# Particle-Hole Symmetric Luttinger Liquids in a Quantum Hall Circuit

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We report current transmission data through a split-gate constriction fabricated onto a two-dimensional electron system in the integer quantum Hall (QH) regime. Split-gate biasing drives interedge backscattering and is shown to lead to suppressed or enhanced transmission, in marked contrast to the expected linear Fermi-liquid behavior. This evolution is described in terms of particle-hole symmetry and allows us to conclude that an unexpected class of gate-controlled particle-hole-symmetric chiral Luttinger liquids (CLLs) can exist at the edges of our QH circuit. These results highlight the role of particle-hole symmetry on the properties of CLL edge states.

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Quantum Hall (QH) states [1] are created at integer and particular rational fractions of the filling factor $\nu$, defined as the ratio between the electron density $n$ and the magnetic flux density $n_\phi$ measured in units of $\phi_0 = h/e$. Charge excitations confined at the edge are the only charged modes that can propagate in the QH phase. These edge excitations at the fractional $\nu = 1/m$, with $m$ odd integer, form a one-dimensional liquid that was predicted to be equivalent to a chiral Luttinger liquid (CLL) [2] with an interaction parameter $g = \nu$ [3–5]. These predictions were tested by a large number of experiments [6–12] even if many open issues remain, in particular, for the case $\nu \neq 1/m$.

A split-gate (SG) technique [13] can be exploited to define a nanofabricated constriction in order to induce a controllable scattering between counterpropagating edge channels that are locally brought in close proximity [see Fig. 1(a)]. The constriction thus realizes an artificial impurity and can be used to test one of the most significant manifestations of CLL behavior: the complete suppression of the (low-temperature, low-bias) transmission through the impurity and its related power-law behavior [3–5]. Backscattering at the constriction is controlled by the split-gate voltage $V_g$: by increasing $|V_g|$ the interedge distance is decreased; at larger $|V_g|$ values, in addition, the density of the two-dimensional electron system in proximity to the SG is appreciably reduced. In the presence of a uniform external magnetic field, this leads to a reduced filling factor $\nu^*$ within the constriction region [see Fig. 1(a)].

Here we show that the SG voltage $V_g$ not only modifies the backscattering strength but also defines unexpected robust CLLs that are related by particle-hole symmetry. In order to demonstrate this we study the constriction transmission in the QH regime at bulk *integer* filling factor $\nu = 1$. The measured low-energy conductance displays a nonlinear behavior determined by the SG voltage. Both suppression and enhancement of the transmission through the constriction were observed, in disagreement with the usual expectation of linear tunneling for $g = \nu = 1$. We argue that these data reveal the existence of peculiar CLLs with an effective $g$ determined by the SG voltage and that

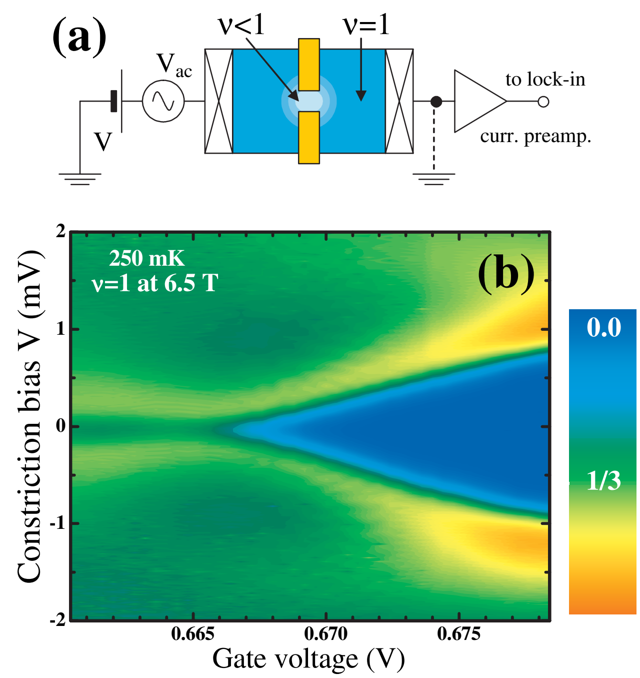


FIG. 1 (color). (a) The split-gate constriction induces a controllable interedge backscattering in the quantum Hall regime. Local charge depletion close to the metal gates creates a region with a reduced filling factor $\nu^*$. The differential conductance $G(V)$ is measured adopting a finite-bias, phase-locked technique ($V$ is the dc bias that corresponds to the voltage difference between the two interacting edge states, $V_{ac}$ is the ac voltage modulation). (b) Finite-bias transmission curves [$t(V) = hG(V)/e^2$] as a function of split-gate voltage at $T = 250$ mK and $\nu = 1$.

 

the intrinsic particle-hole symmetry [14,15] of the QH system influences the link between edge states and CLLs. These experimental results thus suggest that CLL behavior can be accessed in regimes that were not considered in previous theoretical evaluations in which charge depletion associated with the SG and particle-hole symmetry of the host QH system play significant roles.

Devices were realized starting from a high-mobility ($4.6 \times 10^6$ cm$^2$/Vs) AlGaAs/GaAs single heterojunction. The two-dimensional electron system (2DES) was buried 100 nm below the surface, with a sheet density of $1.5 \times 10^{11}$ cm$^{-2}$. Gates were realized by thermal evaporation of a metal bilayer Al/Au (5/25 nm) with constriction gaps between 600 and 800 nm. Measurements were performed in a $^3$He refrigerator with base temperature of 250 mK. The filling factor was kept to $\nu = 1$ by tuning the magnetic field (at 6.5 T) to the center of the vanishing longitudinal resistivity $\rho_{xx}$ measured outside the constriction. A finite dc bias $V$, added to a small ac modulation (20 $\mu$V), was applied to the device as shown in Fig. 1(a). In the QH regime shown in Fig. 1(a), $V$ is the voltage difference between the edge states entering the constriction from the two sides. The transmission coefficient $t(V)$ was obtained by two-wire differential conductance $G(V)$ measurements and analyzed as a function of the negative SG voltage $V_g$. At $\nu = 1$ the constriction transmission $t(V)$ is linked to $G(V)$ by $t(V) = G(V)h/e^2$ [16].

Figure 1(b) reports a color plot of measured transmission characteristics at different values of $V_g$ close to SG pinch-off. Curves display a marked nonlinearity and evolve from a small zero-bias suppression of the transmission ($V_g \approx -0.665$ V) to a deep minimum that saturates at zero (blue central region). The observed nonlinear behavior indicates that the relevant interaction $g$ is not equal to 1 but is determined by the SG voltage, confirming similar results obtained in the fractional QH regime [12,17]. Figure 2(a) reports a representative set of experimental curves in which at higher bias $t \approx 1/3$. This suggests that in the constriction region $\nu < 1$ and interedge tunneling is dominated by scattering between edge states in the reduced filling $\nu^* = 1/3$. Consistent with this interpretation, the experimental data are successfully described by the theory introduced by Fendley *et al.* [5] for CLLs with $g = 1/3$. Notably, when the transmission dip is small (i.e., far from the pinch-off limit $t = 0$), the nonlinearity width is in parameter-free agreement with this theory. The $V_g$ evolution can be captured by using a single adjustable parameter: the so-called impurity interaction strength $T_B$. CLL $g = 1/3$ behavior is also confirmed by the temperature evolution. Figures 2(b) and 2(c) report experimental and calculated transmission curves at different temperatures. In the calculations the interaction strength was set to $T_B = 2.8$ K in order to fit the measured $V = 0$ transmission values. This one adjustment yields agreement in the whole voltage-bias and temperature range tested. Significantly the fixed point observed at $t \approx 0.2$ is present also in the theory.

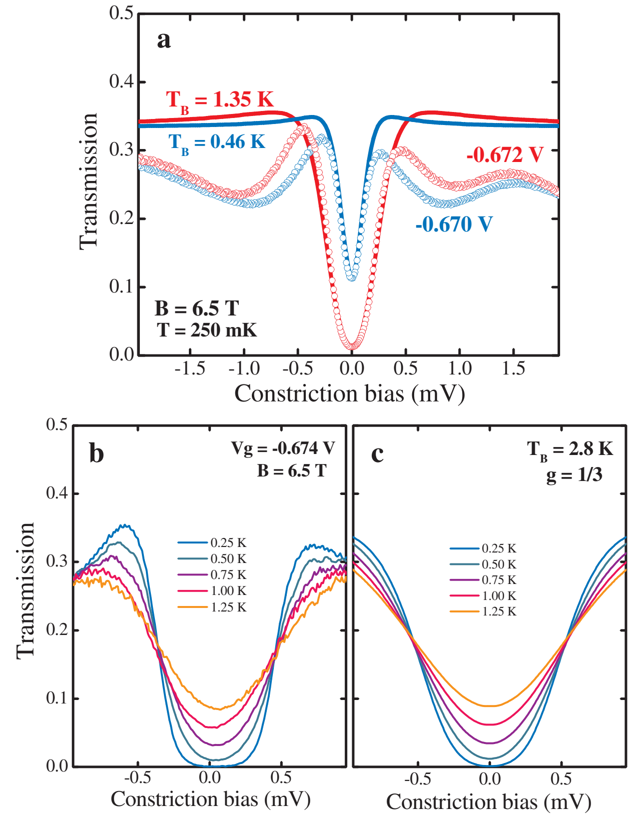


FIG. 2 (color). (a) Comparison between experimental (circles) and theoretical (lines) transmission characteristics calculated from the CLL theory of interedge tunneling at $g = 1/3$ as developed in Ref. [5]. The width of the dip before saturation is in *parameter-free* agreement with theory. The saturation to $t = 0$ is well described by adjusting the single fit parameter $T_B$ (tunneling strength). (b) Evolution of finite-bias transmission curve at $V_g = -0.674$ V as a function of temperature. (c) CLL theory prediction of the temperature dependence of the transmission at $g = 1/3$. Agreement is obtained with a *single* adjustable fit parameter ($T_B = 2.8$ K) for *all* curves.

This agreement confirms that the nonlinear transmission of the constriction is determined by the local filling factor $\nu^*$ and is consistent with CLL predictions when at higher bias $t \approx 1/3$ even if the region characterized by the filling factor $\nu^*$ is rather small. Single-particle effects can be ruled out as the Landau level gap $\hbar\omega_c \approx 10$ meV, is 1 order of magnitude bigger than the observed nonlinearity range. Coulomb blockade is inconsistent with the observed peak width and high bias. In order to support our interpretation of the data, we point out that $\rho_{xx}$ and Hall resistivity $\rho_{xy}$ measurements performed on depleted regions of the 2DES (we adopted a $100 \times 100$ nm$^2$ top gate for this analysis) ensure that the observation of fractional states at $\nu^* = 1/3$ and 2/3 is well within sample quality, base temperature, and magnetic field values. The observed nonlinearity is not related to any imperfection of the measured constriction: similar data were obtained on different devices with different constriction geometries. Devices displayed a sharp quantization of the conductance at zero magnetic field, without resonant features.

Having established that the CLL behavior is associated with the fractional QH edge states with $\nu^*$, we now show that our circuit implements a QH geometry which is self-dual under charge conjugation in the first Landau level. Indeed, a partially filled Landau level (with $\nu < 1$) can be described either in terms of electrons or in terms of holes with filling $1 - \nu$ over a completely filled Landau level. As long as interlevel mixing can be neglected, the complementary hole picture is governed by a time-reversed version of the electronic Hamiltonian and a mapping between states at $\nu \leftrightarrow 1 - \nu$ can be established [14,15,18]. To understand the impact of this particle-hole symmetry on the experiments here presented, we show in Fig. 3(a) a cartoon of an ideal QH constriction at "local" filling factor $\nu^*$ and biased at $V = 2V_i$. Chiral edge states propagate at the boundary between different filling factors in the directions indicated by the arrows: incoming edge states are polarized [19] at $\pm V_i$ and emerge after interaction or equilibration at a potential $\pm V_o$. The same circuit is described in terms of holes in Fig. 3(b) with constriction filling factor $1 - \nu^*$. Following particle-hole symmetry, the hole dynamics in Fig. 3(b) is the same of time-reversed electrons or, equivalently, of electrons in a reversed external electromagnetic field [Fig. 3(c)]. A final 180° rotation (around the axis defined by outgoing edge states) maps the system back to a biased constriction with central filling $1 - \nu^*$.

This simple particle-hole argument sets a correspondence between the behavior of the QH circuit at constriction filling factor $\nu^*$ and that with $1 - \nu^*$. The main consequence of the mapping is that the meaning of "crossing the constriction" changes in the transformation Fig. 3(a) → Fig. 3(d): transmitted trajectories of Fig. 3(a) map on reflected trajectories of Fig. 3(d) and vice versa. This suggests that transmission $t$ amplitudes in Fig. 3(a) are equal to reflection $r$ amplitudes in Fig. 3(d). More specifically the currents flowing in the constrictions of Figs. 3(a) and 3(d) are

$$I_{\nu^*}(V) = G_0(V_i + V_o), \quad (1)$$

$$I_{1-\nu^*}(V) = G_0(V_i - V_o), \quad (2)$$

where $G_0 = e^2/h$. The sum of these two currents is $I_{1-\nu^*}(V) + I_{\nu^*}(V) = G_0 V$, so that in terms of differential transmission $t = G_0^{-1} dI/dV$ and reflection $r$ we have the following duality

$$t_{\nu^*}(V) = 1 - t_{1-\nu^*}(V) = r_{1-\nu^*}(V). \quad (3)$$

The circuit of Fig. 3(a) is therefore dual respect to the one of Fig. 3(d), while the fixed point at $t = 1/2$ (corresponding to $\nu^* = 1/2$) realizes a self-dual configuration. Since the local filling factor $\nu^*$ can be tuned by the SG voltage, our QH circuit [Fig. 1(a)] allows us to test directly the occurrence of this duality in the interedge tunneling at the constriction and to verify the presence of unexpected CLLs. Figure 4 shows the measured evolution of the transmission curves versus $V_g$ and presents a behavior that is consistent with the charge-conjugation argument presented above. Starting from the pinch-off limit, curves become flat when $|V_g|$ is reduced to a crossover point at $t \approx 1/2$. A further reduction of the tunneling strength leads to symmetric curves at $t \approx 2/3$ characterized by an *enhanced* zero-bias transmission. This behavior was verified on a set of different constrictions with different geometries and crossover of the transmission was always found consistent with $t = 1/2$.

Curves with a zero-bias transmission enhancement seem to contradict any available CLL prediction for $g < 1$. This behavior, however, can be explained following the charge-conjugation argument. Let us consider, for instance, the $\nu^* = 2/3$ case. The electron circuit can be analyzed in terms of its dual hole system at $\nu^* = 1/3$ [Fig. 3(a) → Fig. 3(b)]. In this picture, the constriction induces a backscattering between edge states at $\nu^* = 1/3$ that should thus be described in terms of a CLL with $g = 1/3$. The behavior of such a circuit is well known: enhanced CLL reflection is expected at zero bias leading to increased zero-bias transmission at the constriction. The apparent contradiction between this result and CLL theory raises from the existence of two complementary and nonequivalent ways to ideally join chiral edges and form the complete non-

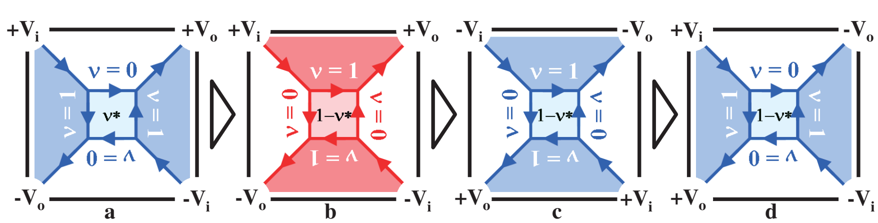


FIG. 3 (color). Particle-hole symmetry in a quantum Hall circuit with constriction. Arrows indicate the direction of propagation of edge states, and voltages at the corners show the potentials of ingoing and outgoing (after interaction or equilibration) edge channels. The constriction of (a) can be described in terms of holes (b). Because of particle-hole symmetry, holes in (b) follow the same dynamics of time-reversed electrons or, equivalently, of electrons in reversed external electromagnetic field (c). The circuit (c) can be mapped back to the original constriction geometry by a 180° rotation around the axis defined by outgoing edge states. This mapping establishes a correspondence between circuits (a) and (d): finite-bias transmission $t(V)$ and reflection $r(V)$ are related by $t_{\nu^*}(V) = r_{1-\nu^*}(V)$.

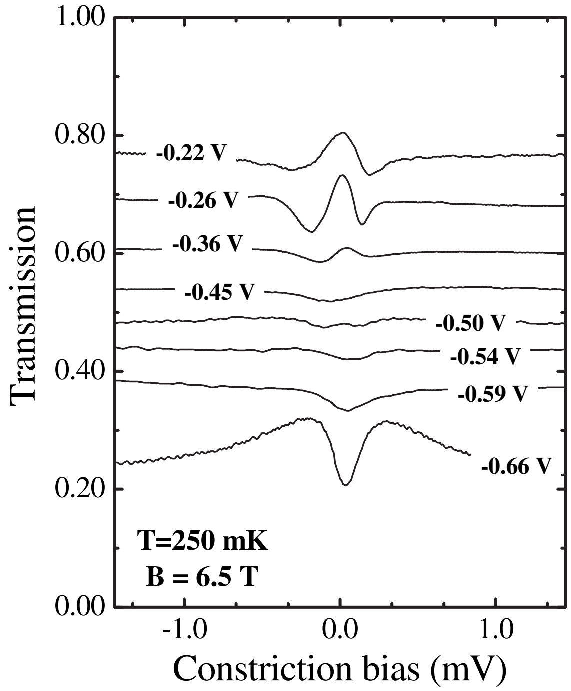


FIG. 4. Complete evolution of finite-bias transmission as a function of the gate voltage $V_g$. Minima observed for $t \approx 1/3$ evolve in a flat region at $t \approx 1/2$ and finally to zero-bias peaks at $t \approx 2/3$.

chiral Tomonaga Luttinger liquid (TLL). Both are realized in the $\nu = 1$ circuit, where they are related by charge conjugation. Indeed, while TLL theory always predicts a zero-bias suppression of the transmission through an impurity for $g < 1$, the constriction transmission can be suppressed or enhanced depending on which edge actually plays the role of the $L$ (left) and $R$ (right) branches of the TLL. The case $\nu^* = 1/3$ is well established: the $g = 1/3$ TLL develops *horizontally* [following the geometry of Fig. 1(a)] when top-bottom chiral edges are joined. The $\nu^* = 2/3$ case follows by charge conjugation. In this case the excitations living at the boundary between $\nu^* = 2/3$ and $\nu = 1$ form, by symmetry, a CLL with $g = 1/3$ and the nonchiral TLL develops *vertically* when left-right edges are joined. In this context the transmission suppression of the TLL corresponds to a *reflection* suppression at the constriction. Similar experimental results were obtained at a bulk filling factor $\nu = 2$ (data not shown) that display two minimum-to-maximum crossovers and linear characteristics at $t \approx 1/4$, 1/2, and 3/4 indicative of independent effects of the two channels of the $\nu = 2$ edge.

The creation and gate-voltage control of CLLs in a QH circuit has recently attracted a significant theoretical interest [20,21]. The possibility of a duality in the tunneling characteristics due to residual edge-edge interactions in an X-shaped constriction varying the angle was also theoretically evaluated [22]. Our symmetry argument provides a general description of the experimental features that does not depend on the detailed geometry of the constriction in agreement with the experiment findings. QH particle-hole symmetry was already invoked to explain electrical transport close to the QH liquid-to-insulator transition [23]. Our experiments highlight the role of QH particle-hole symmetry in the correspondence between edge backscattering and TLLs.

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