Hyperelasticity, Viscoelasticity, and Nonlocal Elasticity Govern Dynamic Fracture in Rubber

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Dynamic cracks in rubber can spontaneously oscillate under certain biaxial strain conditions [R. D. Deegan *et al.,* Phys. Rev. Lett. **88**, 014304 (2002)]. We have found that this unusual phenomenon can be understood from the unique mechanical properties of rubber: hyperelasticity, viscoelasticity, and nonlocal elasticity. While all these are important, the decisive role of nonlocality needs to be particularly emphasized. Through numerical simulations with a lattice model, we have quantitatively reproduced the experimental results.

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Introduction.—Upon a balloon popping, the edges of some of the broken pieces are often rippled. This suggests that the cracks tearing the material must be oscillating during propagation. This simple yet amazing phenomenon constitutes a fracture pattern distinct from the more common one where cracks in brittle materials branch upon losing stability. Despite its fundamental importance, however, it had not been investigated seriously until four years ago when Deegan and co-workers systematically carried out the first fracture experiments with rubber sheets [1]. It was found that the fracture pattern in rubber is solely determined by the applied strain condition, and at certain biaxial strains cracks would propagate along sine-wave-like paths.

This very unusual instability of dynamic fracture poses a new challenge to the study of crack dynamics. Henry and Levine first investigated this problem using a phase field model [2]. The phenomenon was qualitatively reproduced, but a quantitative comparison to the experiments was lacking. Furthermore, the details of the fracture process cannot be obtained from this kind of model, and some of the results are in doubt in our opinion. Recently, Marder developed a shock wave theory that can explain the supersonic rupture but not the crack oscillation [3].

This Letter presents a numerical study of a lattice model, from which we have found some clues for the first time to quantitatively explain the oscillating instability of fracture in rubber.

Lattice model.—Lattice models have been frequently used to study fracture and phase transition problems since the seminal work of Slepyan [4–13]. The inherent discreteness makes this type of model naturally suitable for these problems where the continuum approximation might be inappropriate.

Our lattice model takes into account the intrinsic mechanical properties of rubber: hyperelasticity, viscoelasticity, and nonlocal elasticity. The first two terms have already been included in Marder's lattice model [3]. However, it seems to us that, although the importance of the nonlocal effect has been recognized in a variety of mechanical problems such as micron-scale plasticity [14–16] and elastic or plastic fracture [16–19], it has never been considered in the studies of fracture of hyperelastic materials.

The model for a rubber sheet is a triangular lattice on which the mass points are connected with massless bonds to nearest (NN) and next-to-nearest neighbors (NNN). The interactions between the NNNs represent nonlocal effects. At the continuum level, such a model corresponds to a strain gradient nonlocal theory [13,20,21] which takes into account the contributions to the strain energy from both strain and strain gradient (thus introducing a material length scale). To account for the hyperelasticity and viscoelasticity, the massless bonds are taken as rubbery springs with dashpots in parallel (Kelvin viscosity). The force of a bond is the sum of the elastic and viscous components, the values of which are $F_{\mathrm{elastic}} = A_0[a(\lambda - 1/\lambda^2) + b(1 - 1/\lambda^3)]$ and $F_{\mathrm{viscous}} = A_0 \eta \nu / l_0$, respectively [22]. The resultant force on a mass point is obtained by adding up all the forces on it by its neighbors. The motion of mass points is governed by Newton's second law. The equations of motion are integrated numerically using the velocity Verlet algorithm.

To simulate a fracture experiment, the lattice strip is stretched to a desired biaxial strain state (ε_x, ε_y) with the four edges being fixed, a seed crack is then made by simply removing several mass points somewhere, and at each time step, once the elongations of any bonds exceed a critical value λ_f, they are deleted permanently. In addition, we follow Marder [3] to increase λ_f behind the crack to keep its back surface intact. But even with this, a fraction of mass points will still detach from an oscillating crack. To account for their possible collisions with any other parts of the lattice, additional neighbor relations are found using the link-cell algorithm from molecular dynamics.

Simulation results.—Figure 1 shows one of our simulated cases where an oscillating fracture path has been

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quantitatively reproduced [23]. In fact, through extensive computer experiments on our model material, we have found some clues to the understanding of the fracture oscillation. And by adjusting the model parameters, we are able to control not only the fracture path and speed but also the amplitude and wavelength of the oscillation. In this way, we have quantitatively reproduced most of the experimental results [1] (maximum error is less than 20%, while some data are in perfect agreement).

Mechanisms of crack oscillation.—In order to understand how the crack oscillation takes place, we have carefully investigated the near-tip stress and velocity fields and the issue of supersonic propagation. The opening stress fields in both the oscillating and the nonoscillating cases are kidney shaped, showing no qualitative difference. The velocity fields appear more suggestive (see Fig. 2). We have found in all the simulated cases (oscillating and nonoscillating) a negative velocity zone (NVZ) ahead of the crack tip in which the mass points move in the opposite direction of the crack propagation. The NVZ appears to induce shear motion that should be important for the tip oscillation. Furthermore, the velocity field is symmetric for a straight crack, but becomes asymmetric upon the crack starts to oscillate (in some cases there are even vortices near the tip). Another thing worthy of noting is a small zone behind the crack tip in which the mass points move rapidly along the crack direction. Such a zone can be regarded as a process zone and exists only in the case of oscillation. It fades for a straight crack.

Previous studies have guessed that the crack oscillation may be connected to the supersonic propagation [3,24]. The wave speeds used to compare with the crack speeds were either measured or calculated at the rubbery state with no viscous effect, and it was shown that both the oscillating and nonoscillating cracks were faster than the shear wave. But the limited experimental data cannot exclude the possibility of subsonic rupture (oscillating or not). Indeed, at least in our numerical studies (despite the model being imperfect [25]), both oscillating and nonoscillating cracks can be supersonic as well as subsonic. Therefore, we tend to believe that the supersonic state is neither a necessary nor a sufficient condition for the oscillation. However, it is hard to draw a conclusion without experimental proof of subsonic oscillation.

Effects of material properties.—To see the effect of nonlocality, we have tried removing the NNN interactions from our model. Then there is no way to induce oscillations, and cracks can only propagate straight just as in Marder's study [3] where only the NN interactions were included. In this situation, we have observed a qualitative change in the velocity and stress fields. That is, the negative velocity zone disappears [26], and the opening stress field is less concentrated and no longer kidney shaped. As mentioned earlier, a lattice model with NNN interactions is equivalent to a strain gradient nonlocal theory at the continuum level. The large strain gradient near a crack tip can elevate the strain energy and stress near the tip [19], which in this study should be crucial for the crack oscillation.

Regarding the effect of hyperelasticity, it is found that the nonlinearity of the deformation is not crucial in reproducing the oscillation. In fact, the bonds in our lattice models are just weakly nonlinear at large strains, and even when they are made ideally linear, crack oscillation still can occur. In contrast, the amount of the deformation appears much more decisive. It is found that at small strains (e.g., several percent) unstable cracks always branch other than oscillate, which is in accord with the more common fracture pattern in brittle materials.

For viscoelasticity, we find its effect is strongly coupled with that of the discreteness of the lattice. We define a dimensionless parameter $\zeta = \tau/T$ to represent this combined effect. Here $\tau = \eta/a$ is the retardation time that has a broad spectrum for a viscoelastic material, and $T = d/(a/\rho)^{1/2}$ is the time for stress wave traveling a distance of d (lattice spacing). Our results show that both the normalized amplitude and the wavelength (by d) of the oscillation decrease with decreasing ζ (by decreasing η or increasing d), and when ζ becomes smaller than a critical value, the oscillation is inhibited absolutely and the crack will travel straight. Here we note that, according to the

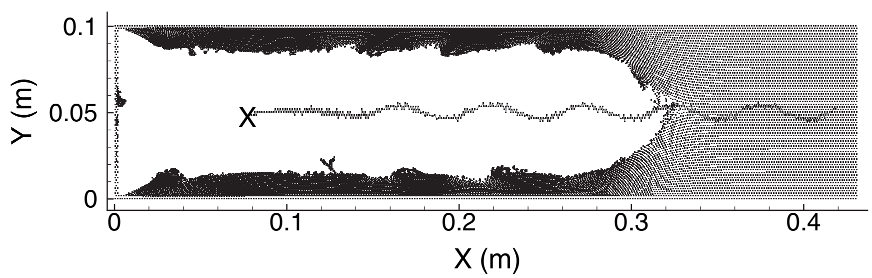

FIG. 1. Simulation results at applied strains $\varepsilon_x = 1.3$, $\varepsilon_y = 1.8$. Shown here is the lattice strip in full size at 5.56 ms. Together shown is the oscillating fracture path that in steady state is a sinusoid. The crack tip is defined on the rightmost mass point that has lost any 3 of its 12 neighbors. The cross marks the seed crack. The input parameters are $\rho = 944$ kg/m^3, $a = 0.1$ MPa, $b = 0.15$ MPa, $\eta_{NNN} = 3\eta_{NN} = 196.875$ Pa · s, $d = 0.625$ mm, and $\lambda_f = 3.05$. The simulated wavelength and amplitude of the oscillation are 2.3 and 0.17 cm (converted to the unstrained state), respectively, while the experimental results were about 2.1 and 0.175 cm [1]. The steady crack speed is 48 m/s (or 20.9 m/s in Lagrangian frame), being in the range of the experimental measurements [1].

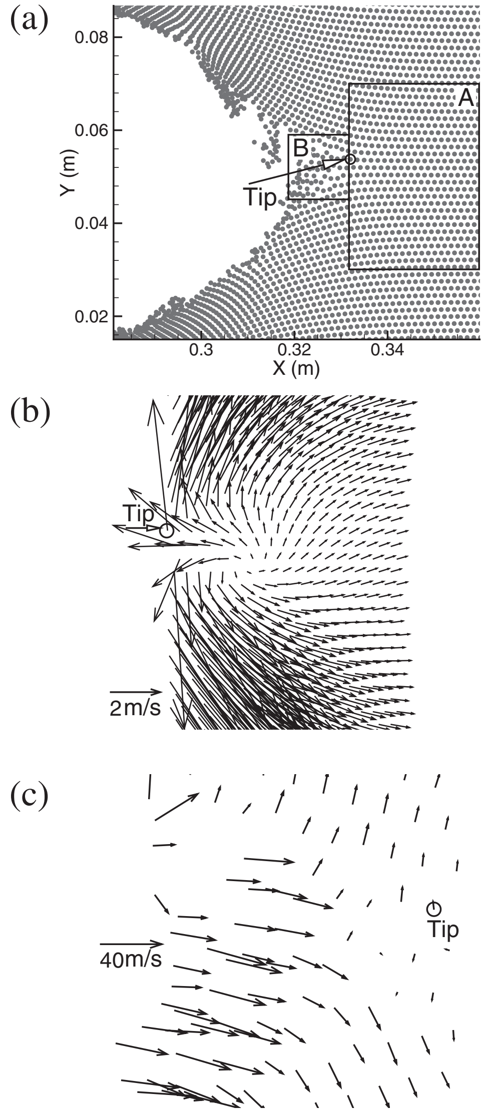

FIG. 2. (a) Magnification of the near-tip region of Fig. 1. Note the wedgelike shape of the crack. (b) The velocity field of the region inside box A in (a). Note the backward motion and the vortex. (c) The velocity field of the region inside box B in (a). We regard this region as the process zone. Note the rapid forward motion.

phase field model [2], the wavelength of the oscillation was not significantly affected by the viscosity, which we think physically unreasonable.

The effect of the viscosity can be understood as the following. de Gennes [27] has proposed three distinct zones along with the cohesive zone at a crack tip propagating in a viscoelastic solid, i.e., an unrelaxed glassy zone, a viscous dissipation zone, and a fully relaxed rubbery zone. The size of the glassy zone equals the crack speed multiplied by time τ; thus with τ decreasing the glassy zone shrinks and the dissipation zone surrounding it extends closer to the crack tip. The result is that more energy is dissipated near the tip and the oscillation is suppressed.

The dependence of the oscillation upon d seems more confusing, as normally the ''mesh size'' is just a numerical quantity and the simulation results should converge if the ''mesh'' is fine enough. Nevertheless, the lattice spacing in lattice model must not be read in this way. In fact, the situation here is very much similar to the virtual internal bond (VIB) modeling of fracture where the ''mesh size'' is a physical quantity representing a certain physical length scale [28,29]. In conventional finite element simulation of fracture, a length scale is usually introduced via a cohesive zone so the mesh size is a numerical quantity only [30]. However, in the lattice or VIB model for fracture no physical length scale is explicitly assigned, so it must adhere to the ''mesh size.'' In fact, the lattice spacing in this study corresponds to the characteristic length (or material length) that is present in any nonlocal continuum theory [13,20,21].

At this point, the physical meaning of ζ becomes evident. That is, there are two characteristic times (or lengths) in our model (associated with viscosity and nonlocality, respectively), and it is the ''competition'' between them that determines the crack oscillation. It also becomes clear why a lattice model containing only NN interactions cannot reproduce crack oscillation. Such a model and its continuum counterpart (e.g., in Ref. [3]) are scale independent and incorporate no material lengths, hence cannot bring out the apparent length scales (wavelength and amplitude) in an oscillating fracture path.

Discussion.—In this study, η, d, and λ_f are the key parameters associated with the fracture. At first glance, they seem to be material constants so that a single set of values will suffice to reproduce all the experiments under different strain conditions. Nevertheless, we have had to adjust the values of them with the applied strains to reproduce the experimental results. To understand this, we note that both the viscoelastic and nonlocal responses are intrinsically multiscale, so η and d indeed can have multiple values. Obviously, if a model includes a sufficient number of η's and d's that cover the entire time and length scales, there will be no need to modify their values with the applied strains. However, any realistic model can take only a limited number of η's and d's. Fortunately, under a given strain condition, it appears that only a few η's and d's (probably only one η and one d) are dominant, so a model including even a single η and d can be adequate. Of course, then their values must be different at different strain conditions. Concerning the dependence of λ_f on the strain state, Marder has given some explanation [3]. We believe that a certain applied strain condition will induce certain microstructural changes, which then determine certain λ_f and the dominant η and d.

At this point, the roles of the applied strain condition can be summarized as follows: providing the energy for fracture, building large deformation that is necessary for the oscillation, and ''selecting'' the ''right'' values of η, d, and

λ_f. It is through all of these that a given strain condition "selects" a unique fracture pattern from many possible patterns [1].

The last issue to note is the influence of the sample width on the crack propagation. A problematic result from the phase field model [2] is that the wavelength scaled linearly with the sample width, whereas the relation we have seen is not this simple. In fact, it is found that the crack will oscillate only when the sample is wide enough, and then the wavelength and amplitude indeed increase with increasing sample width, but not linearly, till saturation.

Conclusion.—We have created a lattice model for explaining the oscillating fracture paths in rubber. It is established that the dynamic fracture of rubber is essentially governed by hyperelasticity, viscoelasticity, and nonlocal elasticity. While all these are important, the most important finding of this work is the crucial role of nonlocal effects in at least some aspects of dynamic fracture.

Our study calls for the development of a gradient-type nonlocal theory of rubber. The theory should reduce to a local theory when the strain gradient is zero. Perhaps the simplest way to do so is modifying the "local" Mooney-Rivlin theory by adding gradient terms to the strain energy. With such nonlocal theory at the continuum level, one will be able to construct an even better lattice model for studying the dynamic rupture of rubber.

Finally, even though we know how to control the motion of a crack, it still is unclear why it would spontaneously oscillate under the right condition. We believe that it would be profitable to further study this problem with the theory of pattern formation [31].

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[23] See EPAPS Document No. E-PRLTAO-95-026541 for a movie of this simulation. This document can be reached via a direct link in the online article's HTML reference section or via the EPAPS homepage (http://www.aip.org/pubservs/epaps.html).
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