Resolving the Discrepancy of 135 MeV *pd* Elastic Scattering Cross Sections and Relativistic Effects

K. Sekiguchi,[1] H. Sakai,[1,2,3] H. Witała,[2,4] W. Glöckle,[5] J. Golak,[4] K. Hatanaka,[6] M. Hatano,[2] K. Itoh,[7] H. Kamada,[8] H. Kuboki,[2] Y. Maeda,[3] A. Nogga,[9] H. Okamura,[10] T. Saito,[2] N. Sakamoto,[1] Y. Sakemi,[6] M. Sasano,[2] Y. Shimizu,[6] K. Suda,[3] A. Tamii,[6] T. Uesaka,[3] T. Wakasa,[11] and K. Yako[2]

[1]*RIKEN, the Institute of Physical and Chemical Research, Wako, Saitama 351-0198, Japan*
[2]*Department of Physics, University of Tokyo, Bunkyo, Tokyo 113-0033, Japan*
[3]*Center for Nuclear Study, University of Tokyo, Bunkyo, Tokyo 113-0033, Japan*
[4]*Institute of Physics, Jagiellonian University, PL-30059 Cracow, Poland*
[5]*Institut für theoretische Physik II, Ruhr-Universität Bochum, D-44780 Bochum, Germany*
[6]*Research Center for Nuclear Physics, Osaka University, Ibaraki, Osaka 567-0047, Japan*
[7]*Department of Physics, Saitama University, Saitama 338-8570, Japan*
[8]*Department of Physics, Faculty of Engineering, Kyushu Institute of Technology, Kitakyushu 804-8550, Japan*
[9]*Institut für Kernphysik, Forschungszentrum, Jülich, D-52425 Jülich, Germany*
[10]*Cyclotron and Radioisotope Center, Tohoku University, Sendai, Miyagi 980-8578, Japan*
[11]*Department of Physics, Kyushu University, Fukuoka 812-8581, Japan*
(Received 30 January 2005; published 12 October 2005)

Three precise measurements for elastic *pd* scattering at 135 MeV/*A* have been performed with the three different experimental setups. The cross sections are described well by the theoretical predictions based on modern nucleon-nucleon forces combined with three-nucleon forces. Relativistic Faddeev calculations show that relativistic effects are restricted to backward angles. This result supports the two measurements recently reported by RIKEN and contradicts the KVI data.

DOI: 10.1103/PhysRevLett.95.162301 PACS numbers: 21.30.−x, 21.45.+v, 24.10.−i, 24.70.+s

A hot topic of present day few-nucleon system studies is to explore the properties of three-nucleon forces (3*N*Fs) acting in systems with more than $A = 2$ nucleons. These forces appear for the first time in the three-nucleon (3*N*) system where they provide an additional contribution to a predominantly pairwise potential energy of three nucleons. A meson-exchange picture can undoubtedly lead to such forces; however, they are relatively weak compared to nucleon-nucleon (*NN*) forces, and therefore it is hard to approach and also to find evidences for them experimentally. The first evidence of the 3*N*F is found in the 3*N* bound systems, ^{3}H and ^{3}He. The binding energies of these nuclei are not described by exact solutions of the three-nucleon Faddeev equations employing modern *NN* forces, e.g., AV18 [1], CDBonn [2], Nijmegen I, II and 93 [3] (see, e.g., Ref. [4]). The discrepancy between data and theory is explained by adding 3*N*F, mostly based on a 2π exchange between three nucleons with the Δ-isobar excitation, such as the Tucson-Melbourne (TM) [5] and the Urbana IX 3*N*F [6]. The binding energies show the significant contributions of the 3*N*F; however, they could only constrain the overall strength. In order to study the momentum and/or spin dependence of the 3*N*F, the 3*N* scattering is one attractive approach; others are to study the spectra of nuclear systems up to $A = 10$ [7].

An indication of 3*N*F for the 3*N* scattering was first pointed out in the cross section minima for nucleon-deuteron (*Nd*) elastic scattering at energies of the incoming nucleon above ≈ 60 MeV by Witala *et al.* in 1998 [8]. Since then experimental studies of higher-energy proton-deuteron (*pd*) elastic scattering covering incident energies of up to ≈ 250 MeV have been performed extensively and provided precise data of cross sections [9–13] and spin observables, such as analyzing powers [9–11,14], spin correlation coefficients [15], and polarization transfer coefficients [12,16]. Precise cross section data for the elastic *dp* scattering taken at RIKEN with 135 MeV/*A* deuterons have shown large disagreement between data and rigorous Faddeev calculations with modern *NN* forces [9–11]. Combination of these *NN* forces and 3*N*Fs such as the TM99 [17] (TM99 is a version of the TM force which is more consistent with chiral symmetry [18,19]) and the Urbana IX removed this discrepancy and led to a good description of the measured cross sections. This result can be taken as a clear signature of the 3*N*F effects in *Nd* elastic scattering. However, spin observables are not always explained by the addition of the 3*N*Fs, showing the defects in spin parts of the 3*N*Fs [16].

The recent measurement of the elastic *pd* scattering with a 135 MeV proton beam and a mixed solid CD_2-CH_2 target at KVI Groningen [13] provided cross sections which were in disagreement with the data at RIKEN. The KVI data were larger than the RIKEN data by about 10–40% and also differed in shape. *If* the KVI data were correct, the presently available 2π exchange 3*N*Fs would be insufficient to explain the difference. Therefore one should look for other sources which have not been considered to fill the difference, such as 3*N*Fs other than 2π exchange types, and/or relativistic effects, or something completely new. According to the recent theoretical predictions in the framework of the coupled channel approach [20], the π-ρ and ρ-ρ type 3*N*Fs cause little effect. Therefore relativistic effects could be a candidate to fill the difference. We estimate for the first time the magnitude of relativistic effects in the 3*N* scattering. Of highest importance, however, is to clarify by experiment which are correct, the RIKEN data or the KVI data.

Aiming to resolve the discrepancy by experiment, we performed the following three measurements for elastic *pd* scattering. First, we made a measurement at RIKEN with the proton beam and a CD_2-CH_2 sandwiched solid target at the angles where the *pp* and *pd* elastic scattering were simultaneously measured with the magnetic spectrograph SMART. Using the well-known elastic *pp* cross sections we can estimate the overall systematic uncertainty for the *pd* cross section. We used the H_2^+ ions of 270 MeV as the 135 MeV proton beam for convenience of acceleration. Second, to confirm the angular distribution we measured again with 135 MeV/*A* deuterons. This measurement was performed just after the previous *pp* scattering experiment with the same experimental setup in order to minimize the systematic uncertainties. Note the mass of H_2^+ is almost identical to that of deuteron so that we did not need to change any parameters of the accelerators or beam transport system. We tried to check the fluctuations of the target thickness during the experiment by measuring the *dp* scattering at the fixed angle $\theta_{c.m.} = 69.7°$, where the scattered deuterons and recoil protons were detected in coincidence in the scattering chamber. For the same purpose, the cross section at $\theta_{c.m.} = 165.1°$ was measured with the SMART system over several times during the experiment. We also measured the carbon background events which were not obtained in the previous measurement [10,11]. Last, we performed a totally independent measurement at the Research Center for Nuclear Physics (RCNP) of Osaka University, using a 135 MeV proton beam and deuterated polyethylene target. The absolute normalization of the cross sections has been performed by taking data with a D_2 gas target and the double-slit system for which the RCNP group has already established the procedure to obtain the absolute *pd* cross section [12]. In this paper we would like to present these new data and compare them to the old ones and to theoretical predictions including relativistic effects to clarify firmly the 3*N*F effects.

The measurement of the cross sections with 270 MeV H^{2+} beam (*p* on ^{2}H) was carried out at the RIKEN accelerator research facility with the SMART system. The target was a sandwich of the polyethylene (CH_2) with thickness of 18.7 mg/cm^2 and the self-supporting 99% isotopically enriched deuterated polyethylene foil (CD_2) with thickness of 21 mg/cm^2 [12,21]. The relative deviation of the CD_2 target thickness was estimated to be within about 2.5% and was attributed to the inhomogeneity of the CD_2 foil. At the laboratory angles 10°–14° which correspond to the c.m. angles 16°–20° for the elastic *pd* scattering the momentum difference of the scattered protons from the *pp* and *pd* elastic scattering is within 4%. In this angular range the protons emitted from each of these reactions were measured simultaneously with the SMART spectrograph and their energy spectra were completely resolved. The yields for the elastic *pd* scattering were obtained by subtracting carbon contributions in the excitation energy spectra. The elastic *pp* scattering yields were obtained by subtracting the backgrounds from the $p + d \rightarrow p + p + n$ breakup reaction and from the proton scattering on the carbon. For interpolation purposes the breakup background contribution was assumed to be a third-order polynomial in the proton energy. The measured cross sections for *pp* elastic scattering were compared with the values calculated by the phase-shift analysis code SAID [22] and found to be consistent within 2%. Thus we estimated the overall systematic uncertainty of the measured cross section data to be 3% at most.

We made the cross section measurement with 270 MeV deuteron beam (*d* on ^{1}H) in the angular range $\theta_{c.m.} = 10°–180°$. The CH_2 target used in the proton beam experiment was employed as a hydrogen target. For the forward scattering ($\theta_{c.m.} \leq 90°$) the scattered deuterons were detected, while for the backward scattering ($\theta_{c.m.} \geq 90°$) the recoil protons were measured. The statistical errors of the cross sections are within 1.6%. The fluctuation of the target thickness is within 3%. The uncertainty due to carbon background subtraction is less than 5%. The overall systematic uncertainties which include also the uncertainties by *pp* experiment are estimated to be 6%.

The experiment performed at RCNP used a proton beam in conjunction with the high resolution spectrometer Grand Raiden. The proton beam was accelerated up to 135 MeV by the AVF and ring cyclotrons and bombarded the same CD_2 foil we used in the experiment at RIKEN. The proton beam was stopped in a Faraday cup in the scattering chamber, except for the gaseous target measurement at the angle $\theta_{lab} = 25.5°$. In this case the beam was stopped in a Faraday cup located downstream outside the scattering chamber. The scattered protons or deuterons were momentum analyzed by the Grand Raiden. The protons were measured at the angles $\theta_{c.m.} \leq 90°$ and the recoil deuterons were detected at the angles $\theta_{c.m.} \geq 90°$. The measured angles were $\theta_{c.m.} = 17.0°–157.7°$. The yields from D_2 were obtained by subtracting carbon contributions in the excitation energy spectra. To normalize cross sections taken with the CD_2 target a measurement with a D_2 gas target was performed at the laboratory angles 25.5° and 60° which corresponded to the c.m. angles 39.0° and 87.4°, respectively. The D_2 gas target was contained in the cell of a cylinder of 40 mm diameter made of 200 μm-thick aluminum. The absolute gas pressure was continuously monitored with a precision better than 0.2%.

The temperature of the target cell was checked during the measurement and kept at room temperature. A double-slit system was used to determine precisely the target volume and the solid angle of the Grand Raiden spectrometer. The effective target thickness and the solid angle were calculated by Monte Carlo simulations. Spectra with the empty gaseous cell were also measured to determine background contributions from the aluminum cell. An additional measurement was performed with hydrogen gas replacing the deuteron one in order to cross check the experimental setup at the angle $\theta_{\text{lab.}} = 25.5°$. The measured cross section of pp scattering is consistent within 3% of the results calculated by the SAID program. The statistical errors of the pd elastic scattering cross sections are smaller than 1.4%. The absolute normalization was estimated to be 3% by elastic pp scattering measurement. The uncertainty due to carbon background subtraction for the excitation energy spectrum is 3%. There is also the uncertainty of 2.5% attributed to the inhomogeneity of the CD_2 foil. The overall systematic uncertainties are estimated to be 5% at most.

All the experimental results are shown in Fig. 1. The data taken at RCNP are shown with solid diamonds. The open squares (circles) are the data measured with the proton (deuteron) beam at RIKEN. The data published in Refs. [9–11] are shown with open diamonds. The KVI data reported in Ref. [13] are shown with open triangles. Only statistical errors are presented. The very good agreement between the independent measurements allows us to conclude that the systematic uncertainty due to the detection setup is small. Comparison of the new three sets of data to the previously reported ones [9–11] supports the previous measurements and shows a clear disagreement with the KVI data [13].

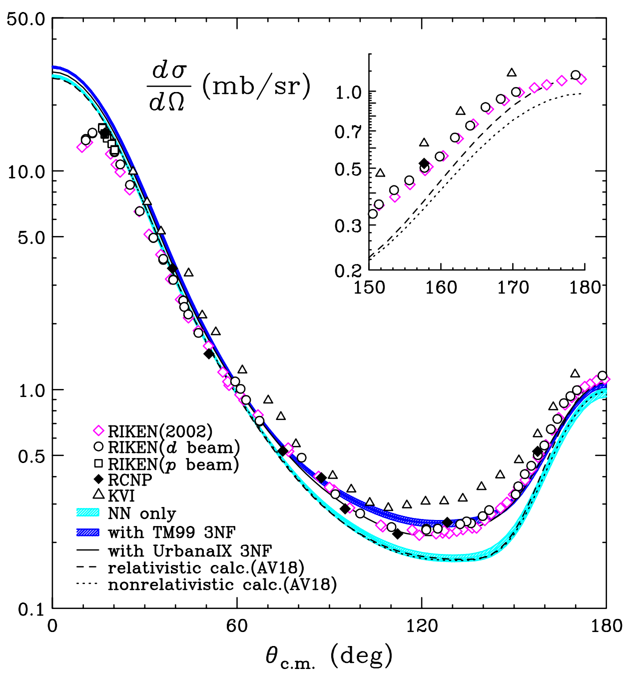

FIG. 1 (color online). Differential cross section for elastic Nd scattering at 135 MeV/nucleon. The light shaded band contains NN force predictions (AV18, CDBonn, Nijmegen I and II). The dark shaded band results when they are combined with the TM99 3NF. The solid line is the AV18 + Urbana IX prediction. The dashed and dotted lines are the results of relativistic and nonrelativistic Faddeev calculations based on the NN forces AV18. The symbols are data from different measurements (see text).

In Fig. 1 we compare the data to theoretical predictions based on modern NN forces and their combinations with 3NFs [23–26]. We used the modern NN forces AV18, CDBonn, and Nijmegen I and II combined with the 3NF TM99 [17] with the cutoff Λ values which lead for a particular NN force combined with the 3NF TM99 to a reproduction of the ^{3}H binding energy. In case of the AV18 NN force we also combined it with the 3NF Urbana IX.

As is seen in Fig. 1 various NN force predictions (light shaded band) clearly underestimate the cross section data for the angles $\theta_{\text{c.m.}} \geq 90°$. The narrow band of predictions reflects the weak dependence on the choice of the nearly on-shell equivalent NN interaction. The inclusion of the TM99 (the dark shaded band) or, in the case of AV18, of the 3NF Urbana IX (solid line), leads to a very good description of the data.

In view of such a good description of the cross section it is interesting to find out how large relativistic effects are at 135 MeV/A. This was done assuming that only NN forces are acting. We followed a formalism for treating the relativistic three-body Faddeev equations of Ref. [27] with a boosted two-nucleon potential V expressed in terms of the relativistic potential v given in the NN c.m. system,

$$V(\vec{P}) \equiv \sqrt{[\omega(\vec{k}) + v]^2 + \vec{P}^2} - \sqrt{\omega(\vec{k})^2 + \vec{P}^2}. \tag{1}$$

The momentum $\vec{P}$ is the total momentum of the two-nucleon system, and $\vec{k}$ and $-\vec{k}$ are the individual momenta of the nucleons in their NN c.m. ($\omega(\vec{k}) = 2\sqrt{\vec{k}^2 + m^2}$). We did not calculate the matrix elements of the boosted potential in all its complexity [28] but restricted only to the leading order terms in a P/ω and v/ω expansion,

$$V(\vec{k}, \vec{k}'; \vec{P}) = v(\vec{k}, \vec{k}')\left\{1 - \frac{\vec{P}^2}{2\omega(\vec{k})\omega(\vec{k}')}\right\}. \tag{2}$$

We checked the quality of the approximation of Eq. (2) by calculating the deuteron wave function $\phi_d(\vec{k})$ when the deuteron is moving with a momentum corresponding to 135 MeV/A. The resulting deuteron binding energy and the deuteron D-state probability for the deuteron in such a motion are close to the values for the deuteron at rest. Choosing the exact expression Eq. (1) those properties come out exactly. A relativistic potential v was generated from the nonrelativistic NN potential AV18 by performing the scale transformation of Ref. [29]. This should be improved in future studies by allowing for dynamically dictated relativistic features in the potential. Using $V(\vec{k}, \vec{k}'; \vec{P})$ the boosted t-matrix elements $t(\vec{k}, \vec{k}'; \vec{P})$ are calculated and they form the dynamical input for the $3N$ Faddeev equation with the relativistic form of the free propagator G_0 [30]. To solve this equation in the relativistic case it is most convenient to use instead of the standard Jacobi momenta [23] the relative momentum $\vec{k}$ in the NN c.m. subsystem and the momentum $\vec{q}$ of the spectator nucleon in the $3N$ c.m. system. In the nonrelativistic limit the momentum $\vec{k}$ reduces to the standard Jacobi momentum $\vec{p}$ [26]. The relativistic formulation applied is of the Bakamjian-Thomas type and belongs to the instant form of relativistic dynamics [31].

Nowadays partial wave decomposition is still required to solve numerically $3N$ Faddeev equations. The standard partial wave states [26], however, are generalized due to the choice of the NN-subsystem momentum $\vec{k}$ and the total spin s both defined in the NN c.m. system. This lead to Wigner spin rotations when boosting to the $3N$ c.m. system [31,32], resulting in a more complex form for the permutation matrix element [32] than used in Ref. [26]. The details of our relativistic formulation and its numerical performance are given in Ref. [32].

A restricted relativistic calculation with $j < 2$ partial waves states showed that Wigner spin rotations have only negligible effects on the cross section at 135 MeV/A. Thus when performing the fully converged calculation ($j \leq 5$, $J \leq 25/2$) we neglected the Wigner rotations completely. The resulting cross sections are shown in Fig. 1. It is seen that the effects of relativity are visible only in the backward angular region for $\theta_{\mathrm{c.m.}} \geq 160°$ where they increase the cross section by up to about 15%. For $\theta_{\mathrm{c.m.}} < 160°$ the effects of relativity are practically negligible.

Summarizing, we performed measurements of the cross sections for elastic pd scattering using 135 MeV proton and 270 MeV deuteron beams. Present experimental arrangements allowed us to get precise cross section data. The three sets of data presented here taken with three different experimental setups completely support the previous measurement and disagree with the KVI data. The agreement of our new sets and our old set, measured with different experimental setups, gives confidence that the systematic errors are small.

Comparison of our data with theory based on different NN forces combined with current $3N$Fs revealed clear evidence for the action of $3N$Fs. The discrepancies between the 135 MeV cross section data and the pure two-nucleon force predictions can be removed by including the $3N$Fs TM99 or Urbana IX. The conclusion that $3N$F effects are seen in the region of the cross section minimum is further supported by including relativity in the instant form of relativistic dynamics as proposed by Bakamjian and Thomas. This leads to small relativistic effects at backward angles, but negligible contributions in the minimum, leaving $3N$Fs as the only plausible mechanism to resolve the discrepancies between NN theory and data. Our results clearly indicate the usefulness of Nd elastic scattering for the study of $3N$Fs. Nd elastic scattering cross sections together with spin observables at higher energies, where there are still discrepancies, will provide an important information to test forthcoming additional $3N$ force mechanisms.

This work was supported financially in part by the Grants-in-Aid for Scientific Research No. 04402004 and No. 10304018 of the Ministry of Education, Culture, Sports, Science, and Technology of Japan, and by the Polish Committee for Scientific Research under Grant No. 2P03B00825, and by US-DOE grants No. DE-FC02-01ER41187 and No. DE-FG02-00ER41132. The numerical calculations have been performed on the CRAY SV1 and the CRAY T3E of the NIC in Jülich, Germany.

[1] R. B. Wiringa *et al.*, Phys. Rev. C **51**, 38 (1995).
[2] R. Machleidt, Phys. Rev. C **63**, 024001 (2001).
[3] V. G. J. Stoks *et al.*, Phys. Rev. C **49**, 2950 (1994).
[4] A. Nogga *et al.*, Phys. Rev. C **65**, 054003 (2002).
[5] S. A. Coon *et al.*, Nucl. Phys. **A317**, 242 (1979).
[6] B. S. Pudliner *et al.*, Phys. Rev. C **56**, 1720 (1997).
[7] S. C. Pieper *et al.*, Phys. Rev. C **66**, 044310 (2002).
[8] H. Witała *et al.*, Phys. Rev. Lett. **81**, 1183 (1998).
[9] N. Sakamoto *et al.*, Phys. Lett. B **367**, 60 (1996).
[10] H. Sakai *et al.*, Phys. Rev. Lett. **84**, 5288 (2000).
[11] K. Sekiguchi *et al.*, Phys. Rev. C **65**, 034003 (2002).
[12] K. Hatanaka *et al.*, Phys. Rev. C **66**, 044002 (2002).
[13] K. Ermisch *et al.*, Phys. Rev. C **68**, 051001 (2003).
[14] E. J. Stephenson *et al.*, Phys. Rev. C **60**, 061001 (1999); R. Bieber *et al.*, Phys. Rev. Lett. **84**, 606 (2000); K. Ermisch *et al.*, *ibid.* **86**, 5862 (2001).
[15] R. V. Cadman *et al.*, Phys. Rev. Lett. **86**, 967 (2001).
[16] K. Sekiguchi *et al.*, Phys. Rev. C **70**, 014001 (2004).
[17] S. A. Coon and H. K. Han, Few-Body Syst. **30**, 131 (2001).
[18] J. L. Friar *et al.*, Phys. Rev. C **59**, 53 (1999).
[19] D. Hüber *et al.*, Few-Body Syst. **30**, 95 (2001).
[20] A. Deltuva *et al.*, Phys. Rev. C **68**, 024005 (2003).
[21] Y. Maeda *et al.*, Nucl. Instrum. Methods Phys. Res., Sect. A **490**, 518 (2002).
[22] R. A. Arndt *et al.*, Phys. Rev. C **56**, 3005 (1997), and references therein.
[23] H. Witała *et al.*, Few-Body Syst. **3**, 123 (1988).
[24] D. Hüber *et al.*, Acta Phys. Pol. B **28**, 1677 (1997).
[25] D. Hüber *et al.*, Few-Body Syst. **14**, 171 (1993).
[26] W. Glöckle *et al.*, Phys. Rep. **274**, 107 (1996).
[27] W. Glöckle *et al.*, Phys. Rev. C **33**, 709 (1986); F. Coester, Helv. Phys. Acta **38**, 7 (1965).
[28] H. Kamada *et al.*, Phys. Rev. C **66**, 044010 (2002).
[29] H. Kamada and W. Glöckle, Phys. Rev. Lett. **80**, 2547 (1998).
[30] H. Kamada, Few-Body Syst., Suppl. **12**, 433 (2000).
[31] B. D. Keister and W. N. Polyzou, Adv. Nucl. Phys. **20**, 225 (1991).
[32] H. Witała *et al.*, Phys. Rev. C **71**, 054001 (2005).