

Einstein-Podolsky-Rosen Correlations via Dissociation of a Molecular Bose-Einstein Condensate

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Recent experimental measurements of atomic intensity correlations through atom shot noise suggest that atomic quadrature phase correlations may soon be measured with a similar precision. We propose a test of local realism with mesoscopic numbers of massive particles based on such measurements. Using dissociation of a Bose-Einstein condensate of diatomic molecules into bosonic atoms, we demonstrate that strongly entangled atomic beams may be produced which possess Einstein-Podolsky-Rosen (EPR) correlations in field quadratures in direct analogy to the position and momentum correlations originally considered by EPR.

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The recent demonstrations of atomic correlation measurements at the shot noise level [1,2] are a significant step towards true quantum-atom optics. Quantum optics, which began with photon correlation measurements, has allowed for many fundamental tests of quantum mechanics. Importantly, the availability of lasers allowed the development of techniques to perform quadrature phase measurements. In quantum-*atom* optics, Bose-Einstein condensates (BEC) play the role of the laser. However, homodyne noise correlation measurements of *atomic* field quadratures have not been available.

In this Letter, we suggest one route to achieve this, and outline a scheme which would allow for fundamental tests of quantum mechanics with massive particles. We base our proposal on the Einstein-Podolsky-Rosen (EPR) paradox [3]. The EPR paper introduced two particles with perfect correlations (entanglement) in momenta and positions, these persisting with spatial separation. Depending on which property of one particle we choose to measure, we can predict with certainty the same observable of the other particle. EPR concluded that local realism was inconsistent with the completeness of the quantum mechanical description of nature. As suggested by Reid in 1989 [4], products of variances of inferred optical quadratures can demonstrate the paradox by seeming to violate the Heisenberg uncertainty relation, although this is impossible for directly observed quadratures. This is applicable to realistic correlations, and was demonstrated experimentally by Ou *et al.* [5] in 1992.

We show here that dissociation [6–9] of a molecular BEC can also exhibit EPR correlations in atomic quadratures. Such tests of quantum mechanics [see also Ref. [10]] are a step toward understanding the properties of mesoscopic superpositions of massive particles, since they introduce couplings to gravitational fields, not previously tested in quantum measurement experiments.

There has been much experimental progress [11,12] and intense theoretical interest [13–23] in the production of molecular dimers from Bose condensed atoms. We will

assume that this can be created from bosonic constituents and propose a realization of the EPR paradox via dissociation, which can automatically yield two counterpropagating beams through momentum conservation. Starting from a three-dimensional (3D) molecular condensate, Dürr *et al.* [9] have used Feshbach resonance techniques to create a quasi-mono-energetic expanding wave of bosonic atoms, in close analogy to successful fermionic correlation experiments [2]. Using well-known optical trapping techniques [24], a 1D bosonic experiment would give the two beams needed here.

We consider an initially phase-coherent atomic BEC confined to one spatial dimension. [Reference [25] gives the necessary conditions.] This is then divided into three parts. The two outside parts are moved away from the central core, and stored for use as local oscillators. The central core is coherently converted to a molecular BEC. Our theory describes the result of a subsequent dissociation of the molecular BEC into two energetic “daughter” condensates [7], which interfere with the local oscillators to provide the measured quadrature signals.

The quantum-field-theory effective Hamiltonian describing this process is [7,13]

$$\hat{H} = \hat{H}_0 + \hbar \int dz \left\{ \sum_i V_i(z) \hat{\Psi}_i^\dagger \hat{\Psi}_i + \sum_{i,j} \frac{U_{ij}}{2} \hat{\Psi}_i^\dagger \hat{\Psi}_j^\dagger \hat{\Psi}_j \hat{\Psi}_i - i \frac{\chi(t)}{2} [\hat{\Psi}_2^\dagger \hat{\Psi}_1^2 - \hat{\Psi}_1^{\dagger 2} \hat{\Psi}_2] \right\}. \quad (1)$$

Here, the atomic and molecular fields are, respectively, described by the bosonic operators $\hat{\Psi}_1$ and $\hat{\Psi}_2$, \hat{H}_0 is the kinetic energy, $V_i(z)$ ($i = 1, 2$) are the trapping potentials (including internal energies), and the U_{ij} are the strengths of the one-dimensional intra- and cross-species s -wave interactions. The term $\chi(t) = \chi_0 \theta(t_1 - t)$ is responsible for coherent conversion of molecules into atom pairs, where $\chi_0 > 0$ and $\theta(t_1 - t)$ is the Heaviside function that turns off the coupling χ at $t > t_1$.

In what follows, we assume that the atom-atom s -wave scattering term is negligible. This condition is imposed in order to minimize the phase diffusion [26] of the two local oscillators required to access the atomic quadrature correlations. This requires either a low density and short interaction times, or else an effective renormalized interaction near a magnetic Feshbach resonance which is tuned to give a zero effective scattering ($U_{11} = 0$). The dissociation coupling χ_0 would be caused in the first case by a Feshbach sweep, or in the second case [27] by a coherent Raman transition with an overall detuning 2Δ [7]. This gives an energy mismatch $2\hbar\Delta < 0$ between the atomic and molecular fields, which is converted into kinetic energy [$2\hbar|\Delta| \rightarrow 2\hbar^2k^2/(2m_1)$] of atom pairs with opposite momenta around $k_0 = \pm\sqrt{2m_1|\Delta|/\hbar}$, where m_1 is the mass.

To gain some analytic insight, we will first analyze a simple model, beginning with a uniform molecular BEC in a coherent state with 1D (linear) density n_2 . The condensate extends from $-L/2$ to $L/2$, with periodic boundary conditions. The dissociation coupling χ is turned on suddenly, and subsequently assumed to be constant. Because we are interested in evolution over short times, the molecular field depletion is assumed at this stage to be negligible so that the amplitude $\Psi_2 = \sqrt{n_2}$ (assumed real) can be absorbed into an effective gain constant $g = \chi_0\sqrt{n_2}$. The dimensionless form of the equations is achieved by introducing characteristic time and length scales, $t_0 = 1/g$ and $d_0 = \sqrt{\hbar t_0/(2m_1)}$, and transforming to dimensionless time $\tau = t/t_0$, coordinate $\xi = z/d_0$, detuning $\delta = \Delta t_0 = \Delta/g$, and dimensionless fields $\hat{\psi}_i(\xi, \tau) = \hat{\Psi}_i(\xi d_0, \tau t_0)/\sqrt{n_2}$. The dimensionless initial molecular field density is now scaled to one.

We expand $\hat{\psi}_1(\xi, \tau)$ in terms of single-mode bosonic operators: $\hat{\psi}_1(\xi, \tau) = \sum_q \hat{a}_q(\tau) e^{iq\xi}/\sqrt{L}$, where $q = d_0k$ is a dimensionless momentum [$l = L/d_0$, $k = (2\pi/L)n$, $n = 0, \pm 1, \pm 2, \dots$]. The corresponding Heisenberg equations have the solutions $\hat{a}_q(\tau) = A_q(\tau)\hat{a}_q(0) + B_q(\tau)\hat{a}_{-q}^\dagger(0)$, and $\hat{a}_{-q}^\dagger(\tau) = B_q(\tau)\hat{a}_q(0) + A_q^*(\tau)\hat{a}_{-q}^\dagger(0)$, where $A_q(\tau) = \cosh(g_q\tau) - i\lambda_q \sinh(g_q\tau)/g_q$, $B_q(\tau) = \sinh(g_q\tau)/g_q$, $\lambda_q \equiv q^2 + \delta$, and $g_q \equiv (1 - \lambda_q^2)^{1/2}$. The coefficients A_q and B_q satisfy $|A_q|^2 - B_q^2 = 1$. The detuning δ is the only parameter that characterizes the dynamics of this dimensionless model. For dissociation to proceed, δ must be negative, which can be achieved by appropriate tuning of the frequencies of the Raman lasers.

In the above solutions, coupling is between opposite momentum components only. In quantum optics similar solutions have been studied by Reid [4] in the context of parametric down-conversion. In that case, the parameter λ_q would be identified with an effective phase mismatch term, which was set to zero. We note here that, unlike photons, the correlated atom pairs are not distinguishable by frequency or polarization, but by different momenta or spatial locations. We can now calculate any operator moments at time τ , given a vacuum initial state of the atomic fields.

We now consider the measurements that must be made to demonstrate the EPR paradox. It is readily seen that correlations exist between atomic quadratures of the beams with opposite momenta. For example, a measurement of $\hat{X}_q = \hat{a}_q + \hat{a}_q^\dagger$ allows us to infer, with some error, the value of $\hat{X}_{-q} = \hat{a}_{-q} + \hat{a}_{-q}^\dagger$, and vice versa. The same holds for the $\hat{Y}_{\pm q} = -i(\hat{a}_{\pm q} - \hat{a}_{\pm q}^\dagger)$ quadratures. This allows us to define, depending on which beam we measure, four inferred variances,

$$V^{\text{inf}}(\hat{X}_{\pm q}) = V(\hat{X}_{\pm q}) - [V(\hat{X}_{+q}, \hat{X}_{-q})]^2/V(\hat{X}_{\mp q}), \quad (2)$$

with similar expressions for $V^{\text{inf}}(\hat{Y}_{\pm q})$, where $V(a, b) = \langle ab \rangle - \langle a \rangle \langle b \rangle$. These quadrature correlations can be studied using balanced homodyne detection, which mixes the signal with a strong local oscillator on the matter-wave analog of a 50-50 beam splitter and is a well-known technique in quantum optics. Quadratures are measured via measuring the density differences, after combining the signal and local oscillator [28].

As an example demonstrating the EPR paradox we consider the correlations between the momentum components $\pm q_0 = \pm\sqrt{|\delta|}$ corresponding to perfect phase matching with $\lambda_q = 0$. In this case we obtain $V^{\text{inf}}(\hat{X}_{+q_0}) \times V^{\text{inf}}(\hat{Y}_{+q_0}) = \cosh^{-2}(2\tau) < 1$, and the same result for $V^{\text{inf}}(\hat{X}_{-q_0})V^{\text{inf}}(\hat{Y}_{-q_0})$. Since the products of the noninferred variances are bound by the Heisenberg uncertainty relation $V(\hat{X}_{\pm q_0})V(\hat{Y}_{\pm q_0}) \geq 1$, this is similar to the EPR paradox. However, as the plane-wave momentum components are not localized, this system does not allow the required spatial separation of the EPR *gedanken* experiment.

We now return to the more realistic case described by the full Hamiltonian (1). The dissociated atoms are assumed untrapped longitudinally yet confined transversely, so that they can be treated as a free 1D field, initially in a vacuum state. The absolute detuning $|\Delta|$ must not exceed the trap radial oscillation frequency ω_\perp in order to maintain the 1D condition [25] with “frozen” transverse motion of the dissociated atoms. For completeness, the atom-molecule and molecule-molecule scattering terms are all taken into account. Dimensionless interaction couplings are introduced according to $u_{i2} = U_{i2}\sqrt{n_2}/\chi_0$, where $n_2 = n_2(0)$ is the initial 1D peak density of the molecular BEC.

To solve for the resulting quantum dynamics, we use stochastic differential equations in the positive- P representation [26,29]. The essence of the positive- P method is in mapping the operator equations of motion into stochastic c -number differential equations that can be solved numerically. This requires four independent stochastic fields, ψ_i and ψ_i^\dagger , corresponding to the operators $\hat{\psi}_i$ and $\hat{\psi}_i^\dagger$, while $v_1(\xi, \tau) = u_{12}\psi_2^\dagger\psi_2$ and $v_2(\xi, \tau) = -u_{22}(1 - \xi^2/\xi_0^2) + \sum_i u_{i2}\psi_i^\dagger\psi_i$ represent the effective potentials including the atom-molecule and molecule-molecule s -wave interactions. Here, $\xi_0 = z_0/d_0$ is the dimensionless Thomas-Fermi (TF) radius. We include linear losses of atoms and

molecules at rates γ_i . The stochastic variables have a correspondence with normally ordered operator moments in the sense of an average over a large number of trajectories. The equations are

$$\begin{aligned} \frac{\partial \psi_1}{\partial \tau} &= i \frac{\partial^2 \psi_1}{\partial \xi^2} - (\gamma_1 + i\delta + iv_1)\psi_1 + \kappa\psi_2\psi_1^+ \\ &\quad + \sqrt{\kappa\psi_2}\eta_1 + \sqrt{-iu_{12}\psi_1\psi_2/2}(\eta_2 + i\eta_3), \\ \frac{\partial \psi_2}{\partial \tau} &= \frac{i}{2} \frac{\partial^2 \psi_2}{\partial \xi^2} - (\gamma_2 + iv_2)\psi_2 - \frac{\kappa}{2}\psi_1^2 + \sqrt{-iu_{22}\psi_2^2}\eta_4 \\ &\quad + \sqrt{-iu_{12}\psi_1\psi_2/2}(\eta_2 - i\eta_3), \\ \frac{\partial \psi_1^+}{\partial \tau} &= i \frac{\partial^2 \psi_1^+}{\partial \xi^2} - (\gamma_1 - i\delta - iv_1)\psi_1^+ + \kappa\psi_2^+\psi_1 \\ &\quad + \sqrt{\kappa\psi_2^+}\eta_5 + \sqrt{iu_{12}\psi_1^+\psi_2^+}/2(\eta_6 + i\eta_7), \\ \frac{\partial \psi_2^+}{\partial \tau} &= \frac{i}{2} \frac{\partial^2 \psi_2^+}{\partial \xi^2} - (\gamma_2 - iv_2)\psi_2^+ - \frac{\kappa}{2}\psi_1^{+2} \\ &\quad + \sqrt{iu_{22}\psi_2^{+2}}\eta_8 + \sqrt{iu_{12}\psi_1^+\psi_2^+}/2(\eta_6 - i\eta_7). \end{aligned} \quad (3)$$

Here, η_j ($j = 1, \dots, 8$) are real, independent Gaussian noise terms with the correlations $\overline{\eta_j(\xi, \tau)\eta_k(\xi', \tau')} = \delta_{jk}\delta(\xi - \xi')\delta(\tau - \tau')$, and $\overline{\eta_j} = 0$. To numerically integrate these equations, we consider that the molecular BEC is initially in a coherent state, represented spatially by the TF solution. The time duration for the molecule-atom conversion is controlled via $\kappa(\tau) = \theta(\tau_1 - \tau)$, so that $\kappa(\tau) = 0$ for $\tau > \tau_1$. Once the dissociation stops, we continue the evolution of the resulting atomic field in free space to allow spatial separation of atoms with positive and negative momenta. At the same time we set the molecular fields to zero for $\tau > \tau_1$. This models selective removal of the molecules by a ‘‘blast’’ pulse with a resonant laser [8] and is aimed at minimizing the effect of atom-molecule scattering and atom losses due to inelastic collisions, which can potentially reduce the atom-atom correlations. We note that the losses due to inelastic collisions are neglected altogether in our model. With typical loss rate coefficients of the order of $5 \times 10^{-17} \text{ m}^3/\text{s}$ [8] and peak molecular density $\sim 10^{20} \text{ m}^{-3}$, their disruptive effect should be negligible on submillisecond time scales used here. Similar considerations apply to the role of losses due to molecule-molecule inelastic collisions and three-body losses [24].

In the nonuniform treatment, due to the mode mixing of different momentum components, there is a difference in the way the necessary quadratures must be defined in comparison to our analytic plane-wave treatment [see also Ref. [30]]. With the quadratures defined in terms of individual Fourier components as above, we obtain no inferred violation of the uncertainty principle and hence no EPR correlation signature. This is because of the assumption of uniform local oscillators, implicitly built into this definition of the quadratures. This problem also arises

in the measurement of pulsed optical squeezing [31] and is overcome using pulsed local oscillators that are mode matched with the signals. Accordingly, we define four mode-matched quadrature operators as

$$\begin{aligned} \hat{X}_{\pm}(\tau) &= \int d\xi [\phi_{\pm}^*(\xi)\hat{\Psi}_1(\xi, \tau) + \phi_{\pm}(\xi)\hat{\Psi}_1^{\dagger}(\xi, \tau)], \\ \hat{Y}_{\pm}(\tau) &= -i \int d\xi [\phi_{\pm}^*(\xi)\hat{\Psi}_1(\xi, \tau) - \phi_{\pm}(\xi)\hat{\Psi}_1^{\dagger}(\xi, \tau)]. \end{aligned} \quad (4)$$

Here, $\phi_{\pm}(\xi) = |\phi_{\pm}(\xi)| \exp(\mp iq_0\xi - i\vartheta)$ are two nonuniform local oscillator fields having the same center-of-mass momenta, $\pm q_0 = \pm\sqrt{|\delta|}$, as the two respective atomic-beam signals. These are described classically, with Gaussian profiles, $|\phi_{\pm}(\xi)|^2 = (2\pi\sigma_{\pm}^2)^{-1/2} \exp(-\xi^2/2\sigma_{\pm}^2)$, with $\int d\xi |\phi_{\pm}(\xi)|^2 = 1$, and are centered at the locations of the twin atomic beams at the time of measurement τ . The phase ϑ of the local oscillators can be optimized to compensate for the molecular mean field phase shift.

The new quadrature operators have the same commutation relations as before so that the EPR criterion, $V^{\text{inf}}(\hat{X}_{\pm})V^{\text{inf}}(\hat{Y}_{\pm}) < 1$, and the expressions for the inferred variances are the same. Experimentally, the fields ϕ_{\pm} should be larger than the atomic signals, but they have been normalized to one here for convenience.

Figure 1(a) shows the final atomic and local oscillator densities, obtained from simulation of the full Eq. (3), averaged over 50 000 stochastic trajectories. In this example [32], the average number of atoms in each beam after dissociation is 52, with about 10% of them being lost during the subsequent free expansion stage. Note that, while the two local oscillators should share the same phase, they can have slightly different shapes, or atom numbers, without a strong destructive effect on the correlations. As we mentioned earlier, the local oscillators can in principle be prepared by splitting a single atomic BEC and then ‘‘stored’’ at spatial locations away from the molecular

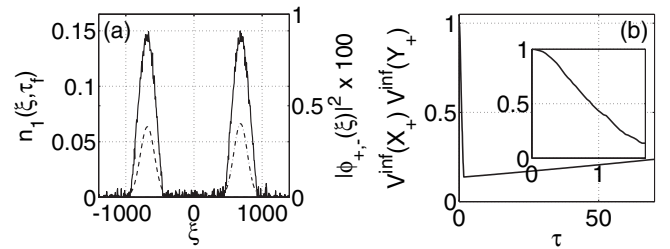


FIG. 1. (a) Final average atomic density distribution $n_1(\xi, \tau_f)$ (solid line) and the densities of the normalized local oscillator fields $|\phi_+(\xi)|^2$ and $|\phi_-(\xi)|^2$ (dash-dotted and dashed lines, with $\sigma_+ = 1.45$ and $\sigma_- = 1.50$, respectively). We used a time window of $\tau_f = 70$, with dissociation on from $\tau_0 = 0$ to $\tau_1 = 1.65$. Other parameter values are $\delta = -25$, $u_{22} = 0.2$, $u_{12} = 0.1$, $\xi_0 = 238$, and $\gamma_{1(2)} = 1.2 \times 10^{-3}$. (b) Product of the resulting inferred variances as a function of time τ for $\vartheta = 0.19$. The inset shows the same quantity while the dissociation is on.

BEC. The zero relative phase can be maintained by Feshbach tuning of the magnetic field to the zero crossing of the effective atom-atom scattering length. In this case, the relative phase drift due to possible unequal number of atoms in the two local oscillators is minimized.

Figure 1(b) shows the product of the inferred variances, giving a clear demonstration of the EPR paradox as $V^{\text{inf}}(\hat{X}_+)V^{\text{inf}}(\hat{Y}_+) < 1$. We note that quadrature correlations studied here require simultaneous measurement of many particles, and that repeated measurements of single atomic pairs would not yield the same results.

To summarize, we have shown that dissociation of a molecular BEC into bosonic atoms can provide a simple yet robust demonstration of the EPR paradox with massive particles. The effects of molecular condensate trapping and depletion, atom- and molecule-molecule s -wave interactions, and possible one-body losses of atoms and molecules have all been included in our numerical calculations. An experimental realization of our proposal would be the first step towards testing fundamental quantum mechanics with mesoscopic numbers of massive particles.

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- [1] S. Fölling *et al.*, Nature (London) **434**, 481 (2005).
 [2] M. Greiner *et al.*, Phys. Rev. Lett. **94**, 110401 (2005).
 [3] A. Einstein, B. Podolsky, and N. Rosen, Phys. Rev. **47**, 777 (1935); N. Bohr, Phys. Rev. **48**, 696 (1935).
 [4] M. D. Reid, Phys. Rev. A **40**, 913 (1989); M. D. Reid and P. D. Drummond, Phys. Rev. Lett. **60**, 2731 (1988).
 [5] Z. Y. Ou *et al.*, Phys. Rev. Lett. **68**, 3663 (1992).
 [6] U. V. Poulsen and K. Mølmer, Phys. Rev. A **63**, 023604 (2001).
 [7] K. V. Kheruntsyan and P. D. Drummond, Phys. Rev. A **66**, 031602 (2002); K. V. Kheruntsyan, Phys. Rev. A **71**, 053609 (2005).
 [8] T. Mukaiyama *et al.*, Phys. Rev. Lett. **92**, 180402 (2004).
 [9] S. Dürr, T. Volz, and G. Rempe, Phys. Rev. A **70**, 031601(R) (2004).
 [10] H. Pu and P. Meystre, Phys. Rev. Lett. **85**, 3987 (2000); L.-M. Duan *et al.*, *ibid.* **85**, 3991 (2000).
 [11] R. H. Wynar *et al.*, Science **287**, 1016 (2000).
 [12] E. A. Donley *et al.*, Nature (London) **417**, 529 (2002); J. Herbig *et al.*, Science **301**, 1510 (2003); K. Xu *et al.*, Phys. Rev. Lett. **91**, 210402 (2003); S. Dürr *et al.*, *ibid.* **92**, 020406 (2004).
 [13] P. D. Drummond, K. V. Kheruntsyan, and H. He, Phys. Rev. Lett. **81**, 3055 (1998); K. V. Kheruntsyan and P. D. Drummond, Phys. Rev. A **61**, 063816 (2000).
 [14] D. J. Heinzen *et al.*, Phys. Rev. Lett. **84**, 5029 (2000); P. D. Drummond, K. V. Kheruntsyan, D. J. Heinzen and R. H. Wynar, Phys. Rev. A **65**, 063619 (2002).
 [15] E. Timmermans *et al.*, Phys. Rev. Lett. **83**, 2691 (1999); Phys. Rep. **315**, 199 (1999).
 [16] J. Javanainen and M. Mackie, Phys. Rev. A **59**, R3186 (1999); M. Mackie, R. Kowalski, and J. Javanainen, Phys. Rev. Lett. **84**, 3803 (2000).
 [17] V. A. Yurovsky *et al.*, Phys. Rev. A **60**, R765 (1999).
 [18] F. A. van Abeelen and B. J. Verhaar, Phys. Rev. Lett. **83**, 1550 (1999).
 [19] M. Holland, J. Park, and R. Walser, Phys. Rev. Lett. **86**, 1915 (2001); S. J. J. M. F. Kokkelmans and M. J. Holland, *ibid.* **89**, 180401 (2002).
 [20] J. J. Hope and M. K. Olsen, Phys. Rev. Lett. **86**, 3220 (2001); M. K. Olsen and L. I. Plimak, Phys. Rev. A **68**, 031603 (2003).
 [21] K. Góral, M. Gajda, and K. Rzazewski, Phys. Rev. Lett. **86**, 1397 (2001).
 [22] T. Köhler, T. Gasenzer, and K. Burnett, Phys. Rev. A **67**, 013601 (2003).
 [23] R. A. Duine and H. T. C. Stoof, Phys. Rev. Lett. **91**, 150405 (2003); Phys. Rep. **396**, 115 (2004).
 [24] B. L. Tolra *et al.*, Phys. Rev. Lett. **92**, 190401 (2004); T. Kinoshita *et al.*, Science **305**, 1125 (2004).
 [25] K. V. Kheruntsyan *et al.*, Phys. Rev. Lett. **91**, 040403 (2003); Phys. Rev. A **71**, 053615 (2005).
 [26] M. J. Steel *et al.*, Phys. Rev. A **58**, 4824 (1998).
 [27] In a different context, the strategy of combining a Feshbach resonance and Raman transitions for atom-molecule conversion has been considered by S. J. J. M. F. Kokkelmans *et al.*, Phys. Rev. A **63**, 031601 (2001) and M. Mackie, *ibid.* **66**, 043613 (2002).
 [28] H.-A. Bachor, *A Guide to Experiments in Quantum Optics* (Wiley-VCH, Weinheim, 1998).
 [29] P. D. Drummond and C. W. Gardiner, J. Phys. A **13**, 2353 (1980); P. D. Drummond and J. F. Corney, Phys. Rev. A **60**, 2661(R) (1999).
 [30] V. A. Yurovsky and A. Ben-Reuven, Phys. Rev. A **67**, 043611 (2003).
 [31] M. G. Raymer, P. D. Drummond, and S. J. Carter, Opt. Lett. **16**, 1189 (1991).
 [32] Considering a $^{87}\text{Rb}_2$ experiment as a guideline, these parameters relate to a molecular BEC confined by a harmonic trap with an axial (radial) frequency $\omega_z/2\pi = 1$ Hz ($\omega_\perp/2\pi = 12$ kHz). We chose the molecule-molecule scattering length $a_{22} = 1$ nm, giving the 1D coupling $U_{22} \approx 2\hbar a_{22}/m_2 l_\perp^2 = 1.5 \times 10^{-4} \text{ ms}^{-1}$, where $m_2 = 2m_1$ and $l_\perp = \sqrt{\hbar/m_2\omega_\perp}$. In addition, we take the atom-molecule s -wave interaction strength $U_{12} = 0.75 \times 10^{-4} \text{ m s}^{-1}$; the relatively small values of these couplings are chosen here for making the computation feasible. The initial number of molecules is $N_2 = 380$, at 1D peak density of $n_2(0) = \pi l_\perp^2 n_2^{(3D)}(0) = 3.1 \times 10^6 \text{ m}^{-1}$ ($n_2^{(3D)}(0) = 2 \times 10^{20} \text{ m}^{-3}$), while the TF radius is $z_0 = \sqrt{2\hbar U_{22} n_2(0)/(m_2 \omega_z^2)} = 93 \text{ }\mu\text{m}$. We take the detuning $\Delta = -6.1 \times 10^4 \text{ s}^{-1}$, and the atom-molecule coupling $\chi^{(3D)} = 3.4 \times 10^{-7} \text{ m}^{3/2} \text{ s}^{-1}$, so that the 1D value is $\chi \approx \chi^{(3D)}/\sqrt{4\pi l_\perp^2} = 1.4 \text{ m}^{1/2} \text{ s}^{-1}$. The characteristic time scale is $t_0 = 0.41 \text{ ms}$, so that the duration of dissociation ($\tau_1 = 1.65$) corresponds to $t_1 = 0.68 \text{ ms}$, while the total time window ($\tau_f = 70$) is $t_f = 29 \text{ ms}$.
 [33] P. T. Cochrane, G. Collett, P. D. Drummond, and J. J. Hope, <http://www.xmnds.org>.