General Nonextremal Rotating Black Holes in Minimal Five-Dimensional Gauged Supergravity

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We construct the general solution for nonextremal charged rotating black holes in five-dimensional minimal gauged supergravity. They are characterized by four nontrivial parameters: namely, the mass, the charge, and the two independent rotation parameters. The metrics in general describe regular rotating black holes, providing the parameters lie in appropriate ranges so that naked singularities and closed timelike curves (CTCs) are avoided. We calculate the conserved energy, angular momenta, and charge for the solutions, and show how supersymmetric solutions arise in a Bogomol'nyi-Prasad-Sommerfield limit. These have naked CTCs in general, but for special choices of the parameters we obtain new regular supersymmetric black holes or smooth topological solitons.

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The discovery of the remarkable anti–de Sitter/conformal field theory (AdS/CFT) correspondence showed that bulk properties of solutions in the five-dimensional gauged supergravities that result from compactification of the type IIB string are related to properties of strongly coupled conformal field theories on the four-dimensional boundary of five-dimensional anti–de Sitter spacetime [1–3]. It therefore becomes of great importance to study the solutions of the five-dimensional gauged supergravity theories. One of the most important classes of such solutions are those that describe black holes in five dimensions. In particular, it has been argued that the boundary conformal field theory dual to rotating five-dimensional black holes should describe a system in a four-dimensional rotating Einstein universe [4].

The rotating five-dimensional solutions found in [4] were neutral Kerr–(anti-)de Sitter black holes. In order to be able to make contact with supersymmetric Bogomol'nyi-Prasad-Sommerfield (BPS) configurations, for which the AdS/CFT correspondence is more solidly founded, it is of considerable interest to generalize the neutral solutions to include electric charge too. In the analogous problem in ungauged supergravity, it is straightforward to generate charged solutions from neutral ones, by using the global symmetries of the ungauged supergravities as solution-generating transformations. By this means, the general charged rotating black holes of five-dimensional ungauged supergravity were obtained in [5], starting from the neutral rotating Ricci-flat black holes found in [6]. For the solutions in gauged supergravity there are no surviving global symmetries that can be used to provide solution-generating transformations, and one has little option but to resort to brute-force calculations, starting from an appropriate ansatz, to construct the charged rotating solutions. One way to simplify the problem is to specialize to the case where the two independent rotation parameters of the generic five-dimensional rotating black hole are set equal, since this reduces the problem from cohomogeneity-2, with partial differential equations, to cohomogeneity-1, with ordinary differential equations. Supersymmetric rotating black holes with two equal angular momenta were obtained in [7], and it was shown in [7] that the rotation is necessary for the solution to be free of naked singularities and closed timelike curves (CTCs). The nonextremal charged rotating solutions of gauged five-dimensional supergravity with equal rotation parameters were constructed in [8,9]. Recently, some special cases involving unequal rotation parameters were also constructed in [10]. However, these latter arose as solutions of $\mathcal{N} = 2$ gauged supergravity coupled to two vector multiplets, with a specific relation between the three electric charges, and did not, in general, admit a specialization to solutions of pure minimal $\mathcal{N} = 2$ gauged supergravity. The purpose of this Letter is to present the general solution for charged rotating nonextremal black holes in minimal five-dimensional gauged supergravity, with independent rotation parameters in the two orthogonal 2-planes (two-dimensional planes).

We have found the general solution for charged rotating black holes in five-dimensional minimal gauged supergravity, with unequal angular momenta, by a process involving a considerable amount of trial and error, followed by an explicit verification that the equations of motion are satisfied. In doing this, we have been guided by the previously obtained special case found in [8], where the two angular momenta were set equal, and the general charged rotating solutions in ungauged minimal supergravity, which are contained within the results in [5]. In this Letter, we begin by presenting our new solutions, and then we calculate the conserved angular momenta and electric charge. By integrating the first law of thermodynamics, we also obtain the conserved mass, or energy, of the solutions. By considering the conditions under which the anticommutator of supercharges in the AdS superalgebra has zero eigenvalues, we then show how a BPS limit of our general nonextremal solutions gives rise to new supersymmetric configurations. These include new supersymmetric rotating black holes, with two independently specifiable angular momenta, and new topological solitons that are nonsingular on complete manifolds.

In terms of Boyer-Lindquist–type coordinates $x^\mu = (t, r, \theta, \phi, \psi)$ that are asymptotically static (i.e., the coordinate frame is nonrotating at infinity), we find that the metric and gauge potential for our new rotating solutions can be expressed as

$$\begin{aligned} ds^2 = & -\frac{\Delta_\theta[(1+g^2r^2)\rho^2 dt + 2q\nu]dt}{\Xi_a\Xi_b\rho^2} + \frac{2q\nu\omega}{\rho^2} \\ & + \frac{f}{\rho^4}\left(\frac{\Delta_\theta dt}{\Xi_a\Xi_b} - \omega\right)^2 + \frac{\rho^2 dr^2}{\Delta_r} + \frac{\rho^2 d\theta^2}{\Delta_\theta} \\ & + \frac{r^2+a^2}{\Xi_a}\sin^2\theta d\phi^2 + \frac{r^2+b^2}{\Xi_b}\cos^2\theta d\psi^2, \end{aligned} \quad (1)$$

$$A = \frac{\sqrt{3}q}{\rho^2}\left(\frac{\Delta_\theta dt}{\Xi_a\Xi_b} - \omega\right), \quad (2)$$

where

$$\begin{aligned} \nu &= b\sin^2\theta d\phi + a\cos^2\theta d\psi, \\ \omega &= a\sin^2\theta\frac{d\phi}{\Xi_a} + b\cos^2\theta\frac{d\psi}{\Xi_b}, \\ \Delta_\theta &= 1 - a^2g^2\cos^2\theta - b^2g^2\sin^2\theta, \\ \Delta_r &= \frac{(r^2+a^2)(r^2+b^2)(1+g^2r^2) + q^2 + 2abq}{r^2} - 2m, \\ \rho^2 &= r^2 + a^2\cos^2\theta + b^2\sin^2\theta, \\ \Xi_a &= 1 - a^2g^2, \quad \Xi_b = 1 - b^2g^2, \\ f &= 2m\rho^2 - q^2 + 2abqg^2\rho^2. \end{aligned} \quad (3)$$

A straightforward calculation shows that these configurations solve the equations of motion of minimal gauged five-dimensional supergravity, which follow from the Lagrangian

$$\mathcal{L} = (R + 12g^2) * \mathbb{1} - \frac{1}{2} * F \wedge F + \frac{1}{3\sqrt{3}} F \wedge F \wedge A, \quad (4)$$

where $F = dA$, and g is assumed to be positive, without loss of generality.

For some purposes, it is useful to note that the nonvanishing metric components are given by

$$\begin{aligned} g_{00} &= -\frac{\Delta_\theta(1+g^2r^2)}{\Xi_a\Xi_b} + \frac{\Delta_\theta^2(2m\rho^2 - q^2 + 2abqg^2\rho^2)}{\rho^4\Xi_a^2\Xi_b^2}, \\ g_{03} &= -\frac{\Delta_\theta[a(2m\rho^2 - q^2) + bq\rho^2(1+a^2g^2)]\sin^2\theta}{\rho^4\Xi_a^2\Xi_b}, \\ g_{04} &= -\frac{\Delta_\theta[b(2m\rho^2 - q^2) + aq\rho^2(1+b^2g^2)]\cos^2\theta}{\rho^4\Xi_b^2\Xi_a}, \\ g_{33} &= \frac{(r^2+a^2)\sin^2\theta}{\Xi_a} + \frac{a[a(2m\rho^2 - q^2) + 2bq\rho^2]\sin^4\theta}{\rho^4\Xi_a^2}, \\ g_{44} &= \frac{(r^2+b^2)\cos^2\theta}{\Xi_b} + \frac{b[b(2m\rho^2 - q^2) + 2aq\rho^2]\cos^4\theta}{\rho^4\Xi_b^2}, \\ g_{34} &= \frac{[ab(2m\rho^2 - q^2) + (a^2+b^2)q\rho^2]\sin^2\theta\cos^2\theta}{\rho^4\Xi_a\Xi_b}, \\ & g_{11} = \frac{\rho^2}{\Delta_r}, \qquad g_{22} = \frac{\rho^2}{\Delta_\theta}. \end{aligned} \quad (5)$$

The Killing vector

$$\ell = \frac{\partial}{\partial t} + \Omega_a\frac{\partial}{\partial\phi} + \Omega_b\frac{\partial}{\partial\psi} \quad (6)$$

becomes null on the outer Killing horizon at $r = r_+$, the largest positive root of $\Delta_r = 0$, where the angular velocities on the horizon are given by

$$\begin{aligned} \Omega_a &= \frac{a(r_+^2+b^2)(1+g^2r_+^2) + bq}{(r_+^2+a^2)(r_+^2+b^2) + abq}, \\ \Omega_b &= \frac{b(r_+^2+a^2)(1+g^2r_+^2) + aq}{(r_+^2+a^2)(r_+^2+b^2) + abq}. \end{aligned} \quad (7)$$

One can then easily evaluate the surface gravity

$$\kappa = \frac{r_+^4[(1 + g^2(2r_+^2 + a^2 + b^2)] - (ab+q)^2}{r_+[(r_+^2+a^2)(r_+^2+b^2) + abq]}, \quad (8)$$

and hence the Hawking temperature $T = \kappa/(2\pi)$. The entropy is given by

$$S = \frac{\pi^2[(r_+^2+a^2)(r_+^2+b^2) + abq]}{2\Xi_a\Xi_b r_+}. \quad (9)$$

The angular momenta can be evaluated from the Komar integrals $J = 1/(16\pi)\int_{S^3} *dK$, where $K = \partial/\partial\phi$ or $K = \partial/\partial\psi$, yielding

$$\begin{aligned} J_a &= \frac{\pi[2am + qb(1+a^2g^2)]}{4\Xi_a^2\Xi_b}, \\ J_b &= \frac{\pi[2bm + qa(1+b^2g^2)]}{4\Xi_b^2\Xi_a}. \end{aligned} \quad (10)$$

The electric charge follows from the Gaussian integral $Q = 1/(16\pi)\int_{S^3}(*F - F \wedge A/\sqrt{3})$, yielding

$$Q = \frac{\sqrt{3}\pi q}{4\Xi_a \Xi_b}. \tag{11}$$

Using the technique introduced in [11], the easiest way to calculate the conserved mass, or energy, is to integrate the first law of thermodynamics $dE = TdS + \Omega_a dJ_a + \Omega_b dJ_b + \Phi dQ$, where $\Phi = \ell^\mu A_\mu$ is the electrostatic potential on the horizon. Doing this, we find

$$E = \frac{m\pi(2\Xi_a + 2\Xi_b - \Xi_a \Xi_b) + 2\pi q a b g^2(\Xi_a + \Xi_b)}{4\Xi_a^2 \Xi_b^2}. \tag{12}$$

The BPS limit can be found by looking at the eigenvalues of the Bogomol'nyi matrix coming from the anticommutators of the supercharges, as discussed in [12]. We have BPS solutions if $E - gJ_a - gJ_b - \sqrt{3}Q = 0$. (We have, without loss of generality, made specific sign choices here.) From the expressions for (E, J_a, J_b, Q), we find that the BPS limit is then achieved if

$$q = \frac{m}{1 + (a+b)g}. \tag{13}$$

The supersymmetry of the solutions in this limit can be confirmed by calculating the norm of the Killing vector

$$K_+ \equiv \frac{\partial}{\partial t} + g\frac{\partial}{\partial \phi} + g\frac{\partial}{\partial \psi}, \tag{14}$$

which, as discussed in [12], arises as the square of the Killing spinor η, in the sense that $K_+^\mu = \bar{\eta}\gamma^\mu \eta$. We find that its norm is given by

$$K_+^2 = -\frac{[h - m(1 + ag\cos^2\theta + bg\sin^2\theta)]^2}{h^2}, \tag{15}$$

where

$$h = (1 + ag)(1 + bg)[1 + (a+b)g]\rho^2. \tag{16}$$

Thus indeed the norm of K_+ is, as it should be since it has a spinorial square root, manifestly negative definite. The fraction of supersymmetry preserved is in general $\frac{1}{4}$, except when $a = -b$, in which case the preserved supersymmetry is doubled to become $\frac{1}{2}$. The latter solution was previously obtained in [13].

We now discuss the global structure of the rotating AdS_5 black hole. To do this, we first note that the metric can be expressed as

$$ds^2 = -\frac{\Delta_r \Delta_\theta r^2 \sin^2 2\theta}{4\Xi_a^2 \Xi_b^2 B_\phi B_\psi} dt^2 + \rho^2\left(\frac{dr^2}{\Delta_r} + \frac{d\theta^2}{\Delta_\theta}\right) + B_\psi(d\psi + v_1 d\phi + v_2 dt)^2 + B_\phi(d\phi + v_3 dt)^2, \tag{17}$$

where the functions B_ϕ, B_ψ, v_1, v_2, and v_3 can be straightforwardly found by comparing (17) with the metric in (1). The absence of naked CTCs requires that B_ϕ and B_ψ be non-negative outside the horizon. We shall focus on the discussion of supersymmetric solutions, satisfying the condition (13). It can be seen from (15) that the identity

$$-\frac{\Delta_r \Delta_\theta r^2 \sin^2 2\theta}{4\Xi_a^2 \Xi_b^2 B_\phi B_\psi} + B_\psi(v_2 + g + g v_1)^2 + B_\phi(v_3 + g)^2 = -\frac{[h - m(1 + ag\cos^2\theta + bg\sin^2\theta)]^2}{h^2} \tag{18}$$

holds. It follows that in general, at the Killing horizon where $\Delta_r = 0$, we have $B_\phi \cdot B_\psi < 0$, implying the existence of naked CTCs. There are two special cases where naked CTCs can be avoided, leading to either supersymmetric black holes or topological solitons.

Supersymmetric black holes.—The first way to avoid naked CTCs is if the right-hand side of (18) vanishes on the Killing horizon. This occurs when the parameters in the supersymmetric solutions satisfy the further restriction

$$gm = (a+b)(1+ag)(1+bg)(1+ag+bg). \tag{19}$$

Remarkably, when this extra condition is satisfied, the function Δ_r has a double root; Δ_r is now given by

$$\Delta_r = r^{-2}(r^2 - r_0^2)^2[g^2 r^2 + (1 + ag + bg)^2], \tag{20}$$

where $r_0^2 = g^{-1}(a + b + abg)$. At the Killing horizon $r = r_0$, we find that the determinant of the metric in the (θ, ϕ, ψ) directions is given by

$$\det g_{(\theta,\phi,\psi)} = \frac{(a+b)^2(a+b+abg)\sin^2 2\theta}{4g^3(1-ag)^2(1-bg)^2}. \tag{21}$$

This implies that naked CTCs are avoided if the remaining free parameters a and b satisfy the inequality

$$a + b + abg > 0. \tag{22}$$

The Killing horizon $r = r_0$ is then the event horizon of a well-defined supersymmetric black hole that is regular on and outside the event horizon. The occurrence of the double root of Δ_r at $r = r_0$ implies that the black hole has zero temperature. The various conserved and thermodynamic quantities for these new supersymmetric black holes are given by

$$\begin{aligned}
E &= \frac{\pi(a+b)}{4g(1-ag)^2(1-bg)^2}[(1-ag)(1-bg) \\
&\quad + (1+ag)(1+bg)(2-ag-bg)], \\
S &= \frac{\pi^2(a+b)\sqrt{a+b+abg}}{2g^{3/2}(1-ag)(1-bg)}, \\
J_a &= \frac{\pi(a+b)(2a+b+abg)}{4g(1-ag)^2(1-bg)}, \\
J_b &= \frac{\pi(a+b)(a+2b+abg)}{4g(1-ag)(1-bg)^2}, \\
Q &= -\frac{\pi\sqrt{3}(a+b)}{4g(1-ag)(1-bg)}.
\end{aligned} \tag{23}$$

Note that supersymmetric black holes cannot arise when

$a = -b$. Our new supersymmetric black holes have cohomogeneity-2, reducing to 1 if $a = b$. The supersymmetric $a = b$ cases were obtained in [7], and extended to include additional vector multiplets in [14].

Topological solitons.—The second way to avoid naked CTCs is if $B_\phi = 0$ at $r = r_0$. This can happen when the free parameters in the general supersymmetric solutions obey the further restriction

$$m = -(1 + ag)(1 + bg)(1 + ag + bg)(2a + b + abg) \times (a + 2b + abg). \tag{24}$$

Now r_0, the outer root of Δ_r, is given by

$$r_0^2 = -(a + b + abg)^2. \tag{25}$$

Defining a new radial coordinate $R = r^2 - r_0^2$, we find that the metric describes a smooth topological soliton, with R running from 0 to ∞. The requirement of the absence of a conical singularity when B_ϕ vanishes at $R = 0$ implies the quantization condition

$$\frac{(a + b + abg)(3 + 5ag + 5bg + 3abg^2)}{(1 - ag)(a + 2b + abg)} = 1. \tag{26}$$

In the cohomogeneity-1 special cases $a = b$ or $a = -b$, these topological solitons are encompassed within the soliton solutions obtained in [12].

Aside from the above two possibilities, the supersymmetric solutions in general have naked CTCs. As in the examples discussed in [10,12], a conical singularity at the Killing horizon can be avoided by periodically identifying the asymptotic time coordinate t with an appropriate period. However, if the Killing horizon is associated with a double root of Δ_r, then such an identification is unnecessary, analogous to the ungauged rotating solution obtained in [15]. The geodesic analysis of analogous time machines can be found in [16,17].

In the general case where the charged rotating metrics that we have found are nonextremal, they describe regular black holes provided the parameters lie in appropriate ranges that are easily determinable using the same techniques we have used above for analyzing the BPS limits.

As discussed in [4], rotating black hole solutions in five-dimensional gauged supergravity provide backgrounds whose AdS/CFT duals describe four-dimensional field theories in the rotating Einstein universe on the boundary of anti–de Sitter spacetime. With the general solutions in minimal gauged supergravity that we have now found, this aspect of the AdS/CFT correspondence can be studied in a framework that also allows one to take a BPS or near-BPS limit, where the mapping from the bulk to the boundary is better controlled. In particular, it is of great interest to provide the microscopic interpretation from the boundary CFT for the entropy (23) of the supersymmetric black holes with two general rotations. We plan to report further on these considerations in forthcoming work.

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