

## Difference in $B^+$ and $B^0$ Direct $CP$ Asymmetry as an Effect of a Fourth Generation

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Direct  $CP$  violation in  $B^0 \rightarrow K^+ \pi^-$  decay has emerged at the  $-10\%$  level, but the asymmetry in  $B^+ \rightarrow K^+ \pi^0$  mode is consistent with zero. This difference points towards possible new physics in the electroweak penguin operator. We point out that a sequential fourth generation, with sizable  $V_{t's}^* V_{t'b}$  and near maximal phase, could be a natural cause. We use the perturbative QCD factorization approach for  $B \rightarrow K\pi$  amplitudes. While the  $B^0 \rightarrow K^+ \pi^-$  mode is insensitive to  $t'$ , we critically compare  $t'$  effects on direct  $CP$  violation in  $B^+ \rightarrow K^+ \pi^0$  with  $b \rightarrow s\ell^+ \ell^-$  and  $B_s$  mixing. If the  $K^+ \pi^0 - K^+ \pi^-$  asymmetry difference persists, we predict  $\sin 2\Phi_{B_s}$  to be negative.

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Direct  $CP$  violation (DCPV) in  $B^0 \rightarrow K^+ \pi^-$  decay has recently been observed [1,2] at the  $B$  factories. The combined asymmetry is  $\mathcal{A}_{K\pi} = -0.114 \pm 0.020$ . However, the asymmetry in  $B^+ \rightarrow K^+ \pi^0$  decay is found to be [2,3]  $\mathcal{A}_{K\pi^0} = +0.049 \pm 0.040$ , which differs from  $\mathcal{A}_{K\pi}$  by

$$\mathcal{A}_{K\pi^0} - \mathcal{A}_{K\pi} = +0.163 \pm 0.045, \quad (1)$$

with  $3.6\sigma$  significance. All existing models have predicted  $\mathcal{A}_{K\pi^0} \sim \mathcal{A}_{K\pi}$ , as this basically follows from isospin symmetry. The large difference of Eq. (1), if it persists, could indicate isospin breaking new physics (NP), likely [4] through the electroweak penguin (EWP) operator.

In this Letter we point out a natural source for such EWP effects: the existence of a fourth generation. The  $t'$  quark can modify the EWP coefficients, but leaves the strong and electromagnetic penguin coefficients largely intact. Equation (1) can be accounted for, provided that  $m_{t'} \sim 300$  GeV, and the quark mixing elements  $V_{t's}^* V_{t'b}$  are not much smaller than  $V_{cb}$  and have near maximal  $CP$  phase. Independently,  $b \rightarrow s\ell^+ \ell^-$  and  $B_s$  mixing constraints can allow large  $t'$  effects only if [5] the associated  $CP$  phase is near maximal.

Precision electroweak data imply that  $|m_{t'} - m_{b'}|$  cannot be too large [6]. Unitarity of quark mixing requires  $|V_{ub'}| < 0.08$  [6], while constraining  $V_{t's}^* V_{t'b}$  is the subject of this Letter. Since  $b \rightarrow d$  transitions appear standard model (SM) like, we set  $V_{t'd} \sim 0$ . We thus decouple from  $s \rightarrow d$  constraints such as  $\epsilon_K$  and  $K \rightarrow \pi\nu\nu$  as well [7].

Adding a fourth generation modifies short distance coefficients. Defining  $\lambda_q = V_{qs}^* V_{qb}$ , the effective Hamiltonian relevant for  $B \rightarrow K\pi$  can be written as

$$H_{\text{eff}} \propto \lambda_u (C_1 O_1 + C_2 O_2) + \sum_{i=3}^{10} (\lambda_c C_i - \lambda_{t'} \Delta C_i) O_i, \quad (2)$$

where  $O_{1,2}$  are the tree operators,  $\lambda_c C_i$  are the usual SM penguin terms, and  $-\lambda_{t'} \Delta C_i$  with  $\Delta C_i \equiv C_i^{t'} - C_i$  is the fourth generation effect. We have used  $\lambda_u + \lambda_c + \lambda_t + \lambda_{t'} = 0$ , simplified by ignoring  $|\lambda_u| \lesssim 10^{-3}$ , such that  $\lambda_t \equiv -\lambda_c - \lambda_{t'}$  [8]. The penguin coefficients  $\lambda_t C_i +$

$\lambda_{t'} C_i^{t'}$  at scale  $\mu$  are then put [5] in the form of Eq. (2), which respect the SM limit for  $\lambda_{t'} \rightarrow 0$  or  $m_{t'} \rightarrow m_t$ . Explicit forms for  $C_i$  and  $O_i$  can be found, for example, in Ref. [9].

The  $K\pi$  amplitudes are dominated by  $C_{4,6}^{t'}$ . To illustrate  $t'$  sensitivity, in Fig. 1 we plot  $-\Delta C_i / |C_4^{t'}|$  at  $m_b$  scale versus  $m_{t'}$ . The effect is clearly most prominent for the EWP  $C_9$  coefficient, with linear  $x_{t'} \equiv m_{t'}^2 / M_W^2$  dependence arising from  $Z$  and box diagrams [8].  $\Delta C_7$  has similar dependence but has weaker strength. For the strong penguin  $\Delta C_{4,6}$ , the  $t'$  effect in the QCD penguin loop is weaker than logarithmic [10] and is very mild. As we shall see, the  $B^0 \rightarrow K^+ \pi^-$  amplitude does not involve the EWP. In contrast, the  $B^+ \rightarrow K^+ \pi^0$  amplitude is sensitive to the EWP via  $\Delta C_9 - \Delta C_7$  (virtual  $Z$  materializing as  $\pi^0$ ).

We see that it is natural for the fourth generation to show itself through the EWP. The effect depends also on the quark mixing matrix product, parametrized as [5]

$$\lambda_{t'} = V_{t's}^* V_{t'b} = r_s e^{i\phi_s}. \quad (3)$$

The phase  $\phi_s$  is needed to affect the CPV observables, Eq. (1). Most works on the fourth generation have ignored the phase in  $V_{t's}^* V_{t'b}$ , making the fourth generation effect far less flexible hence uninteresting.

Let us first see how  $\mathcal{A}_{K\pi} < 0$  can be generated. In the usual QCD factorization (QCDF) approach [11], strong phases are power suppressed, while strong penguin  $C_4$  and  $C_6$  coefficients pick up perturbative absorptive parts.

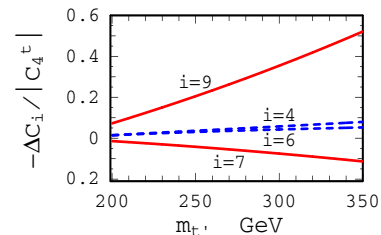


FIG. 1 (color online). The  $t'$  correction  $-\Delta C_i$  normalized to the strong penguin coefficient  $|C_4^{t'}|$  (both at  $m_b$  scale) vs  $m_{t'}$ .

Thus, the predicted  $\mathcal{A}_{K\pi}$  is small, and turns out to be positive. For the perturbative QCD factorization (PQCDF) [12] approach, one has an additional absorptive part coming from the annihilation diagram, which arises from a cut on the two quark lines in  $B \rightarrow \bar{s}q \rightarrow K\pi$  decay. In this way, the PQCDF approach predicted [12] the sign and order of magnitude of  $\mathcal{A}_{K\pi}$ . By incorporating annihilation contributions as in PQCDF, however, QCDF can also [13] give negative  $\mathcal{A}_{K\pi}$ .

We adopt PQCDF as a definite calculational framework. The  $\bar{B}^0 \rightarrow K^- \pi^+$  amplitude for the 3 generation SM is roughly given by

$$\mathcal{M}_{K^- \pi^+}^{\text{SM}} \propto \lambda_u f_K F_e + \lambda_c (f_K F_e^P + f_B F_a^P), \quad (4)$$

where  $F_e^{(P)}$  is the color-allowed tree (strong penguin) contribution and is real, and  $F_a^P$  is the strong penguin annihilation term that has a large imaginary part. We have dropped subdominant nonfactorizable effects for sake of presentation. Details cannot be given here, but these factorizable contributions can be computed by following Ref. [12], convoluting the hard part (related to short distance coefficients  $C_i$ ) and the soft, nonperturbative meson wave functions. Basically, all the  $F_j^{(P)}$ s are integrals over Bessel functions, and, in particular, a Hankel function for  $F_a^P$  [12]. We give the SM numbers for  $F_e$ ,  $F_e^P$ , and  $F_a^P$  in Table I, which leads to  $\mathcal{A}_{K\pi} = -0.16$  for  $\phi_3 \equiv \arg \lambda_u^* = 60^\circ$  [value used throughout [14]], compared to the experimental value of  $-0.114 \pm 0.020$ .

For  $B^- \rightarrow K^- \pi^0$ , the difference with  $K^- \pi^+$  is

$$\sqrt{2} \mathcal{M}_{K^- \pi^0}^{\text{SM}} - \mathcal{M}_{K^- \pi^+}^{\text{SM}} \propto \lambda_u f_\pi F_{ek} + \lambda_c f_\pi F_{ek}^P, \quad (5)$$

where  $F_{ek}$  is the color-suppressed tree term, while  $F_{ek}^P$  is the color-allowed EWP, and both are real. A negligible tree annihilation term  $\lambda_u f_B F_a$  has been dropped. Since both the  $F_{ek}$  and  $F_{ek}^P$  terms are subdominant compared to  $F_e^P$  in the 3 generation SM,  $\mathcal{A}_{K\pi^0}$  and  $\mathcal{A}_{K\pi}$  cannot be far apart. From the values of  $F_{ek}$  and  $F_{ek}^P$  given in Table I, we get  $\mathcal{A}_{K\pi^0} = -0.10$ , which is less negative than  $\mathcal{A}_{K\pi}$ , but at some variance with Eq. (1).

Adding the  $t'$  quark, one finds  $\mathcal{M}_{K^- \pi^+} \cong \mathcal{M}_{K^- \pi^+}^{\text{SM}}$ . The difference is proportional to  $\lambda_{t'} (f_K \Delta F_e^P + f_B \Delta F_a^P)$ , which is small unless  $\lambda_{t'}$  is very large. This is because  $F_{e,a}^P$  are strong penguins, hence  $\Delta F_{e,a}^P$  depends very weakly on  $m_{t'}$ , as can be seen from Table I (for  $m_{t'} = 300$  GeV) and Fig. 1. Thus,  $\mathcal{A}_{K\pi}$  is insensitive to the fourth generation. For  $K^- \pi^0$ , one finds

$$\sqrt{2} \mathcal{M}_{K^- \pi^0} - \sqrt{2} \mathcal{M}_{K^- \pi^+}^{\text{SM}} \propto -\lambda_{t'} f_\pi \Delta F_{ek}^P, \quad (6)$$

where again  $\Delta F_{e,a}^P$  terms have been dropped, and  $\Delta F_{ek}^P$  is the  $t'$  correction to the EWP, which is generated by  $\Delta C_9 - \Delta C_7$  at short distance.

Let us put the  $K^- \pi^+$  and  $K^- \pi^0$  amplitudes in more heuristic form. Equation (4) can be put in the form

$$\mathcal{M}_{K^- \pi^+} \approx \mathcal{M}_{K^- \pi^+}^{\text{SM}} \propto r e^{-i\phi_3} + e^{i\delta}, \quad (7)$$

and the fourth generation effect is minor. The ratio  $r = |\lambda_u| f_K F_e / \lambda_c |f_K F_e^P + f_B F_a^P|$  parametrizes the relative strength of tree ( $T$ ) versus strong penguins ( $P$ ), and  $\delta$  is the strong phase of  $f_K F_e^P + f_B F_a^P$  arising from  $F_a^P \equiv |F_a^P| e^{i\delta_a}$ . Analogously, for  $K^- \pi^0$  one roughly has

$$\mathcal{M}_{K^- \pi^0} \propto r \left( 1 + \frac{f_\pi F_{ek}}{f_K F_e} \right) e^{-i\phi_3} + \frac{f_\pi F_{ek}^P}{|f_K F_e^P + f_B F_a^P|} + e^{i\delta} - \frac{f_\pi \Delta F_{ek}^P}{|f_K F_e^P + f_B F_a^P|} \left| \frac{V_{t's}^* V_{t'b}}{V_{cs}^* V_{cb}} \right| e^{i\phi_s}, \quad (8)$$

where  $F_{ek}$  and  $F_{ek}^P$  terms come from SM [see Eq. (5)], and the  $\Delta F_{ek}^P$  term comes from the  $t'$  effect of Eq. (6). Since  $r \sim 1/5$ , we see from Table I that, for  $m_{t'} \sim 300$  GeV and  $|V_{t's} V_{t'b}| \equiv r_s$  not much smaller than  $|V_{cb}| \sim 0.04$ , the impact of  $t'$  on  $\mathcal{A}_{K\pi^0}$  could be significant.

We have presented in the above the major contributions in PQCDF framework. Performing a detailed calculation following Ref. [12], we plot  $\mathcal{A}_{K\pi}$  and  $\mathcal{A}_{K\pi^0}$  in Fig. 2(a) for  $m_{t'} = 300, 350$  GeV and  $r_s = 0.01$  and  $0.03$ . We see that, indeed,  $\mathcal{A}_{K\pi}$  is almost independent of  $t'$ , while it is clear that the largest impact on  $\mathcal{A}_{K\pi^0}$  is for  $\phi_s \sim \pm \pi/2$  and large  $m_{t'}$  and  $r_s$ . To maximize  $\mathcal{A}_{K\pi^0} - \mathcal{A}_{K\pi} > 0$ ,  $\phi_s \sim +\pi/2$  is selected, and Eq. (1) can in principle be accounted for.

The  $\mathcal{A}_{K\pi} \sim -0.16$  value is at some variance with the experimental value of  $-0.114 \pm 0.020$ . This number depends crucially on the strong penguin phase. Rather than varying detailed model parameters, we vary  $\delta \equiv \arg(f_K F_e^P + f_B F_a^P)$ . The sign difference between tree and strong penguin constitutes a phase of  $\pi$ , and  $\pi - \delta \sim 24^\circ$  is perturbative. We plot  $\mathcal{A}_{K\pi}$  and  $\mathcal{A}_{K\pi^0}$  versus  $\phi_s$  in Fig. 2(b) for  $m_{t'} = 300$  GeV and  $r_s = 0.03$ , for  $\delta = 155^\circ, 156^\circ$  (nominal), and  $160^\circ$ . We see that a slightly smaller  $\pi - \delta$  lowers  $|\mathcal{A}_{K\pi}|$  and is preferred. Note that  $\mathcal{A}_{K\pi^0} \sim 0$  around  $\phi_s \sim 90^\circ$  is due to a near cancellation between the  $\phi_3$  (tree) and  $\phi_s$  (EWP) contributions. Thus, we think PQCDF can account for  $\mathcal{A}_{K\pi} = -0.114 \pm 0.020$  without

TABLE I. Factorizable contributions for  $B^{0[+]} \rightarrow K^+ \pi^{-[0]}$  in standard model, and for  $m_{t'} = 300$  GeV. The difference between the  $t'$  and  $t$  penguin contributions gives  $\Delta F_j^P$ . “N/A” stands for “not applicable.”

	Tree	$t$ penguin	$t'$ penguin
$F_e^{(P)}$	0.841 [0.843]	-0.074 [-0.075]	-0.076 [-0.078]
$F_a^{(P)}$	N/A [0.001 + 0.002 <i>i</i> ]	0.003 + 0.026 <i>i</i> [0.003 + 0.026 <i>i</i> ]	0.003 + 0.026 <i>i</i> [0.003 + 0.026 <i>i</i> ]
$F_{ek}^{(P)}$	N/A [-0.105]	N/A [-0.014]	N/A [-0.029]

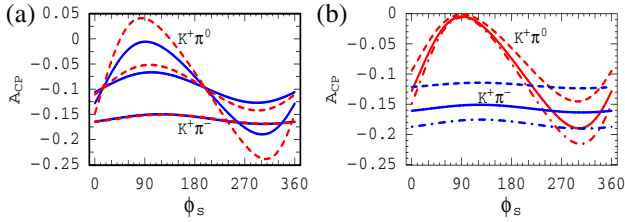


FIG. 2 (color online). Direct CPV asymmetries  $\mathcal{A}_{K\pi}$  and  $\mathcal{A}_{K\pi^0}$  vs  $\phi_s \equiv \arg V_{t's}^* V_{t'b}$ . In (a) the solid and dashed curves are for  $m_{t'} = 300$  and  $350$  GeV, respectively, and for  $r_s \equiv |V_{t's}^* V_{t'b}| = 0.01$  and  $0.03$ . All curves for  $\mathcal{A}_{K\pi}$  coalesce, but for  $\mathcal{A}_{K\pi^0}$ , the  $r_s = 0.03$  curves are steeper. For (b) the strong penguin absorptive phase  $\delta$  is varied from  $155^\circ$  (dot-dashed line),  $156^\circ$  (solid line) to  $160^\circ$  (dashed line) for  $m_{t'} = 300$  GeV and  $r_s = 0.03$ .

affecting  $\mathcal{A}_{K\pi^0}$ , but the NP phase  $\phi_s$  should be rather close to  $90^\circ$ .

To entertain a large EWP effect in CPV in  $b \rightarrow s$  decay, one needs to be mindful of the closely related  $b \rightarrow s\ell^+\ell^-$  and  $B_s$  mixing constraints, as well as the usually stringent  $b \rightarrow s\gamma$  constraint. We have checked that the  $b \rightarrow s\gamma$  rate constraint is well satisfied for the range of parameters under discussion. This is because on-shell photon radiation is generated by the  $b \rightarrow s$  transition operator  $O_{7\gamma}$ , and the associated coefficient  $\Delta C_{7\gamma}$  has weaker  $m_{t'}$  dependence than  $\Delta C_7$  shown in Fig. 1. However,  $b \rightarrow s\ell^+\ell^-$  is generated by EWP [8] operators very similar to  $O_{7-10}$  in Eq. (2) for  $b \rightarrow s\bar{q}q$ . The difference is basically just in the  $Z$  charge of  $q$  versus  $\ell$ , hence with same  $m_{t'}$  dependence. The box diagram for  $B_s$  mixing also has similar  $m_{t'}$  dependence. Taking the formulas from Ref. [5], we plot  $b \rightarrow s\ell^+\ell^-$  rate ( $m_{\ell\ell} > 0.2$  GeV) and  $\Delta m_{B_s}$  versus  $\phi_s$  in Figs. 3(a) and 3(b), for  $m_{t'} = 300, 350$  GeV and  $r_s = 0.01$  and  $0.03$ .

We can understand the finding of Ref. [5] that  $\phi_s \sim 90^\circ$  is best tolerated by the  $b \rightarrow s\ell^+\ell^-$  and  $\Delta m_{B_s}$  constraints. For  $\cos\phi_s < 0$ , the  $b \rightarrow s\ell^+\ell^-$  rate gets greatly enhanced [5], and would run against recent measurements. One is therefore forced to the  $\cos\phi_s > 0$  region, where  $t'$  effect is destructive against SM  $t$  effect. For  $\Delta m_{B_s}$ , the effect gets destructive for  $\cos\phi_s > 0$  when  $r_s$  is sizable. Since one just has a lower bound [6] of  $14.4$  ps $^{-1}$ ,  $\Delta m_{B_s}$  tends to push one away from the  $\cos\phi_s > 0$  region. The combined effect is to settle around  $\phi_s \sim \pm\pi/2$ , i.e., imaginary [5]. This result is independent of the discrepancy of Eq. (1).

For sake of discussion we have plotted, as horizontal solid straight lines in Fig. 3(a), the  $1\sigma$  range of  $\mathcal{B}(B \rightarrow X_s\ell^+\ell^-) = (6.1^{+2.0}_{-1.8}) \times 10^{-6}$  [6] for  $m_{\ell\ell} > 0.2$  GeV. This is the Particle Data Group (PDG) 2004 average over Belle and BaBar results [15,16], with a combined total of  $154$  M  $B\bar{B}$  pairs. Belle has recently measured [17] with  $152$  M  $B\bar{B}$  pairs the value  $\mathcal{B}(B \rightarrow X_s\ell^+\ell^-) = (4.11 \pm 0.83^{+0.74}_{-0.70}) \times 10^{-6}$  for  $m_{\ell\ell} > 0.2$  GeV, which would be more stringent. However, this lower result should be confirmed by BABAR, hence we use the more conservative [18] PDG 2004 range.

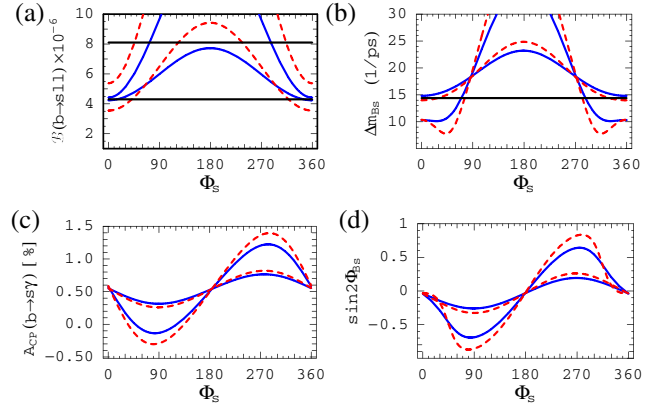


FIG. 3 (color online). (a)  $\mathcal{B}(b \rightarrow s\ell^+\ell^-)$ , (b)  $\Delta m_{B_s}$ , (c)  $A_{CP}(b \rightarrow s\gamma)$ , and (d)  $\sin 2\Phi_{B_s}$  vs  $\phi_s = \arg V_{t's}^* V_{t'b}$ . Notation is the same as Fig. 2(a), with effect strongest for larger  $r_s$  and  $m_{t'}$ . Horizontal solid band in (a) corresponds to  $1\sigma$  experimental range, and solid line in (b) is the lower limit, both from Ref. [6]. The experimental range for (c) is outside the plot.

For  $\Delta m_{B_s}$ , we plot the PDG bound of  $14.4$  ps $^{-1}$  [6] as horizontal solid straight line in Fig. 3(b).

Comparing Figs. 2(a), 3(a), and 3(b), we set  $\mathcal{A}_{K\pi^0} > -0.05$  as a requirement for a solution, for otherwise it is hard to satisfy Eq. (1), and in any case the fourth generation would seem no longer needed. This requirement demands  $r_s > 0.01$ . For  $m_{t'} = 350$  GeV and  $r_s = 0.03$ , which can best bring  $\mathcal{A}_{K\pi^0} \geq 0$ , Figs. 3(a) and 3(b) mutually exclude each other. For  $m_{t'} = 300$  GeV and  $r_s = 0.03$  (the case for  $m_{t'} = 350$  GeV and  $r_s = 0.02$  is very similar), one finds  $\phi_s \simeq 75^\circ$  gives  $\mathcal{A}_{K\pi^0} \sim 0$ . However,  $\mathcal{B}(b \rightarrow s\ell\ell)$  must be close to the maximal value of  $\sim 8 \times 10^{-6}$ , and  $\Delta m_{B_s}$  would be just above the bound. For lower  $r_s$  values, the solution space is broader. For example, for  $m_{t'} = 300$  GeV and  $r_s = 0.02$ , one has  $\mathcal{A}_{K\pi^0} \geq -0.05$  for  $\phi_s \sim 63^\circ$ – $100^\circ$ .  $\mathcal{B}(b \rightarrow s\ell\ell)$  can reach below  $6 \times 10^{-6}$ , but then  $\Delta m_{B_s}$  would again approach the current bound.

We see that for a range of parameter space roughly around  $m_{t'} \sim 300$  GeV and  $0.01 < r_s \leq 0.03$ , solutions to Eq. (1) can be found that do not upset  $b \rightarrow s\ell\ell$  and  $\Delta m_{B_s}$ . Both large  $t'$  mass and sizable  $V_{t's}$  mixing are needed; no solutions are found for  $m_{t'} = 250$  GeV.

As the CPV effect through the EWP is large, one may worry if similar effects may show up already in  $b \rightarrow s\gamma$ . We follow Ref. [19], extend to 4 generations, and plot  $A_{CP}(b \rightarrow s\gamma)$  versus  $\phi_s$  in Fig. 3(c). Like the  $\mathcal{A}_{K\pi^0}$  case, the  $t'$  effect cancels against the SM phase.  $|A_{CP}(b \rightarrow s\gamma)|$  is in general smaller than the SM value of  $\sim 0.5\%$ , and consistent with the current measurement of  $0.004 \pm 0.036$  [20]. In fact, it is below the sensitivity for the proposed high luminosity ‘‘Super B factory.’’

As prediction, we find  $\sin 2\Phi_{B_s} < 0$  for CPV in  $B_s$  mixing, which is plotted versus  $\phi_s$  in Fig. 3(d). We find  $\sin 2\Phi_{B_s}$  in the range of  $-0.2$  to  $-0.7$  and correlating with  $\mathcal{A}_{K\pi^0} - \mathcal{A}_{K\pi}$ . Three generation SM predicts zero.

Note that refined measurements of  $\mathcal{B}(b \rightarrow s\ell\ell)$  and future measurements of  $\Delta m_{B_s}$  and  $\sin 2\Phi_{B_s}$ , together with theory improvements, can pinpoint  $m_{t'}$ ,  $r_s$ , and  $\phi_s$ . We note further that [6]  $14.4 \text{ ps}^{-1} < \Delta m_{B_s} < 21.8 \text{ ps}^{-1}$  cannot yet be excluded because data are compatible with a signal in this region. We eagerly await  $B_s$  mixing and associated CPV measurement in the near future.

It is of interest to predict the asymmetries for the other two  $B \rightarrow K\pi$  modes.  $K^0\pi^-$  is analogous to  $\mathcal{M}_{K^-\pi^+}$  except tree contribution is absent. We find  $\mathcal{M}_{\bar{K}^0\pi^-} \cong \mathcal{M}_{\bar{K}^0\pi^-}^{\text{SM}} \propto \lambda_c(f_K F_e^P + f_B F_a^P)$ , so  $\mathcal{A}_{K^0\pi^-} \approx 0$  and insensitive to  $t'$ . For  $\bar{B}^0 \rightarrow \bar{K}^0\pi^0$ , we have  $\mathcal{M}_{\bar{K}^0\pi^0} \propto \lambda_u f_\pi F_{ek} + \lambda_c(-f_K F_e^P - f_B F_a^P + f_\pi F_{ek}^P) - \lambda_{t'} f_\pi \Delta F_{ek}^P$ . Numerics can still be obtained from Table I, giving  $\mathcal{A}_{K^0\pi^-} - \mathcal{A}_{K^0\pi^0} \sim 0.1$  if  $\mathcal{A}_{K^0\pi^0} - \mathcal{A}_{K\pi^-}$  is of order suggested by Eq. (1). The impact on mixing-dependent CPV in  $\phi_{K_S}$  and  $\eta'K_S$  modes is insignificant [5].

The measurement of  $\mathcal{A}_{K^0\pi^0}$  itself should not yet be viewed as settled, since the recent *BABAR* value of  $+0.06 \pm 0.06 \pm 0.01$  changed sign from the previous [21] value of  $-0.09 \pm 0.09 \pm 0.01$ . But if  $\mathcal{A}_{K^0\pi^0} \sim 0$ , hence Eq. (1), stays, we would need a large effect in the EWP with a new CPV phase. Note that, unlike most treatments of the EWP, our strong phase is not a fitted parameter, but calculated from PQCDF [22].

We have also studied separately the final state rescattering (FSI) model [23] as a different proposed source of strong phase. In this model, one allows  $K^+\pi^-\leftrightarrow K^0\pi^0, \pi^+\pi^-\leftrightarrow K^0\pi^0, \pi^+\pi^-\leftrightarrow K^0\pi^0, \pi^+\pi^-\leftrightarrow K^0\pi^0$  rescattering in the final state (power suppressed in QCDF and PQCDF), and, to avoid double counting, one uses naïve factorization amplitudes as source before rescattering. In this way, one can account [23] for  $\mathcal{A}_{K\pi^-} < 0$ , and also generate a sizable  $\pi^0\pi^0$  via rescattering from  $\pi^+\pi^-$ . Neither QCDF nor PQCDF can account for  $\mathcal{B}(B^0 \rightarrow \pi^0\pi^0) > 10^{-6}$ . However, in contrast to Eq. (1),  $\mathcal{A}_{K^0\pi^0}$  is found [23] to be more negative than  $\mathcal{A}_{K\pi^-}$  for  $\mathcal{A}_{K\pi^-} < 0$ . We find no solution to Eq. (1), even when  $t'$  is considered. Besides the problem that already exists in 3 generation SM, rescattering brings the electro-weak penguin into the  $K^-\pi^+$  amplitude from the  $\bar{K}^0\pi^0$  mode, so adding the  $t'$  does not help.

We have shown that a fourth generation  $t'$  quark can account for  $\mathcal{A}_{K^0\pi^0} \sim 0$ . Using PQCD factorization calculations, one can account for  $\mathcal{A}_{K\pi^-} < 0$  (untouched by  $t'$ ) and generate the needed  $\mathcal{A}_{K^0\pi^0} - \mathcal{A}_{K\pi^-}$  splitting, which repeats in  $\mathcal{A}_{K^0\pi^-} - \mathcal{A}_{K^0\pi^0}$ . The closely related  $b \rightarrow s\ell^+\ell^-$  mode should have a rate not less than  $6 \times 10^{-6}$ , and  $B_s$  mixing should not be far above the current bound of  $14.4 \text{ ps}^{-1}$ . In fact, between the  $b \rightarrow s\ell^+\ell^-$  rate and the bound on  $B_s$  mixing,  $V_{t's}^* V_{t'b}$  should be near imaginary if one wants a large  $t'$  effect. We predict a quite measurable CP violating phase  $\sin 2\Phi_{B_s}$  in the  $-0.2$  to  $-0.7$  range. Refined measurements of the last three measurables can determine  $m_{t'}$  and the strength and phase of  $V_{t's}^* V_{t'b}$ .

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