times 10^{-2} m_\pi^{-1}$. The values for the ratios in (4) from the fit $A_4^+(B_4^+)$ are $\gamma = 2.36(2.36)$, $R_c = 0.628(0.655)$, and $R_n = 0.172(0.195)$. Both fits reproduce data remarkably well, even for higher energies than included in the fit. The fitted parameters from $A_4^+(B_4^+)$ are, in GeV units: $f = 0.080(0.089)$, $b_0 = -0.85(-0.32)$, $b_D = 0.71(-0.10)$, $b_F = -0.04(-0.31)$, $b_1 = 0.60(-0.19)$, $b_2 = 1.07(-0.27)$, $b_3 = -0.19(-0.15)$, $b_4 = -1.25(-0.28)$, $\tilde{a}_1 = 0.37(-0.05)$, $\tilde{a}_2 = 1.14(-0.54)$, $\tilde{a}_5 = 0.22(-1.08)$, $\tilde{a}_7 = 0.00(-0.05)$, $\tilde{a}_8 = 0.31(-0.54)$,

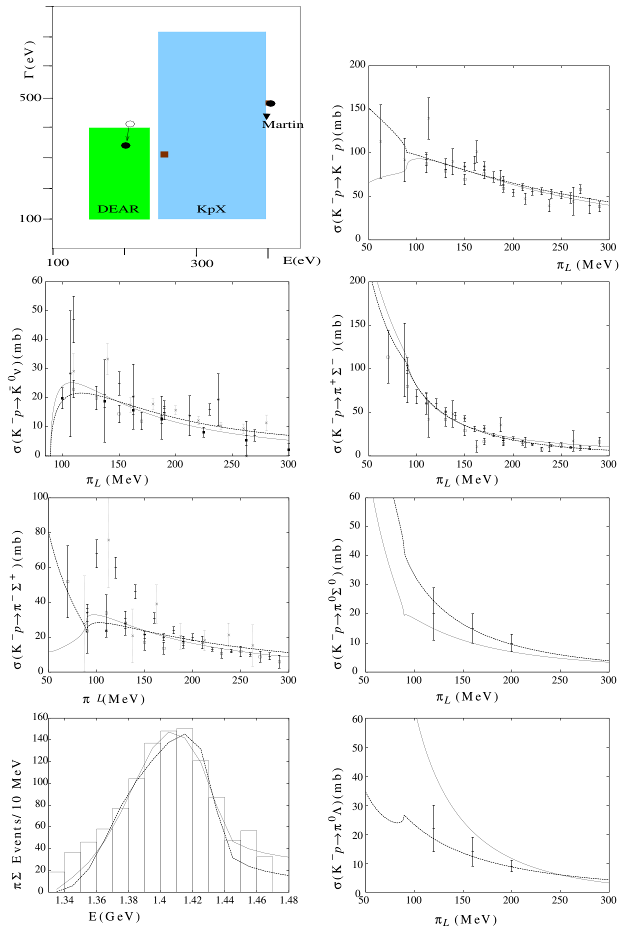

FIG. 1 (color online). First panel: $1s$ kaonic hydrogen strong energy shift and width. In the rest, the solid lines correspond to the fit A_4^+ and the dashed ones to B_4^+. For further details, see the text.

and $\tilde{a}_9 = 1.38(0.64)$. The b_0, b_D, and b_F values from the fit B_4^+ are close to the values obtained from an $\mathcal{O}(p^2)$ CHPT analysis of baryon $1/2^+$ masses, while for the fit A_4^+ this is not the case. Indeed, a pure CHPT calculation of the lightest octet baryon masses is subject to large higher order corrections, and modifications of the $\mathcal{O}(p^2)$ low energy constants can be effectively reabsorbed by changes in the higher order ones within natural bounds [30]. In connection with this, we must stress that we employ the $\mathcal{O}(p^2)$ couplings in UCHPT, which resums large contributions in this sector, so that there is no reason why the values should be the same as in CHPT.

The resulting $K^- p$ scattering length is $a_{K^- p} = (-0.51 + i0.42)$ fm for the fit A_4^+ and $(-1.01 + i0.80)$ fm for the fit B_4^+, i.e., a factor of 2 difference. Notice how the precise DEAR measurement places very severe constraints on the $\bar{K}N$ S-wave at threshold pointing to a less repulsive $K^- p$ interaction. Indeed, this is also reflected by the (two) $\Lambda(1405)$ pole positions which for the fit A_4^+ are at $(1321 - i43.5)$ and $(1402 - i39.6)$ MeV, around 30 to 40 MeV lower than the fit B_4^+ ones located at $(1361 - i29.9)$ and $(1433 - i31.7)$ MeV, respectively.

This is crucial for $\bar{K}$-nucleus potential calculations. We therefore also confirm the presence of two rather narrow poles conforming the $\Lambda(1405)$ [12,13] with this higher order calculation. We agree with the $K^- p$ scattering length in [33] although not for a_0 and a_1 separately. In the isospin limit, we get $a_0 = (-1.23 + i0.45)$ fm and $a_1 = (0.98 + i0.35)$ fm for the fit A_4^+ and $a_0 = (-1.63 + i0.81)$ fm and $a_1 = (-0.01 + i0.54)$ fm for the fit B_4^+, where subindices refer to the $\bar{K}N$ isospin. We notice that the isovector scattering length vanishes for the fit B_4^+ while is large and positive for A_4^+, so that $a_0 + a_1$ tends to cancel in this case.

The $\Lambda\pi$ S-wave and P-wave phase shifts difference at the Ξ^- mass has been recently determined from the measurement of the $\Xi^- \to \Lambda\pi^-$ decay parameters. The results are $\delta_P - \delta_S = (4.6 \pm 1.4 \pm 1.2)°$ [34] and $(3.2 \pm 5.3 \pm 0.7)°$ [35]. Neglecting the tiny P-wave phase shift [36], we obtain 2.5° for the fit A_4^+ and 0.2° for the fit B_4^+. Again the fit A_4 is in better agreement with the new $S = -1$ meson-baryon scattering data.

In summary, we have presented a NLO analysis of S-wave $\bar{K}N$ scattering within UCHPT, that combines the second order SU(3) CHPT meson-baryon amplitudes with a dispersion relation for the inverse of a partial wave amplitude [7]. We have emphasized the strong constraints that these precise data impose on the $\bar{K}N$ S-wave scattering amplitudes, implying a less repulsive $K^- p$ interaction at threshold. This manifests in lower values for the two $\Lambda(1405)$ resonance poles, whose presence we confirm at NLO. As a novelty we find a class of fits (class A) which shows consistency between the DEAR and scattering data, both old and new [34,35]. Further exciting developments are foreseeable with the DEAR/SIDDHARTA experiment [37] which aims at an eV level measurement of the shift and width of kaonic hydrogen.

We would like to thank Daisuke Jido, Ulf-G. Meißner, Eulogio Oset, and Akaki Rusetsky for useful discussions. J. A. O. is grateful to Jürg Gasser for suggesting him this problem. Financial support by the European Commission (EC) RTN Program Network "EURIDICE" Contract No. HPRN-CT-2002-00311, the HadronPhysics I3 Project (EC) Contract No. RII3-CT-2004-506078 (J. A. O.), by MEC (Spain) and FEDER (EC) Grants No. FPA2004-03470 (J. A. O. and M. V.) and No. FPA2003-09298-C02-01 (J. P. and M. V.) is acknowledged.

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