Magnetic Color-Flavor Locking Phase in High-Density QCD

Efrain J. Ferrer and Vivian de la Incera
Department of Physics, Western Illinois University, Macomb, Illinois 61455, USA

Cristina Manuel
Instituto de Física Corpuscular, CSIC-Universitat de València, 46071 València, Spain
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We investigate the effects of an external magnetic field in the gap structure of a color superconductor with three massless quark flavors. Using an effective theory with four-fermion interactions, inspired by one-gluon exchange, we show that the long-range component $\tilde{B}$ of the external magnetic field that penetrates the color-flavor locked phase modifies its gap structure, producing a new phase of lower symmetry. A main outcome of our study is that the $\tilde{B}$ field tends to strengthen the gaps formed by $\tilde{Q}$-charged and $\tilde{Q}$-neutral quarks that coupled among themselves through tree-level vertices. These gaps are enhanced by the field-dependent density of states of the $\tilde{Q}$-charged quarks on the Fermi surface. Our considerations are relevant for the study of highly magnetized compact stars.

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After the suggestion that three-flavor quark matter may actually be the ground state of strong interactions [1], quark stars were postulated as possible astrophysical objects. It is also very likely that quark matter occupies the inner regions of neutron stars. Our present knowledge of QCD at high baryonic density indicates that this new state of matter might be in a color superconducting phase (for reviews, see [2]). On the other hand, it is well known [3] that strong magnetic fields, as large as $B \sim 10^{12}$–10^{14} G, exist in the surface of neutron stars, while in magnetars they are in the range $B \sim 10^{14}$–10^{15} G, and perhaps as high as 10^{16} G [4]. The physical upper limit to the total neutron star magnetic field, as arising from comparing the magnetic and gravitational energies, is of order $B \sim 10^{18}$ G [3]. If quark stars are self-bound rather than gravitational-bound objects this upper limit could go higher. In this Letter we investigate the effect of a strong magnetic field in color superconductivity, with the aim of further studying its possible astrophysical implications.

We will start by considering three massless quarks. In this case, it is well established that the ground state of high-dense QCD corresponds to the color-flavor locked (CFL) phase [5]. In this phase, quarks form spin-zero Cooper pairs in the color- antitriplet, flavor-antitriplet representation, thereby breaking the original $SU(3)_{\rm color} \times SU(3)_L \times SU(3)_R \times U(1)_B$ symmetry to the diagonal subgroup $SU(3)_{{\rm color}+L+R}$. One can now ask how this scenario will change when a magnetic field is switched on. Would the external field affect the pairing phenomena? In a conventional electromagnetic superconductor, since Cooper pairs are electrically charged, the electromagnetic gauge invariance is spontaneously broken and the photon acquires a mass that can screen a weak magnetic field: this is the Meissner effect. In spin-zero color superconductivity, although the color condensate has nonzero electric charge, there is a linear combination of the photon and a gluon that remains massless [5]. The unbroken $U(1)$ group is generated, in flavor-color space, by $\tilde{Q} = Q \times 1 - 1 \times Q$, where Q is the electromagnetic charge generator [6]. Thus a spin-zero color superconductor may be penetrated by a long-range remnant ''rotated magnetic'' field $\tilde{B}$. In the 9-dimensional flavor-color representation that we will use in this Letter, the $\tilde{Q}$ charges of the different quarks are

s_1	s_2	s_3	d_1	d_2	d_3	u_1	u_2	u_3
0	0	-	0	0	-	+	+	0

, (1)

in units of the $\tilde{Q}$ charge of the electron $\tilde{e} = e\cos\theta$, where θ is the mixing angle [7] (we set $\hbar = c = 1$ henceforth).

Although the interaction of an external magnetic field with dense quark matter has been investigated by several authors [8,9], they disregarded the effect of the penetrating $\tilde{B}$ field on the gap structure. However, the $\tilde{B}$ field can change the gap structure and lead to a new superconducting phase. To understand how this can occur notice that due to the coupling of the charged quarks with the external $\tilde{B}$ field, the color-flavor space is augmented by the $\tilde{Q}$ charge operator, and consequently the order parameter of the CFL splits in new independent pieces.

Based on the above considerations, and imposing that the condensates should retain the highest degree of symmetry, we propose the following ansatz for the color-flavor structure of the order parameter of three massless quarks in the presence of a magnetic $\tilde{B}$ field:

$$\Delta^+ = k_1^+(U_0 - N)\Omega_0 + k_2^+ U\Omega_0 + k_n^+ N\Omega_0 + k_c^+ U[\Omega_+ + \Omega_-] \quad (2)$$

with color-flavor matrices defined as: $U_0 = \delta_a^i \delta_b^j$, $U = \delta_a^j \delta_b^i$, $N = (\delta_a^1 \delta_i^1 + \delta_a^2 \delta_i^2)\delta_b^3 \delta_j^3 + (a \leftrightarrow b, i \leftrightarrow j)$; with

a, b and i, j denoting color and flavor indexes, respectively. The matrices $\Omega_0 = \mathrm{diag}(1, 1, 0, 1, 1, 0, 0, 0, 1)$, $\Omega_+ = \mathrm{diag}(0, 0, 0, 0, 0, 0, 1, 1, 0)$, and $\Omega_- = \mathrm{diag}(0, 0, 1, 0, 0, 1, 0, 0, 0)$ are $\tilde{Q}$-charge projectors with algebra $\Omega_\eta \Omega_{\eta'} = \delta_{\eta\eta'}\Omega_\eta$, for $\eta, \eta' = 0, +, -$, and $\Omega_0 + \Omega_+ + \Omega_- = 1$.

An applied magnetic field reduces the flavor symmetries of QCD, as only the d and s quarks have equal electromagnetic charge. Thus, the order parameter (2) implies the following symmetry breaking pattern: $SU(3)_{\mathrm{color}} \times SU(2)_L \times SU(2)_R \times U(1)_B \times U^{(-)}(1)_A \times U(1)_{\mathrm{e.m.}} \rightarrow SU(2)_{\mathrm{color}+L+R} \times \tilde{U}(1)_{\mathrm{e.m.}}$. The $U^{(-)}(1)_A$ symmetry is connected with the current which is an anomaly-free linear combination of s, d, and u axial currents [10]. The locked $SU(2)$ corresponds to the maximal unbroken symmetry, and as such it maximizes the condensation energy. Notice that it commutes with $\tilde{U}(1)_{\mathrm{e.m.}}$.

Therefore, there are 13 broken generators, 8 of which become the longitudinal components of massive gauge bosons, and 5 remain as Goldstone bosons. One is associated to the spontaneous breaking of baryon symmetry, one with the breaking of the anomaly-free $U^{(-)}(1)_A$, and the remaining 3 to the breaking of the chiral $SU(2)$ group. This symmetry breaking pattern suggests that the new phase has quantitative and qualitative differences with respect to the CFL phase. We will call it magnetic CFL (MCFL) phase. In particular, the MCFL phase will possess a distinctive low energy physics.

To trace back the physical origin of the new structures in (2) we should take into account that despite the $\tilde{Q}$ neutrality of all the condensates, they can be composed either by neutral or by charged quarks. Condensates formed by $\tilde{Q}$-charged quarks feel the field directly through the minimal coupling of the background field $\tilde{B}$ with the quarks in the pair. A subset of the condensates formed by $\tilde{Q}$-neutral quarks, can feel the presence of the field through the tree-level vertices that couple them to charged quarks. The gaps $\Delta^B_{A/S} \equiv (k_n \mp k_c)/2$ are antisymmetric/symmetric combinations of condensates composed by charged quarks and condensates formed by this kind of neutral quarks. The gaps $\Delta_{A/S} \equiv (k_1 \mp k_2)/2$, on the other hand, are antisymmetric/symmetric combinations of condensates formed by neutral quarks that do not belong to the above subset. The only way the field can affect $\Delta_{A/S}$ is through the system of highly nonlinear coupled gap equations. The CFL gap matrix is obtained when $\Delta^B_{A/S} = \Delta_{A/S}$. In principle, the symmetries of the problem allow for two extra independent symmetric gaps. But these are only due to subleading color symmetric interactions, and are formed by neutral quarks that are not coupled to charged quarks, so they belong to the same class as Δ_S. Thus, in a first approach to the problem, we will consider that those can as well be described by Δ_S.

To study the MCFL phase we use a Nambu-Jona-Lasinio (NJL) four-fermion interaction abstracted from one-gluon exchange [5]. This simplified treatment, although, disregards the effect of the $\tilde{B}$ field on the gluon dynamics and assumes the same NJL couplings for the system with and without magnetic field, keeps the main attributes of the theory, providing the correct qualitative physics. We will postpone the study within QCD for the future.

The NJL model is defined by two parameters, a coupling constant g and an ultraviolet cutoff Λ. The cutoff should be higher than the typical energy scales in the system, that is, the chemical potential μ and the magnetic energy $\sqrt{\tilde{e}\tilde{B}}$.

The gap equation of the NJL model in coordinate space reads

$$\Delta^+ = i\frac{g^2}{4\Lambda^2}\lambda_A^T \gamma^\mu S_{21}(x, y)\gamma_\mu \lambda_A \delta^{(4)}(x - y), \qquad (3)$$

where λ_A and γ_μ are the Gell-Mann and Dirac matrices, respectively. For simplicity, we have omitted explicit flavor, color, and spinor indexes in the equation. S_{21} is the 21-component of the quark propagators in the Nambu-Gorkov representation.

The computation of the field-dependent quark propagators is laborious (details will be given elsewhere [11]), but it can be managed with the use of the Ritus' method, originally developed for charged fermions [12] and recently extended to charged vector fields [13]. In Ritus' approach the diagonalization in momentum space of charged fermion Green's functions in the presence of a background magnetic field is carried out using the eigenfunction matrices $E_p(x)$. These are the wave functions of the asymptotic states of charged fermions in a uniform magnetic field and play the role in the magnetized medium of the usual plane-wave (Fourier) functions e^{ipx} at zero field. With the help of the $E_p(x)$ functions, we first compute the propagators in momentum space, and then transform to coordinate space adequately. Leaving aside the color-flavor structure, the neutral quark propagators are of the same type as for the CFL phase. The charged (positive/negative) quark propagators in the background of a $\tilde{B}$ field that lies in the $\hat{z}$ axis are

$$S_{21}^{(\pm)} = \frac{\Lambda^+_{(\pm)}\gamma_5 k_c}{p_0^2 - (|\bar{\mathbf{p}}^{(\pm)}| + \mu)^2 - k_c^2} + \frac{\Lambda^-_{(\pm)}\gamma_5 k_c}{p_0^2 - (|\bar{\mathbf{p}}^{(\pm)}| - \mu)^2 - k_c^2} \qquad (4)$$

where $\bar{\mathbf{p}}^{(\pm)} = (0, \pm\sqrt{2|\tilde{e}\tilde{B}|l}, p_3)$ are the spatial components of the momentum for (positive/negative) charged quarks and the integer number l labels the Landau levels. In (4), $\Lambda^\pm_{(\pm)} = (1 \pm \gamma_0 \gamma \cdot \hat{\bar{\mathbf{p}}}^{(\pm)})/2$ are the energy projectors in the ultrarelativistic limit for (positive/negative) charged quarks in the external field, with $\hat{\bar{\mathrm{p}}}^{(\pm)}$ representing the normalized charged-quark three-momentum. On the Fermi surface, the highest occupied Landau level is obtained as $l_{\max} = [\frac{\mu^2}{2|\tilde{e}\tilde{B}|}]$, where the bracket denotes the integer part.

Since color superconductivity, although of type I, allows the penetration of a rotated magnetic field, it is natural to expect that the condensates made up of $\tilde{Q}$-charged quarks will be strengthened by the nonzero $\tilde{B}$, because these paired quarks have opposite $\tilde{Q}$ charges and opposite spins, hence parallel (instead of antiparallel) magnetic moments. The situation here has some resemblance to the magnetic catalysis (MC) of chiral symmetry breaking [14], in the sense that the magnetic field strengthens the pair formation. Despite this similarity, the way the field influences the pairing mechanism in the two cases is quite different. The particles participating in the chiral condensate are near the surface of the Dirac sea. The effect of a magnetic field there is to effectively reduce the dimension of the particles at the lowest Landau level (LLL), which in turn strengthens their effective coupling, catalyzing the chiral condensate. Color superconductivity, on the other hand, involves quarks near the Fermi surface, with a pairing dynamics that is already $(1 + 1)$ dimensional. Therefore, the $\tilde{B}$ field does not yield further dimensional reduction of the pairing dynamics near the Fermi surface and hence the LLL does not have a special significance here. Nevertheless, the field increases the density of states of the $\tilde{Q}$-charged quarks, and it is through this effect that the pairing of the charged particles is reinforced by the penetrating magnetic field. Below we will analytically show that this is indeed the case.

To solve the gap Eq. (3) for the whole range of magnetic-field strengths we need to use numerical methods. We have found, however, a situation where an analytical solution is possible. This corresponds to the case $\tilde{e}\tilde{B} \gtrsim \mu^2/2$. Taking into account that the leading contribution to the gap solution comes from quark energies near the Fermi level, it follows that for fields in this range only the LLL ($l = 0$) contributes.

Using the approximation $\Delta_A^B \gg \Delta_S^B, \Delta_A$; $\Delta_A \gg \Delta_S$, the gap equations decouple and the equation for Δ_A^B is

$$\Delta_A^B \approx \frac{g^2}{3\Lambda^2}\int_\Lambda \frac{d^3q}{(2\pi)^3}\frac{\Delta_A^B}{\sqrt{(q-\mu)^2+2(\Delta_A^B)^2}} + \frac{g^2\tilde{e}\tilde{B}}{3\Lambda^2}\int_{-\Lambda}^{\Lambda}\frac{dq}{(2\pi)^2}\frac{\Delta_A^B}{\sqrt{(q-\mu)^2+(\Delta_A^B)^2}}, \tag{5}$$

where the first (second) term in the right-hand side of Eq. (5) corresponds to the contribution of $\tilde{Q}$-neutral (charged) quark propagators, respectively. For the last one, we dropped all Landau levels but the lowest, as we are interested in the leading term.

The solution of Eq. (5) reads

$$\Delta_A^B \sim 2\sqrt{\delta\mu}\exp\left(-\frac{3\Lambda^2\pi^2}{g^2(\mu^2+\tilde{e}\tilde{B})}\right), \tag{6}$$

with $\delta \equiv \Lambda - \mu$, to be compared with the antisymmetric CFL gap [2]

$$\Delta_A^{\mathrm{CFL}} \sim 2\sqrt{\delta\mu}\exp\left(-\frac{3\Lambda^2\pi^2}{2g^2\mu^2}\right). \tag{7}$$

In this approximation the remaining gap equations read

$$\Delta_S^B \approx -\frac{g^2}{6\Lambda^2}\int_\Lambda \frac{d^3q}{(2\pi)^3}\frac{\Delta_A^B}{\sqrt{(q-\mu)^2+2(\Delta_A^B)^2}} + \frac{g^2\tilde{e}\tilde{B}}{6\Lambda^2}\int_{-\Lambda}^{\Lambda}\frac{dq}{(2\pi)^2}\frac{\Delta_A^B}{\sqrt{(q-\mu)^2+(\Delta_A^B)^2}}, \tag{8}$$

$$\Delta_A \approx \frac{g^2}{4\Lambda^2}\int_\Lambda \frac{d^3q}{(2\pi)^3}\left(\frac{17}{9}\frac{\Delta_A}{\sqrt{(q-\mu)^2+\Delta_A^2}} + \frac{7}{9}\frac{\Delta_A}{\sqrt{(q-\mu)^2+2(\Delta_A^B)^2}}\right), \tag{9}$$

and

$$\Delta_S \approx \frac{g^2}{18\Lambda^2}\int_\Lambda \frac{d^3q}{(2\pi)^3}\left(\frac{\Delta_A}{\sqrt{(q-\mu)^2+\Delta_A^2}} - \frac{\Delta_A}{\sqrt{(q-\mu)^2+2(\Delta_A^B)^2}}\right). \tag{10}$$

We express below the solution of these gap equations as ratios over the CFL antisymmetric and symmetric gaps

$$\frac{\Delta_A}{\Delta_A^{\mathrm{CFL}}} \sim \frac{1}{2^{(7/34)}}\exp\left(-\frac{36}{17x} + \frac{21}{17}\frac{1}{x(1+y)} + \frac{3}{2x}\right), \tag{11}$$

where $x \equiv g^2\mu^2/\Lambda^2\pi^2$, $y \equiv \tilde{e}\tilde{B}/\mu^2$, and

$$\frac{\Delta_S^B}{\Delta_S^{\mathrm{CFL}}} \sim \frac{\Delta_A^B}{\Delta_A^{\mathrm{CFL}}}\left(\frac{3}{4} + \frac{9}{2x\ln 2}\frac{y-1}{y+1}\right), \tag{12}$$

$$\frac{\Delta_S}{\Delta_S^{\mathrm{CFL}}} \sim \frac{\Delta_A}{\Delta_A^{\mathrm{CFL}}}\frac{3}{2}\left(1 - \frac{4}{1+y}\right). \tag{13}$$

Note that our analytic solutions are only valid at strong magnetic fields. The lower value $\tilde{e}\tilde{B} \sim \mu^2/2$ corresponds to $\tilde{e}\tilde{B} \sim (0.8\text{–}1.1) \times 10^{18}$ G, for $\mu \sim 350\text{–}400$ MeV. For fields of this order and larger the Δ_A^B gap is larger than Δ_A^{CFL} at the same density values. Note also that for $\tilde{e}\tilde{B} \gtrsim \mu^2$, $\Delta_{A/S}^B$ grow and $\Delta_{A/S}$ tend to lower, and they clearly split. How fast or slowly they do depends very much on the values of the NJL couplings. For example, for $x \sim 0.3$ [15], one finds $\Delta_A \sim 0.2\Delta_A^B$ for $y = 3/2$, while for $x \sim 1$ then $\Delta_A \sim 0.5\Delta_A^B$.

All the gaps feel the presence of the external magnetic field. As expected, the effect of the magnetic field in Δ_A^B is to increase the density of states, which enters in the argument of the exponential as typical of a BCS solution. The density of states appearing in (6) is just the sum of those of neutral and charged particles participating in the given gap equation (for each Landau level, the density of states around the Fermi surface for a charged quark is $\tilde{e}\tilde{B}/2\pi^2$).

All the $\tilde{Q}$-charged quarks have common gap Δ_A^B. Hence, the densities of the charged quarks are all equal. As two of these quarks have positive $\tilde{Q}$ charge, while the other two have it negative, the $\tilde{Q}$ neutrality of the medium is guaranteed without having to introduce any electron density.

Our zero-temperature results imply that a propagating rotated photon with energy less than the lightest charged-quark mode cannot scatter, since all the $\tilde{Q}$-charged quarks acquire a gap and all the Nambu-Goldstone bosons are neutral. The anisotropy present in the background of an external magnetic field and the existence of charged Goldstone bosons in CFL but not in MCFL indicates a rather different low energy physics, including transport properties. In particular, the MCFL superconductor is transparent and behaves at $T = 0$ as an anisotropic dielectric, as opposed to the isotropic dielectric behavior of the CFL phase [7,16]. However, similar to the CFL, the medium will become optically opaque as soon as leptons are thermally excited [17].

In previous analysis (see the first paper in [8]) the critical magnetic field at which the CFL pairing is destroyed was estimated to be $\sim 10^{20}$ G. This estimate was based in a field-independent CFL pairing energy $\mu^2(\Delta_A^{\mathrm{CFL}})^2$. Considering that in the MCFL phase Δ_A^B increases with the field, it is natural to expect that the critical field will be even larger than 10^{20} G. Since such extremely strong fields will surpass all the energy scales of the system, the quark infrared dynamics will become predominant, and the phenomenon of magnetic catalysis of chiral symmetry breaking [14] will be activated, producing a phase with quark-antiquark condensates but no quark-quark condensate. These two phases will have to be connected by a phase transition, as they have different number of Goldstone bosons due to the breaking of baryon symmetry, which only occurs in the superconducting phase.

Let us stress that in this work we have not considered the implications of finite quark masses. This, together with a careful study of the effects of the magnetic field in the low energy physics, in transport properties, or in neutrino dynamics will be the subject of future investigations.

In conclusion, we have found that a magnetic field leads to the formation of a new color-flavor locking phase, characterized by a smaller vector symmetry than the CFL phase. The essential role of the penetrating magnetic field is to modify the density of states of charged quarks on the Fermi surface. To better understand the relevance of this new phase in astrophysics we need to explore the region of moderately strong magnetic fields $\tilde{e}\tilde{B} < \mu^2/2$, which requires us to carry out a numerical study of the gap equations including the effect of higher Landau levels. Because the total density of states around the Fermi surface for charged particles does not vary monotonically with the number of Landau levels, we still expect to find a meaningful splitting of the gaps at these fields and therefore a qualitative separation between the CFL and MCFL phases.

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