# Casimir-Like Force Arising from Quantum Fluctuations in a Slowly Moving Dilute Bose-Einstein Condensate

D. C. Roberts and Y. Pomeau

*Laboratoire de Physique Statistique de l'Ecole normale supérieure, Paris, France*
(Received 30 March 2005; published 30 September 2005)

We calculate a force due to zero-temperature quantum fluctuations on a stationary object in a moving superfluid flow. We model the object by a localized potential varying only in the flow direction and model the flow by a three-dimensional weakly interacting Bose-Einstein condensate at zero temperature. We show that this force exists for any arbitrarily small flow velocity and discuss the implications for the stability of superfluid flow.

DOI: 10.1103/PhysRevLett.95.145303 PACS numbers: 67.57.De, 03.75.Kk, 47.37.+q

Although there are various definitions of superfluidity [1], one of the defining features of a superfluid is the existence of a critical velocity below which the superfluid flows without dissipation. Landau argued that, by performing a Galilean transformation on the ground state of a uniform superfluid, the superfluid would become unstable above a well-defined critical velocity due to the creation of quasiparticles [2]. If one assumes that it is only through the creation of quasiparticles that dissipation can occur at $T = 0$, then one can infer that a stationary object in a slow-moving superfluid (with a flow velocity well below the critical velocity), would remain in a metastable stationary state as there would be no force acting on this object.

In this Letter, we argue that the phenomenological picture of superfluid flow—namely the existence of a metastable state below a critical velocity—is incomplete and problematic. We illustrate this by examining the case of a localized potential fixed in the flow of a three-dimensional dilute Bose-Einstein condensate. Specifically, we show that a force arises from the scattering of zero-temperature quantum fluctuations, an effect that Landau ignored in his argument for a critical velocity. We demonstrate the existence of this force in an infinitely extended condensate at all nonzero flow velocities (including velocities much lower than Landau's critical velocity).

Casimir [3] first showed that zero-temperature quantum fluctuations in an electromagnetic (EM) vacuum give rise to an attractive force between two closely spaced perfectly conducting plates. A Casimir-like force, $F_{\rm BEC}$, can be shown to arise from the zero-point quantum fluctuations in a dilute Bose-Einstein condensate (BEC), where infinitely thin and infinitely repulsive plates immersed in a zero-temperature three-dimensional dilute BEC replace Casimir's perfect conducting plates. $F_{\rm BEC}$ is given by (to leading order [4]) $F_{\rm BEC} \approx -\frac{\pi^2}{480}\frac{\hbar c_s \sigma}{d^4}$, where $c_s$ is the speed of sound in the dilute BEC, $d$ is the distance between the plates, and $\sigma$ is the area of the plates. Both of these forces (EM and BEC case) arise because boundary conditions are imposed on the quantum fluctuations.

Similarly, we posit that a Casimir-like force exists on an object in a moving dilute BEC. No direct EM analogy can be drawn for this situation because no absolute rest frame (where the relative motion of an object can be measured) exists for an EM vacuum. Nevertheless, we maintain that in a superfluid flow at zero temperature (modeled as a weakly interacting BEC) around a stationary object, a Casimir-like force on the object should arise due to the imposition of boundary conditions on the (BEC) quantum fluctuations. In a specific case of a weak potential varying only in the flow direction, we show that this force exists at all nonzero flow velocities; that is, the effective critical velocity is zero for this system, where critical velocity is defined as the flow velocity below which there is no force on a stationary object in the flow.

We now calculate the force arising from these quantum fluctuations. Momentum is not, in general, conserved in our system because the stationary object, which is described by the potential $\Phi(r)$, breaks the translational symmetry. In general, a force on a moving object described by a potential $\Phi(r)$ can be written in second quantized notation at zero temperature as

$$\vec{F} = -\int d^3r \langle \hat{\psi}^\dagger(r)[\vec{\nabla}\Phi(r)]\hat{\psi}(r)\rangle_{T=0}, \tag{1}$$

where $\hat{\psi}(r)$ and $\hat{\psi}^\dagger(r)$ are field operators that describe the weakly interacting BEC flow and obey the standard boson commutation relations and the expectation value is taken at $T = 0$. $T = 0$ is not well defined in the scattering problem discussed in this Letter, so one can view this simply as a convenient label of the quantum state that we define in detail below.

We model the superfluid as a weakly interacting three-dimensional condensate characterized by an interparticle contact pseudopotential, $g\delta^{(3)}(r)$, where $g$ is determined by the 2-particle positive scattering length $a_{\rm sc}$ and the mass $m$ of the atoms such that $g = 4\pi\hbar^2 a_{\rm sc}/m$. We assume the condensate to be dilute such that $\sqrt{\rho_0 a_{\rm sc}^3} \ll 1$ where $\rho_0$ is the condensate number density.

To calculate the force due to quantum fluctuations on a stationary object in a superfluid flow, we assume for simplicity that the object is described by a weak symmetric potential that varies only in the flow direction (which we take as the $x$ direction), i.e., $\Phi(\mathbf{r}) = \eta\Phi(x)$ where $\Phi(x) = \Phi(-x)$ and $\eta \ll 1$. (The parameter $\eta$ gives the order of magnitude of the external potential.) This situation, which can, in principle, be realized in current dilute BEC experiments, is a specific case chosen to show the existence of a finite Casimir-like force at any arbitrarily small flow velocity. We place further restrictions on this potential in the course of this Letter as needed.

Because we consider a potential varying only in the flow direction, the integrand in Eq. (1) is only a function of the positional variable $x$, which allows the simplification of the force expression to

$$F_x = -A\eta \int_{-\infty}^{\infty} dx \left\langle \hat{\psi}^\dagger(r) \frac{d\Phi(x)}{dx} \hat{\psi}(r) \right\rangle_{T=0}, \qquad (2)$$

where $A$ is the cross-sectional area of the object in the flow.

Although the lack of translational symmetry (due to the presence of the object) makes the existence of a force possible, it does not imply that there will necessarily be a force. For example, if the small quantum fluctuations are ignored, the bosonic field operator $\hat{\psi}$ can be approximated by the classical mean field $\Psi^{(0)}$, whose behavior is determined by the Gross-Pitaevskii equation (GPE). Working, as we do throughout this Letter unless specified otherwise, in dimensionless variables in which the length scale is normalized by the healing length given by $(8\pi\rho_0 a_{\rm sc})^{-1/2}$ and $\Psi$ is normalized by $\sqrt{\rho_0}$, the GPE can be written as

$$[\hat{T} + \Phi(x) - \mu]\Psi^{(0)}(r) + |\Psi^{(0)}(r)|^2\Psi^{(0)}(r) = 0, \qquad (3)$$

where $\hat{T} \equiv -\vec{\nabla}^2 + \sqrt{2}iq\frac{\partial}{\partial x} + q^2/2$, the dimensionless speed is given by $q = c/c_s$, $c$ is the speed of the flow at $x = \infty$, $c_s = \sqrt{\rho_0 g/m}$ is the speed of sound, and $\mu = 1 + q^2$ is the chemical potential [determined by imposing $\Psi^{(0)}(r) = 1$ at $x = \infty$]. The mean field force arising from the potential $\Phi(x)$, given by $-A\eta\int dx|\Psi^{(0)}(r)|^2 \frac{d\Phi(x)}{dx}$, can be shown to be zero below a certain critical flow velocity. This critical flow velocity (as measured far from the potential) in a nonuniform medium is always smaller than Landau's critical velocity in a uniform medium (which in the dilute gas is given by the speed of sound) due to nonlinear effects such as vortex shedding [5] or the creation of gray solitons [6].

If we go beyond the mean field approximation and take into account quantum fluctuations, the bosonic quantum field operator $\hat{\psi}$ can be split into a macroscopic classical field $\Psi^{(1)}$ and a small quantum fluctuation operator $\hat{\phi}$: $\hat{\psi} = \Psi^{(1)} + \hat{\phi}$. $\Psi^{(1)}$ is an improved approximation of the condensate wave function as compared to $\Psi^{(0)}$ because it includes the effects of the quantum fluctuations. Including the fluctuation operator in the analysis leads to a depletion of the ground state and a correction to the ground state energy [7], both on the order of the diluteness parameter $\sqrt{\rho_0 a_{\rm sc}^3}$, which must be small in order for the Bogoliubov theory in this Letter to be valid. While small in dilute gases, quantum depletion and its correlations have measurable effects (see, for example, [8]).

The force due to the boundary conditions imposed by the potential on the quantum fluctuations is, in general, not zero (even below the critical flow velocity in a nonuniform system given by the GPE) and can be written as

$$F_x = -A\eta \int dx(|\Psi^{(1)}(r)|^2 + \langle\hat{\phi}^\dagger(r)\hat{\phi}(r)\rangle_{T=0})\frac{d\Phi(x)}{dx}. \qquad (4)$$

The fluctuation operator can be expanded in terms of $\hat{\alpha}_k$ and $\hat{\alpha}_k^\dagger$—the quasiparticle annihilation and creation operators, respectively—such that $\hat{\phi}(r) = \sum_k[u_k(r)\hat{\alpha}_k - v_k^*(r)\hat{\alpha}_k^\dagger]$, where the sum excludes the condensate mode. For our system of weakly interacting particles to be described by the noninteracting quasiparticles, the quasiparticle amplitudes, $u_k(r)$ and $v_k(r)$, must obey the Bogoliubov–de Gennes (BdG) equations [9],

$$\hat{\mathcal{L}}u_k(r) - (\Psi^{(1)})^2 v_k(r) = E_k u_k(r), \qquad (5)$$

$$\hat{\mathcal{L}}^* v_k(r) - (\Psi^{(1)*})^2 u_k(r) = -E_k v_k(r), \qquad (6)$$

and the normalization condition $\int d^3r[|u_k(r)|^2 - |v_k(r)|^2] = 1$, where $\hat{\mathcal{L}} = \hat{T} + \Phi(x) - \mu + 2|\Psi^{(1)}|^2$ and $k$ is the dimensionless momentum normalized by the healing length. The energy eigenvalue for the moving BEC flow is $E_k = \sqrt{2}qk_x + E_B$ where the Bogoliubov dimensionless dispersion relation for a BEC at rest is given by $E_B = k\sqrt{k^2 + 2}$.

Since the BdG equations describe an effective scattering problem for the quasiparticle amplitudes, we can solve for $u_k(r)$ and $v_k(r)$ by specifying the incoming $u_k(r)$ and $v_k(r)$ (below we consider only scattered states). We impose a condition (see [10] for details) on the incoming $u_k(r)$ and $v_k(r)$ in terms of measurable quantities far from the potential that fully determine the quantum state in this problem, i.e., the quantum state used in the expectation value in Eq. (4). Despite the fact that the state at zero temperature is technically not well defined for this scattering problem, this label remains a convenient way to denote the quantum state relevant to Eq. (4) because the state is annihilated by $\hat{\alpha}$. The expectation values taken with respect to this state (or at "$T = 0$") can be written in terms of the quasiparticle amplitudes, i.e., $\langle\hat{\phi}^\dagger(r)\hat{\phi}(r)\rangle_{T=0} = \sum_k|v_k(r)|^2$.

The condensate wave function modified by the quantum fluctuations is given by the generalized GPE (GGPE) [11]

$$[\hat{T} + \Phi(x) - \mu]\Psi^{(1)}(r) + |\Psi^{(1)}(r)|^2\Psi^{(1)}(r) - \chi(r)\Psi^{(1)}(r) + \sum_k[2|v_k(r)|^2\Psi^{(1)}(r) - u_k(r)v_k^*(r)\Psi^{(1)}(r)] = 0, \qquad (7)$$

where the term proportional to $u_k(r)v_k^*(r)$ is ultraviolet divergent because of the contact potential approximation and must be renormalized (see references in [12]). The term $\chi(r)\Psi^{(1)}(r)$ ensures the orthogonality between the excited modes and the condensate [12] and is given by $\chi(r) = \sum_k c_k v_k^*(r)$ where $c_k = \int d^3r|\Psi^{(1)}(r)|^2[\Psi^{(1)*}(r) \times u_k(r) + \Psi^{(1)}(r)v_k^*(r)]$. The properties of $\Phi(x)$ (described below) make it such that neither $\chi(r)\Psi^{(1)}(r)$ nor the renormalization term contribute to the dominant order of the calculation below. Because we assume the condensate to be dilute, the fluctuation terms are small (on the order of $\sqrt{\rho_0 a_{\rm sc}^3}$) so the GGPE [Eq. (7)] can be approximated by the GPE with an effective complex potential given by $\zeta(x) = \sum_k 2|v_k(r)|^2 - u_k(r)v_k^*(r)$.

To calculate the force arising from zero-point quantum fluctuations $F_x$, we must first solve the Bogoliubov equations for the quantum amplitudes. To obtain a finite force we find it convenient to assume $\int_{-\infty}^{\infty} dx\Phi(x) = \tilde{\Phi}(0) = 0$ where $\tilde{\Phi}(\lambda)$ is the potential in Fourier space. Extracting the trivial phase factor, i.e., $u_k(r) = U(x)e^{i\mathbf{k}\cdot\mathbf{r}}$ and $v_k(r) = V(x)e^{i\mathbf{k}\cdot\mathbf{r}}$, and working in Fourier space defined by $U(x) = \int d\lambda e^{i\lambda x}U(\lambda)$ and $V(x) = \int d\lambda e^{i\lambda x}V(\lambda)$, the Bogoliubov equations can be solved perturbatively to give the quasiparticle amplitudes to first order in $\eta$ as $U_1(k, \lambda) = \tilde{\Phi}(\lambda)\frac{\Gamma_U(k,\lambda)}{C(k,\lambda)}\,\mathrm{sgn}(v_g^R)$ and $V_1(k, \lambda) = \tilde{\Phi}(\lambda)\frac{\Gamma_V(k,\lambda)}{C(k,\lambda)}\,\mathrm{sgn}(v_g^R)$. $\Gamma_U(k, \lambda)$ and $\Gamma_V(k, \lambda)$ are quantities easily derived from the Bogoliubov equations but, for the sake of clarity, we have chosen not to write out their full expressions as they would contribute little to the discussion. The sign of the group velocity of the reflected quantum fluctuation, denoted by $\mathrm{sgn}(v_g^R)$, arises from the boundary conditions where we exclude exponentially growing scattered waves and exclude incoming scattered waves due to causality. The group velocity of the reflected wave is given by $v_g^R = \frac{\partial E_k}{\partial \lambda_R}$ where the wave number of the reflected mode is given by $\lambda_R = k_x + k_R$ and $k_R$ is given by the nontrivial real root of the characteristic equation of the coupled Bogoliubov equations $C(k, \lambda) = \lambda[\lambda^3 + 4k_x\lambda^2 + (4k_x^2 + 2k^2 + 2 - 2q^2)\lambda + 4k_x(k^2 + 1) + 2\sqrt{2}qE_B] = 0$. Assuming $k_x = kf$, then $v_g^R > 0$ if $-1 < f < f_c$ and $v_g^R < 0$ if $f_c < f < 1$ where $f_c = -\frac{q\sqrt{k^2+2}}{\sqrt{2}(k^2+1)}$.

Next, these quasiparticle amplitudes can be used to determine the effective complex potential $\zeta(x)$ in the GGPE to give $\Psi^{(1)}(r)$. Then, integrating over all momenta of the quantum fluctuations, the Casimir-like force due to the quantum fluctuations $F_x$ can be divided into two contributions as seen in Eq. (4), given at the dominant order in $\eta$ as

$$\bar{F}_x = -\int d^3k[F_{\rm cond}(k) + F_{\rm fluc}(k)], \tag{8}$$

where $\bar{F}_x = F_x/\eta^2 p_0 A\sqrt{\rho_0 a_{\rm sc}^3}$ and the zeroth order interaction pressure is given by $p_0 = g\rho_0^2/2$. The contribution to the force due to the condensate modified by the quantum fluctuations is given by

$$F_{\rm cond}(k) = \mathrm{Res}_{\lambda=k_R}\frac{8\sqrt{2}}{\sqrt{\pi}}\tilde{\Phi}(\lambda)\lambda/(\lambda^2+2-2q^2)\left\{\left[U_0(k)\left(1+\frac{\sqrt{2}q}{\lambda}\right)-4V_0(k)\right]\tilde{V}_1(\lambda,k)+V_0(k)\left(1-\frac{\sqrt{2}q}{\lambda}\right)\tilde{U}_1(\lambda,k)\right\}, \tag{9}$$

and the contribution to the force given directly by the quantum fluctuations is given by

$$F_{\rm fluc}(k) = \mathrm{Res}_{\lambda=k_R}\frac{8\sqrt{2}}{\sqrt{\pi}}\tilde{\Phi}(\lambda)V_0(k)\lambda\tilde{V}_1(\lambda, k), \tag{10}$$

where $\mathrm{Res}_{\lambda=k_R}z(\lambda)$ is the residue of $z(\lambda)$ at $\lambda = k_R$. The zeroth order quantum amplitudes (for a homogeneous gas at rest) are given by $U_0(k) = \sqrt{\frac{1}{2}(\frac{k^2+1}{E_B} + 1)}$ and $V_0(k) = \sqrt{\frac{1}{2}(\frac{k^2+1}{E_B} - 1)}$.

Finally, to illustrate the calculation of $F_x$, we define a specific potential describing the stationary object in the flow as $\tilde{\Lambda}(\lambda) \equiv [\tilde{\Phi}(\lambda)]^2 = \frac{1}{\Delta}$ for $|\lambda + k_0| < \Delta/2$, $|\lambda - k_0| < \Delta/2$, and zero otherwise. In real space this can be written as $\Lambda(x) = \frac{\sin(x\Delta/2)}{x\pi\Delta}2\cos(k_0x)$ where $\Delta$ is positive, $1/\Delta$ is a measure of the width of the potential in real space and $k_0$ is a measure of the typical wave number of the potential in real space. Both $\Delta$ and $k_0$ are normalized by the healing length. We assume $k_0 > \Delta/2$ so that the potential satisfies the condition $\tilde{\Phi}(0) = 0$. Note that as $\Delta \to 0$ the potential becomes periodic in real space and delocalized; our analysis would then no longer apply. $\bar{F}_x$ peaks when the width in Fourier space (or real space) is on the order of the healing length, i.e., $\Delta \approx 1$ and disappears in the localized and delocalized limits.

The existence of this force is necessary but not sufficient for the system to be dissipative. In the rest frame of the object described by $\Phi(r)$, the system conserves energy because of the time translational symmetry. Therefore, the usual picture of an "irreversible" phenomenon with dissipation in the ordinary sense does not apply. A numerical solution of the coupled GGPE and BdG equations in the time-dependent lab frame should verify the existence (or nonexistence) of dissipation in this system. Assuming this force calculated in this Letter is dissipative, i.e., a drag force, then the system would want to minimize its energy as a function of flow speed, i.e., relative speed between the flow and the stationary object. In the above example the grand canonical energy decreases with increasing flow speed, the opposite of the usual situation. It follows from this that the flow would accelerate in the presence of dissipation, implying a negative drag force similar to the behavior of a moving gray soliton [13]. At larger $k_0$ such behavior is exhibited by $\bar{F}_x$ as seen in Fig. 1. The behavior of $\bar{F}_x$ at larger flow velocities and at smaller $k_0$ (or, equivalently, larger characteristic wavelengths of the potential in real space) suggests a sign change of the effective mass as occurs in moving matter wave packets in a periodic potential. It is also perhaps instructive to recall the nontrivial and highly geometry-dependent sign of the Casimir force in the EM vacuum (for example, a setup of parallel conductors in an EM vacuum leads to an attractive force while a cubical cavity leads to a repulsive force).

Even though this Casimir-like force exists for speeds much lower than the speed of sound, which in Bogoliubov's theory is equal to Landau's critical velocity

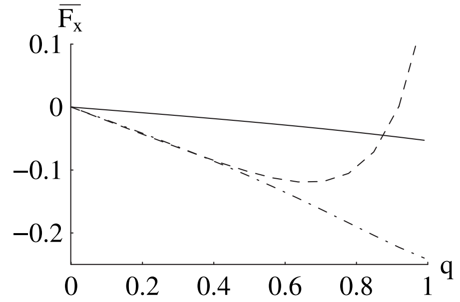


FIG. 1. The scaled force due to quantum fluctuations, $\bar{F}_x = F_x/\eta^2 p_0 A\sqrt{\rho_0 a_{sc}^3}$, acting on a stationary localized potential described by the parameters $k_0$ and $\Delta$ as a function of the mach number of the flow, $q$ (measured far from the potential, i.e., at $x = \infty$), for three different values of $k_0$: $k_0 = 5.0$ is given by the solid line, $k_0 = 2.4$ is given by the dash-dotted line, and $k_0 = 1.8$ is given by the dashed line. The width of the potential in real (and Fourier) space is equal to the healing length, i.e., $\Delta = 1$. ($k_0$ and $\Delta$ are both normalized by the healing length.)

for the onset of dissipation, this force does not explicitly violate the spirit of Landau's principle [14]. Let us recall that Landau's principle states that above a critical velocity a system can lower its energy by the creation of quasiparticles [2]. In our analysis, however, we do not assume that any quasiparticles are created; i.e., the system remains at $T = 0$ in the sense defined above. This force arises from the scattering of zero-temperature quantum fluctuations and is not caused by the creation of quasiparticles, which must satisfy the Landau criterion; instead, it is caused by the changing nature of the eigenstates of the quantum fluctuations, akin to the original Casimir force between two conductors.

In this Letter, we have shown in a specific example that a finite Casimir-like force arises in a moving BEC at $T = 0$. In this case, unlike for the nucleation of vortices, there is no free energy barrier to cross and, at least for this particular situation, the effective critical velocity is zero. Since a nonzero effective critical velocity does exist at the dominant order (on the order of the mean field), one would expect to find the semblance of a nonzero critical velocity as seen in [15], even though, at least in the case considered here, the actual critical velocity for the system might be zero.

We expect this Casimir-like force to act upon any density perturbation—including those created by laser fields [15], untrapped impurities [16], vortices, etc.—moving relative to the condensate. We also expect this force to be more apparent in condensates of lower dimensions due to the enhancement of quantum fluctuations. Finally, although the present analysis assumes a dilute gas and does not strictly apply to dense systems, we do expect a Casimir-like force from quantum fluctuations also to exist for superfluid liquid helium. In fact, the Casimir-like force might have a stronger effect in liquid helium since quantum fluctuations dominate the helium condensate at $T = 0$.

We conclude by noting that although we have discussed a force that exists on a stationary object in a superfluid moving at any arbitrarily small velocity, these results are not inconsistent with the existence of persistent superfluid currents in toroidal geometries. In this Letter where we consider an infinite medium, the Casimir-like force arises from the nonlocal perturbation of the scattered quantum fluctuations. These scattered fluctuations can be seen to transport energy far from the potential similar to wave-drag situations in classical fluids. However, in a finite geometry such as a superflow in a torus the scattered fluctuations will interact with the localized object. In the steady state, we expect these backscattered waves to eventually cancel out the effect discussed in this Letter and thus remain consistent with a persistent superflow. In such a system, this Casimir-like effect would manifest itself not as continual dissipation at arbitrarily low speeds, but rather as a new time scale (expected to be proportional to the characteristic length of the system over the speed of sound) over which the flow becomes dissipationless. The scattered fluctuations should also lead to a local change of temperature (assuming higher order interactions among quasiparticles), which could, in principle, be observed [4].

The authors thank I. Carusotto, K. Burnett, C. Connaughton, D. Guéry-Odelin, A. Fetter, S. Morgan, V. Hakim, and M. Kraemer for valuable discussions.

[1] A. J. Leggett, Rev. Mod. Phys. **71**, S318 (1999).
[2] L. D. Landau, J. Phys. (Moscow) **5**, 71 (1941).
[3] H. B. G. Casimir, Proc. K. Ned. Akad. Wet. **51**, 793 (1948).
[4] D. C. Roberts and Y. Pomeau (to be published).
[5] T. Winiecki, J. F. McCann, and C. S. Adams, Phys. Rev. Lett. **82**, 5186 (1999); T. Frisch, Y. Pomeau, and S. Rica, Phys. Rev. Lett. **69**, 1644 (1992).
[6] V. Hakim, Phys. Rev. E **55**, 2835 (1997).
[7] T. D. Lee, K. Huang, and C. N. Yang, Phys. Rev. **106**, 1135 (1957).
[8] L. Pitaevskii and S. Stringari, Phys. Rev. Lett. **81**, 4541 (1998); D. M. Stamper-Kurn *et al.*, Phys. Rev. Lett. **83**, 2876 (1999); D. C. Roberts, T. Gasenzer, and K. Burnett, J. Phys. B **35**, L113 (2002).
[9] A. L. Fetter, Ann. Phys. (N.Y.) **70**, 67 (1972).
[10] Specifically, we require that the spatial-temporal correlation function—formed by the expectation value of the incoming fluctuation operators taken with respect to this quantum state—be identical to that of a Galilean-transformed uniform ground state. Furthermore, this condition affects only the incoming waves as the correlation functions are defined in such a way that the spatial-temporal difference describes a wave that propagates towards the stationary object.
[11] Y. Castin and R. Dum, Phys. Rev. A **57**, 3008 (1998).
[12] S. Morgan, Phys. Rev. A **69**, 023609 (2004).
[13] P. O. Fedichev, G. V. Shlyapnikov, and J. T. M. Walraven, Phys. Rev. Lett. **80**, 2269 (1998).
[14] Y. Pomeau, Europhys. Lett. **27**, 377 (1994).
[15] R. Onofrio *et al.*, Phys. Rev. Lett. **85**, 2228 (2000).
[16] A. P. Chikkatur *et al.*, Phys. Rev. Lett. **85**, 483 (2000).