Effects of Extreme Magnetic Quadrupole Fields on Penning Traps and the Consequences for Antihydrogen Trapping

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Measurements on electrons confined in a Penning trap show that extreme quadrupole fields destroy particle confinement. Much of the particle loss comes from the hitherto unrecognized ballistic transport of particles directly into the wall. The measurements scale to the parameter regime used by ATHENA and ATRAP to create antihydrogen, and suggest that quadrupoles cannot be used to trap antihydrogen.

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Recently, the ATHENA [1] and ATRAP [2] collaborations produced slow antihydrogen ($\bar{\mathrm{H}}$) atoms at CERN. Both groups made $\bar{\mathrm{H}}$ by storing its constituents, positrons (e^+) and antiprotons ($\bar{p}$), in adjacent sections of Penning-like traps. These traps confine the charged constituents radially with a strong solenoidal field $[B_s(T)]$ and axially with electrostatic wells. When the constituents are mixed by allowing the $\bar{p}$ to traverse the e^+ well, some of the $\bar{p}$ and e^+ combine to make $\bar{\mathrm{H}}$. The traps are similar to the electron trap used in the experiments reported here, shown in Fig. 1(a).

The neutral $\bar{\mathrm{H}}$ atoms are not confined by the charged-particle trapping fields and quickly annihilate on the trap walls. While the production of cold $\bar{\mathrm{H}}$ is remarkable, the most interesting proposed experiments, including tests of *CPT* invariance and gravitational interactions, require trapped $\bar{\mathrm{H}}$. Because $\bar{\mathrm{H}}$ cannot be easily captured in an external trap, most trapping schemes entail superimposed neutral and charged-particle traps.

The most commonly suggested $\bar{\mathrm{H}}$ confinement scheme employs a diamagnetic, minimum-B neutral trap, typically produced by adding quadrupole $[\beta_q(T/m)]$ and mirror fields to the solenoid field [3,4]. In this Letter we will focus on the effects of the quadrupole alone; thus, we use a magnetic field given by $\mathbf{B} = B_s\hat{z} + \beta_q(x\hat{x} - y\hat{y})$.

The depth of the diamagnetic well is proportional to the increase in $|\mathbf{B}|$ from the center of the trap (where the $\bar{\mathrm{H}}$ is created) to the trap wall (where the $\bar{\mathrm{H}}$ will annihilate if the trap is too weak). Hence, the ratio $R_w\beta_q/B_s$, where R_w is the trap radius, must be near unity to develop a significant field increase. Unfortunately, attainable traps are shallow; a 1 T field increase produces a mere 0.67 K trap well. The energy of the $\bar{\mathrm{H}}$ is relatively high [5]. Unless methods are developed [6] to produce colder $\bar{\mathrm{H}}$, only a very small fraction of the $\bar{\mathrm{H}}$ will be trapped; for example, a 1 T well would capture less than 1 $\bar{\mathrm{H}}$ in 10^5 if the $\bar{\mathrm{H}}$ temperature is 2×10^3 K [5].

Even if this shallow an $\bar{\mathrm{H}}$ neutral trap is acceptable, quadrupole traps may still not be usable. Since the quadrupole fields destroy the cylindrical symmetry that underlies the Penning trap's outstanding performance, the $\bar{\mathrm{H}}$ constituents might be lost before they form $\bar{\mathrm{H}}$. In 1999, Gilson and Fajans [7,8] found that very small quadrupole fields ($R_w\beta_q/B_s = 0.0004$) could be quite deleterious, and

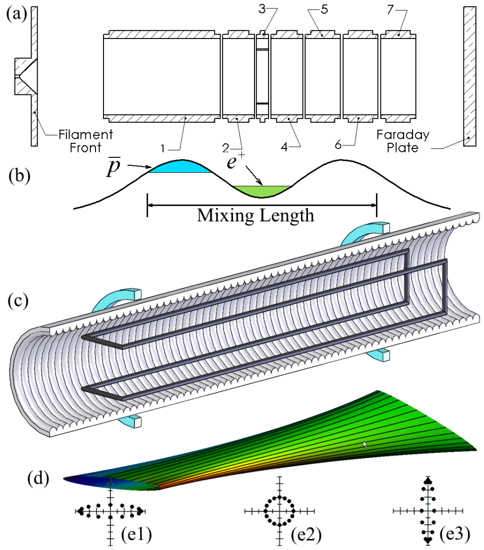

FIG. 1 (color online). (a) Schematic of the trap used in the experiments reported here. The cylinders are 3, 0.84, 0.4, 0.84, 1, 1, and 1 cm long; the wall radius is $R_w = 1$ cm. Electrons are produced by thermionic emission from a filament located to the left of the filament front. The trap is loaded by temporarily flattening the filament-side electrostatic barrier, and the amount of charge remaining in the trap at some later time is measured by briefly flattening the Faraday plate-side electrostatic barrier. The trap vacuum chamber is immersed in liquid helium. (b) A typical double electrostatic well for confining both positrons and antiprotons; in this Letter we use a single positive well to trap electrons. The "mixing length" indicates the distance the $\bar{p}$ travel during the mixing step of the $\bar{\mathrm{H}}$ formation cycle. (c) The solenoidal, quadrupolar, and mirror coils used to simultaneously trap charged and neutral particles. The mirror coils are shown for reference only and are not used in the experiments described here. (d) The shape circumscribed by magnetic field lines emanating from a circle. (e1)–(e3) Poincaré plots of the field lines as they penetrate the ends and center.

they proposed that a diffusive process, possibly resonant particle transport, was responsible for the enhanced losses. However, most of their measurements where taken with far lower (0.004–0.15 T) solenoidal fields than used in the $\bar{\mathrm{H}}$ experiments. It is not clear how to extrapolate their results to the $\bar{\mathrm{H}}$ regime. In 2001, Squires, Yesley, and Gabrielse published [9] single-particle calculations showing that confinement would be unaffected provided that certain resonances could be avoided. Similar resonances were crucial to the earlier experimental work [7,8], and it is not obvious if single-particle analysis is adequate or if resonances could, indeed, be avoided. Later, experimental work at LANL [10] found significant quadrupole-induced confinement degradation with quadrupole fields of $R_w\beta_q/B_s = 0.11$ at axial fields of 0.1 T, but no degradation with the same β_q at 1 T ($R_w\beta_q/B_s = 0.011$). In early 2004, Fajans and Schmidt [11] published a summary of the problems with quadrupoles and showed how these problems would be alleviated by using higher-order multipoles instead of quadrupoles. Later in the year, Gabrielse [12] reported experiments demonstrating little effect from a 16 T/m permanent magnet quadrupole on confinement in a 3 T field ($R_w\beta_q/B_s = 0.03$), but, on the same apparatus, Speck [13] later reported strong effects in a 1 T field ($R_w\beta_q/B_s = 0.09$).

Here we report measurements with a 50 T/m superconducting quadrupole in solenoidal fields up to 1.1 T. We find that for strong quadrupole fields, the plasma is *instantly* transported to the walls by a hitherto unidentified, but very simple, ballistic effect; the particles follow field lines into the trap walls [14]. This ballistic loss is a robust single-particle effect. At somewhat lower quadrupole fields, particles are transported to the walls in 10–1000 s, by what we believe is a combination of diffusive and ballistic effects.

The field lines emanating from a circular ensemble of evenly distributed points [Fig. 1(e2)] circumscribe a twisted bow tie [see Fig. 1(d)] [7,8]: circular in the center and elliptical at both ends, but with the major axes of the end ellipses rotated by 90° [see Figs. 1(e1), 1(e2), and 1(e3)]. There exist four purely radially propagating field lines which follow

$$r(z) = r_0 \exp(\pm\beta_q z/B_s), \tag{1}$$

where r_0 is the field lines' initial radius. All other field lines rotate azimuthally as they propagate, converging towards the fastest outward propagating field line.

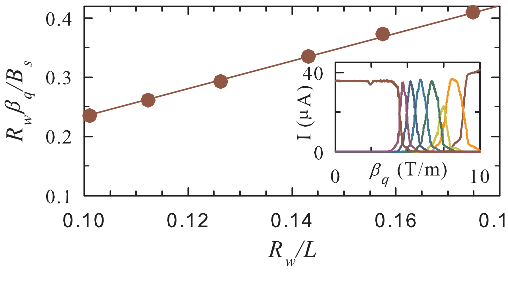

FIG. 2 (color online). (Inset) The beam current collected on successive trap elements, beginning (at $\beta_q = 0$) with the Faraday Plate and ending (at $\beta_q > 9$ T/m) with cylinder 1, as a function of the quadrupole field β_q. (Main panel) The normalized quadrupole field that maximizes the current on cylinders 1–6, as a function of the distance of the cylinders from the filament front. All measurements are at $B_s = 0.2$ T.

When an initially circular beam of radius r_0 is injected from the filament into the combined solenoidal and quadrupolar fields, the beam will follow the field lines. As shown in Fig. 2, the beam hits the wall at the distance L downstream from the filament which satisfies the relation $R_w\beta_q/B_s = (R_w/L)\ln(R_w/r_0)$, from which we can infer the beam (and resulting plasma) radius to be about 0.1 cm. Equilibrium (Boltzmann-Poisson) calculations then indicate that the initial densities of all our trapped plasmas are around 10^8 cm^{-3}, unless otherwise stated. Initial temperatures are measured to be near 1 eV.

The beam does not make it through or, in some cases, even into the trap for relatively modest quadrupole fields. If we were to leave the quadrupole on while we load plasma into the trap, the initial plasma charge and radius would depend on the quadrupole field strength. Consequently, all our confinement experiments begin with the quadrupole field off. After loading the plasma, we ramp the quadrupole field up and hold the plasma for the desired time. Then we ramp the quadrupole field down to zero and measure the remaining charge by dumping it onto our Faraday plate. Both magnetic diffusion and quenches limit the speed with which we can ramp the quadrupole. Typically, we use ramp times obeying $T_{\mathrm{ramp}} = 5.6\beta_q + 1.5$ (in seconds). With

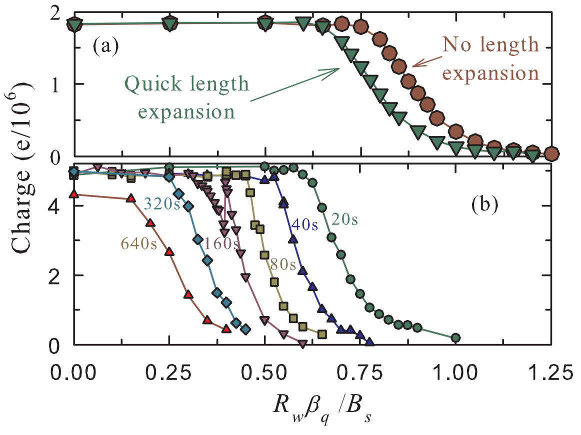

FIG. 3 (color online). (a) Charge remaining in the trap as a function of the quadrupolar field. The "no length expansion" plasma was stored in a single cylinder of length 1 cm; the "quick length expansion" plasma was briefly lengthened from 1 to 4.08 cm while the quadrupole was at full strength. (b) Charge remaining at the indicated times as a function of the quadrupole field. The trap length was 2 cm. The glitch in the 160 s curve is repeatable and is possibly due to a resonance [8]. The solenoidal field was 0.4 T for both (a) and (b).

strong quadrupole and solenoidal fields, quenches force us to ramp slower. We note that as permanent magnet quadrupoles [12,13] cannot be turned off, their use at strengths producing interesting neutral well depths is problematic.

We find that quadrupole fields dramatically degrade confinement. Typical results are shown in Fig. 3(a). For the "No length expansion" data, the plasma was loaded into the trap, the quadrupole ramped up over the time $T_{\rm ramp}$ given above, immediately ramped back down over the same time scale, and the remaining plasma charge measured. Half or more of the charge was lost for fields greater than about $R_w\beta_q/B_s = 0.8$. The measurements shown here use a Malmberg-Penning style well: a grounded center cylinder surrounded by two negative cylinders. Measurements in an equivalent-length harmonic potential well were not appreciably different.

It may be that most of the charge is lost "instantly." Just as the beams in Fig. 2 follow field lines [15] into the wall, particles in trapped plasmas also follow the field lines, and can be guided directly into the wall. Plasmas subject to strong quadrupolar fields are known to be highly distorted [8,11], but that this could lead to direct plasma loss has been unrecognized. This is a serious problem. Given that $R_w\beta_q/B_s$ must be near unity, and since it is difficult to make a trap whose length is shorter than the wall radius, Eq. (1) predicts that the plasma radius will increase substantially. This increase is particularly dangerous during the "mixing" step in the $\bar{\rm H}$ formation cycle, during which the $\bar{p}$ inevitably traverse several cylinders [see Fig. 1(b)]. For example, the mixing length L, normalized to the corresponding wall radius, is greater than about 6.5 in both ATHENA [1] and ATRAP [2], while the radius of the $\bar{p}$ plasmas r_p is about half the wall radius in both experiments [16,17]. For $R_w\beta_q/B_s = 1.39$ (a 1 K trap depth in a 2 T solenoid), the field lines would expand by at least 8000, and all but one $\bar{p}$ in 10×10^6 would be lost.

Since the field lines gather towards the extremal field line [Figs. 1(e1) and 1(e3)], most of this ballistic loss occurs instantly, i.e., on the $\bar{p}$ trap traversal time scale. Even $\bar{p}$ that are on initially inward trajectories will be lost as they rotate around the trap axis [15].

Because the quadrupole ramp takes many seconds, we cannot determine if the particles are lost instantly by simply turning the quadrupole on; the losses might be the result of other, slower mechanisms. However, the loss rate is length dependent. By making a short plasma, ramping the quadrupole field over time $T_{\rm ramp}$ to a value that does not result in much lost plasma, then manipulating the axial confinement well to briefly lengthen the plasma, and finally ramping the quadrupole down over time $T_{\rm ramp}$, we can isolate the ballistic loss mechanism. Much of the plasma is indeed lost instantly. For instance, Fig. 3(a) shows the result of lengthening the grounded trap length by a factor of ~4 for 2 ms, while Fig. 4 demonstrates that roughly the same amount of charge is lost when holding the plasma at the extended length for 20 μs to about 1 s. (Expanding the plasma before the quadrupole is turned on does not diminish the lifetime.)

The data in Figs. 3(a) and 4 suggest two loss mechanisms: instantaneous ballistic loss and a slower diffusive loss, probably similar to the process explored by Gilson and Fajans [7,8]. However, their much weaker quadrupole fields simply increased the diffusion rate. The plasma had to expand significantly before it was lost. In the much stronger fields considered here, field lines already transport the plasma close to the trap wall, so particles need not diffuse far before they are lost.

The ballistic loss mechanism depends only on the ratio of the quadrupole field to the solenoidal field and on the length of the plasma. Experiments, shown in Fig. 5, confirm that the loss is approximately constant when these two parameters are held fixed. As the loss is a single-particle effect, it should not depend on the plasma density. Measurements (not shown) demonstrate that the loss is independent of the initial charge over a range of more than 3 orders of magnitude (inferred densities of approximately 10^6 to 10^9 cm^{-3}). Other measurements demonstrate that the confinement time has no obvious dependence on plasma temperature. Cyclotron radiation was used to cool the plasma; varying the time between the injection of the plasma and the turn-on of the quadrupole controls the plasma temperature. We believe that the temperature ranged from 1 to near 0.0004 eV (4.2 K). However, our temperature diagnostic cannot measure temperatures below 0.05 eV. Application of a rotating wall [18] slightly diminished the loss, probably because the rotating wall compresses the plasma early in the experimental cycle when the quadrupole field is low. Note that the density and temperature regime discussed here, and in all anticipated antihydrogen experiments, is very far from the strongly

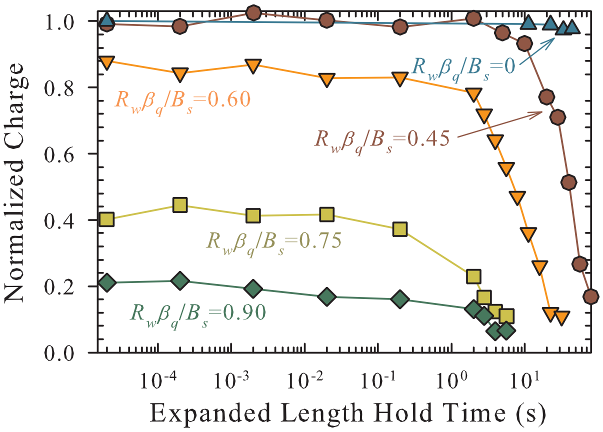

FIG. 4 (color online). The charge remaining in the trap as a function of the time that the grounded trap length is lengthened to 4.08 cm. The charge is normalized by the charge measured when the field is ramped to the strengths indicated on the graph, and then immediately ramped down, without expanding from the initial trap length of 1 cm. This normalization charge is lower than the charge measured with no quadrupole field by factors of 1, 1, 1, 1.1, and 2.1, respectively.

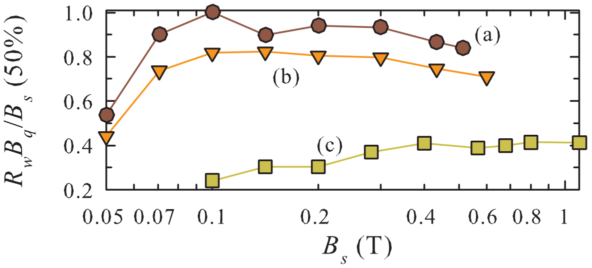

FIG. 5 (color online). The quadrupole field strength at which half the initial ($\beta_q = 0$) charge is lost as a function of the solenoidal field, for a 1 cm grounded trap length that is (a) not briefly lengthened, (b) briefly (2 ms) lengthened to 4.08 cm, and (c) briefly (2 ms) lengthened to 7.08 cm. Once the solenoidal field is greater than about 0.1 T, the loss is roughly independent of the solenoidal field.

correlated crystalline regime, where other effects might come into play.

At quadrupole fields below those that cause ballistic loss, the plasma lasts somewhat longer [see Fig. 3(b)]. Even so, diffusion causes rapid loss. Preliminary measurements suggest that rotating walls can partially ameliorate the diffusive loss. Note that while we have concentrated on long, prolate plasmas, some of our plasmas are quite short; equilibrium calculations predict lengths of ~0.3 cm for the unexpanded curves in Figs. 3(a) and 5. (This is as short as, or shorter than, the unmixed plasmas used in ATHENA and ATRAP [1,2,6].) As these plasmas expand, they become oblate, but are lost nonetheless.

In conclusion, we have discovered a ballistic loss mechanism that, along with the diffusive loss mechanism, suggests that quadrupole magnets cannot be used to trap $\bar{\mathrm{H}}$ while simultaneously confining $\bar{p}$ and positrons in all currently described schemes [1,2,6]. Accumulating the charged $\bar{\mathrm{H}}$ constituents with the quadrupole on appears to be impossible, and it is very likely that the constituents will be lost when the quadrupole is turned on after accumulation. Even if some constituents survive, they will be lost during the mixing process.

The loss mechanisms are robust and are difficult to evade. (1) Weak quadrupole fields would result in less loss, but the neutral trap depths discussed here are, at best, barely adequate. Unless $\bar{\mathrm{H}}$ can be created at, or instantly cooled to, much lower temperatures than currently obtained, decreasing the quadrupole field is not an attractive option. (2) The largest well depth B_{well} that does not result in ballistic loss is given by

$$B_{\mathrm{well}} = B_s\left[\sqrt{1 + \left(\frac{R_w}{L}\ln\frac{R_w}{r_p}\right)^2} - 1\right]. \tag{2}$$

Increasing the ratio R_w/L increases the well depth. However, the potential induced on the axis of a cylinder decreases rapidly once the cylinder length becomes shorter than its radius. Thus, for the mixing operation, L/R_w cannot be made much smaller than 3, and ballistic loss cannot be evaded by manipulating the trap dimensions. (3) The magnetic mirror coils required to confine $\bar{\mathrm{H}}$ axially diminish the radial excursions of the field lines. Unfortunately, physical constraints on the size and location of the coils likely make this beneficial effect insignificant. (4) The charged particles themselves are diamagnetic and, in principle, partially confined by the radial increase in $|\mathbf{B}|$. In practice, we do not observe this effect.

Traps that use higher-order multipoles instead of quadrupoles will have much less ballistic and diffusive loss because their radial fields are low near the trap axis [11]. They are a better choice for trapping antihydrogen.

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