Evolving Dark Energy with $w \neq -1$

Lawrence J. Hall, Yasunori Nomura, and Steven J. Oliver

Department of Physics, University of California, Berkeley, and Theoretical Physics Group, Lawrence Berkeley National Laboratory, Berkeley, California 94720, USA

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Theories of evolving quintessence are constructed that generically lead to deviations from the $w = -1$ prediction of nonevolving dark energy. The small mass scale that governs evolution, $m_\phi \approx 10^{-33}$ eV, is radiatively stable, and the "Why now?" problem is solved. These results rest on seesaw cosmology: Fundamental physics and cosmology can be broadly understood from only two mass scales, the weak scale v and the Planck scale M. Requiring a scale of dark energy $\rho_{\rm DE}^{1/4}$ governed by v^2/M and a radiatively stable evolution rate m_ϕ given by v^4/M^3 leads to a distinctive form for the equation of state $w(z)$. Dark energy resides in the potential of a hidden axion field that is generated by a new QCD-like force that gets strong at the scale $\Lambda \approx v^2/M \approx \rho_{\rm DE}^{1/4}$. The evolution rate is given by a second seesaw that leads to the axion mass $m_\phi \approx \Lambda^2/f$, with $f \approx M$.

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Introduction.—The dominant energy density in the universe has negative pressure, causing a recent acceleration in the expansion of the universe [1], and is known as dark energy. What is the physical picture for this unusual fluid? How can the size of its energy density, $\rho_{\rm DE} \approx (10^{-3}\ \text{eV})^4$, be understood, and how can the underlying physics be probed?

One interpretation of dark energy is in terms of a parameter Λ that determines a fixed energy and pressure for the vacuum—Einstein's cosmological constant. While the size of the small mass scale, 10^{-3} eV, has not been derived from a more basic theory, it could, perhaps, be broadly understood from mild anthropic arguments [2]. Alternatively, dark energy may be associated with the dynamics of some scalar field that is uniform in space, $\phi(t)$ [3,4]. Perhaps the simplest possibility is that the potential for this field, $V(\phi)$, is determined by the single milli-electron-volt mass scale together with dimensionless couplings of order unity. Such theories of "acceleressence" are easy to construct [5], including radiative stability of the meV scale, but lead to generic observational consequences for dark energy identical to those from a cosmological constant. Since the time scale for ϕ evolution, $\text{meV}^{-1} \approx 10^{-12}$ sec, is much less than the present age of the universe, $t_0 \approx 10^{18}$ sec, the field has already evolved to a local minimum of the effective potential.

An equation of state differing from that of the cosmological constant results if the time scale for ϕ evolution is of order t_0. Taylor expanding the potential $V(\phi)$ about ϕ_0, today's value of the field, such theories of quintessence require a dynamical scale

$$m_\phi = \sqrt{V''(\phi_0)} \approx H_0 \approx 10^{-33}\ \text{eV}. \qquad (1)$$

The appearance of such a low mass scale immediately raises questions. Can such a mass scale be protected from radiative corrections? If a mechanism can be found to stabilize m_ϕ to 10^{-33} eV, then presumably it could protect much smaller scales as well, corresponding to a quintessence theory where ϕ is effectively frozen today, with $V(\phi)$ acting as a cosmological constant. It is for these reasons, perhaps, that there is a theoretical expectation that $w = p/\rho$ will be found to be -1 and time independent. However, this expectation ignores the constraints that will be placed on any theory of dark energy by requiring that it solves the radiative stability constraints and the "Dark energy: Why now?" problem.

Why do we live during an era when the energy densities in dark matter and dark energy are comparable? This is the well-known "Dark energy: Why now?" problem. Particle physics provides a simple solution to this problem, at least at the order of magnitude level [6]. Particle physics can be broadly understood in terms of two fundamental mass scales: the reduced Planck scale, $M \approx 10^{18}$ GeV, and the electroweak scale, $v \approx 10^3$ GeV. There is an induced seesaw scale, v^2/M, that is also of great interest. Both the Planck and weak eras were undoubtedly interesting periods in the evolution of the universe, and we expect that the seesaw era, with a temperature of order $v^2/M \approx 10^{-3}$ eV ≈ 10 K, will also be an interesting epoch. It is significant that the observed background radiation temperature is within an order of magnitude of this value—we do indeed live during the seesaw era. During this era, at a temperature of v^2/M, any particle species, or fluid, with an energy density that depends parametrically on M and v as $(v^2/M)^4$ would be expected to contribute a significant fraction to the energy density of the universe. The "Dark energy: Why now?" problem is solved if theories for dark energy and dark matter can be constructed that have this parametric form for their energy densities.

If an evolving quintessence field gives a significant departure of w from -1, there is a "Quintessence: Why now?" problem: Why do we live during an era when the ϕ field is just starting to evolve? From (1) this becomes, why is $m_\phi \approx H_0 \approx 10^{-33}$ eV and not much smaller? In seesaw cosmology the present value of the Hubble parameter is given by $H_0 \approx v^4/M^3$. Once again, seesaw cosmology allows a solution to an otherwise intractable problem: The dynamical mass scale causing the evolution of ϕ must be given parametrically by

$$m_\phi \approx v^4/M^3. \tag{2}$$

In quintessence theories, we can expect to observe deviations from $w = -1$ if the mass scales in $V(\phi)$ are appropriately related to the electroweak scale v. If the mass parameters of $V(\phi)$ are not related to those of known particle physics, it does not appear possible to answer this problem, except perhaps with anthropic arguments [7].

In this Letter we study quintessence in the seesaw cosmology framework. We exhibit a large class of theories that are radiatively stable and automatically solve the "Quintessence: Why now?" problem. It is much more constraining to also solve the usual "Dark energy: Why now?" problem, and we are led to a particular class of axionlike models.

Radiative stability and deviations from $w = -1$.—From a particle physics perspective, the potential $V(\phi)$ is extraordinarily flat [8]. Supersymmetry is commonly used to protect scalar masses at the mass scale v and can even protect certain scalars to v^2/M as needed for acceleressence theories, but this is far from the desired scale of (2). Factors of $1/16\pi^2$ from quantum loops are hardly likely to help. We are thus led to introduce a small parameter μ^4, which explicitly breaks the shift symmetry $\phi \to \phi + c$:

$$V(\phi) = \mu^4 F\left(\frac{\phi}{f}\right) + \text{H.c.} \tag{3}$$

The dimensionless function F is arbitrary, and for simplicity we have assumed that it depends on only a single dimensionful parameter f. Throughout, we assume that the approximate global symmetries of interest are sufficiently protected from any corrections involving nonperturbative quantum gravity. In general, F depends on many dimensionless parameters that are taken to be of order unity. We assume that the initial value of ϕ is of order f and that, since today ϕ is at most slowly evolving, ϕ_0 is also of order f. The observed size of ρ_{DE} then implies that μ must be taken of order the milli-electron-volt scale. To solve the "Dark energy: Why now?" problem we will later seek theories that lead to $\mu \approx v^2/M$. In the limit that $\mu^4 \to 0$, shift symmetry requires the potential to vanish. Hence all radiative corrections to V are proportional to μ^4—the potential is radiatively stable. A pseudo-Goldstone boson provides a well-known example of quintessence with radiative stability, in which case F is a cosine [4,9].

The dynamical mass scale for ϕ evolution is $m_\phi \approx \mu^2/f$. Once the dark energy dominates, the Friedmann equation gives $H_0 \approx \sqrt{G\rho} \approx \mu^2/M$, leading to

$$m_\phi \approx \frac{M}{f} H_0. \tag{4}$$

The slow role condition becomes $f \gtrsim M$. In the framework of seesaw cosmology, there are only two fundamental mass scales M and v, and so we must choose $f \approx M$. This gives $m_\phi \approx H_0$ so that the "Quintessence: Why now?" problem is solved; the slow roll condition is lost during the present era and deviations from $w = -1$ are generically expected. With $f \approx M$, one immediately finds $m_\phi \approx \mu^2/M$, and with $\mu \approx v^2/M$ the double seesaw $m_\phi \approx (v^2/M)^2/M$ leads to the desired relation (2). To explain why $\mu \approx v^2/M$, and to be more precise about the prediction for $w(z)$, we must address the "Dark energy: Why now?" problem.

A dynamical μ^4.—As long as μ^4 appears as an independent free parameter of the theory, the "Dark energy: Why now?" problem will remain unsolved. To make progress, μ^4 must itself be understood to arise dynamically $\mu^4 \to \lambda G(\chi)$, with G a product of fields χ, which may include scalars and fermions. A simple example is $G = \chi^4$, with χ a scalar. The introduction of propagating fields χ changes the radiative structure of the theory—the parameter that explicitly breaks the shift symmetry on ϕ is now λ, which we take to be dimensionless and order unity. For example, integrating over internal χ fields induces a radiative correction to the potential at order $|\lambda|^2$: $\Delta V(\phi) = |\lambda|^2 M^4 |F(\phi/f)|^2$, giving a ϕ mass of order $\lambda M^2/f$. Indeed, treating λ as the spurion for shift symmetry breaking, such a term cannot be forbidden. By making μ^4 dynamical, m_ϕ is generically changed from order H_0 to order λM. Even if the loop integrals are cut off by supersymmetry, m_ϕ can be protected only to v^2/M, sufficient for acceleressence, but very far from the requirements of dynamical quintessence. This disastrous radiative correction, however, is easily removed by taking $F = e^{i\phi/f}$. In this case the potential is periodic, and ϕ is understood to be the pseudo-Goldstone boson of some symmetry $U(1)_\phi$ that is spontaneously broken at scale f near the Planck scale. Our potential V then takes the form

$$V(\phi, \chi) = \lambda G(\chi) e^{i\phi/f} + \text{H.c.} \tag{5}$$

There are other potentially problematic radiative corrections to the potential for ϕ from diagrams involving χ loops. For example, if χ is a scalar and $G = |\chi|^4$, then there are radiative corrections at order λ in which the four χ fields are contracted into a two loop diagram. To avoid such contributions G must carry some charge under some symmetry $U(1)_\chi$. For example, with χ a complex scalar and $G = \chi^4$, it is not possible to contract the χ fields into loops as long as there are no other interactions that violate $U(1)_\chi$. In such theories the interaction (5) explicitly breaks one combination of $U(1)_\phi$ and $U(1)_\chi$. The parameter μ^4 is generated by having χ develop an expectation value f', so that $\lambda G \to \lambda\langle G\rangle e^{i\phi'/f'} \equiv \mu^4 e^{i\phi'/f'}$, giving a potential

$$V(\phi, \phi') = \mu^4 \cos\left(\frac{\phi}{f} + \frac{\phi'}{f'}\right). \tag{6}$$

To obtain a small value for μ^4, we require $f' \ll f \approx M$.

The pseudo-Goldstone boson $\phi' + (f'/f)\phi$ then acquires a mass $\mu^2/f' \gg H_0$, while $\phi - (f'/f)\phi'$ remains an exactly massless Goldstone boson. Therefore, at this point there is no candidate for the dynamical quintessence field.

The situation is radically altered if some additional explicit symmetry breaking interaction is added, $\tilde{V}$, giving a mass to ϕ' that is $\gtrsim \mu^2/f'$. In this case the determinant of the pseudo-Goldstone-boson mass matrix no longer vanishes, so that the previously massless Goldstone boson acquires a mass from (6): $m_\phi = \mu^2/f \approx H_0$. Thus dynamical quintessence theories naturally emerge from theories having the explicit symmetry breaking structure

$$U(1)_\phi \times U(1)_\chi \xrightarrow{\tilde{V}} U(1)_{\phi+\chi} \xrightarrow{\lambda} 0, \tag{7}$$

with the mass of the dark energy field emerging at the final stage of explicit symmetry breaking.

The form of $\tilde{V}$ is itself highly constrained, since radiative corrections involving both λ and $\tilde{V}$ must not introduce further operators that give a large mass to ϕ. To avoid this, the explicit symmetry breaking parameter in $\tilde{V}$ should be dimensionful. For example, the case of $G = \chi^4$ and $\tilde{V} = \eta\chi^4 + \text{H.c.}$ clearly does not work. [An important question is whether theories of the form $m_{\nu_{ij}} \nu_i \nu_j e^{i\phi_{ij}/f_{ij}}$ lead to acceptable potentials for dark energy once the three neutrino fields ν_i are integrated out. If $m_{\nu_{ij}}$ are treated as parameters, one obtains a potential of the form of (6) with μ identified as m_ν [9]. This would be a very interesting understanding of the size of dark energy. However, the simplest such theories do not work: The neutrino mass is not a parameter but depends on electroweak symmetry breaking $m_\nu = m_\nu(h)$, and radiative corrections above the weak scale with internal Higgs fields h destroy the radiative stability of the potential. The schizon models of [10] avoid this by introducing multiple Higgs doublets at the weak scale. But, even in this case, the mass parameters that mix the various Higgs doublets must be set to the weak scale by hand—they cannot arise from vacuum expectation values of other fields. The successful supersymmetric prediction for the weak mixing angle is also destroyed.]

Hidden axions and seesaw cosmology.—To illustrate these ideas, and to see how seesaw cosmology can solve the "Dark energy: Why now?" problem, we consider models with an axion in a hidden sector. Quintessence axions have been considered previously for dark energy [11,12], but not in the context of seesaw cosmology.

The general idea is as follows. Suppose that the fundamental scale of supersymmetry breaking in nature is of order of the TeV scale, v. Any sector of the theory that feels this supersymmetry breaking only indirectly via gravity mediation will have an effective scale of supersymmetry breaking at the seesaw scale $\tilde{m} = v^2/M$. We suppose that such a hidden sector has a supersymmetric QCD-like gauge interaction acting on chiral superfields Q and Q^c. Supersymmetry breaking leads to the corresponding squarks and gluinos acquiring a mass of order $\tilde{m}$, changing the beta function for the gauge coupling and triggering strong dynamics at a scale Λ not far below $\tilde{m}$. A simple example for this behavior arises if the hidden sector is in a conformal window above $\tilde{m}$. We assume that supersymmetry breaking also triggers a mass term for the quarks. If this sector has a Peccei-Quinn symmetry spontaneously broken at f near the Planck scale, then the interaction between the axion, ϕ, and the quarks at the scale Λ has the form

$$\mathcal{L}_{\text{ax}} = m_q q q^c e^{i\phi/f} + \text{H.c.} \tag{8}$$

so that, comparing with (5), $\lambda G = m_q q q^c$. The $U(1)_\phi$ symmetry is the Peccei-Quinn symmetry, $U(1)_{\text{PQ}}$, and is broken near the Planck scale, while the $U(1)_\chi$ symmetry is the axial $U(1)$ symmetry, $U(1)_A$, carried by the quark bilinear qq^c. The interaction (8) explicitly breaks $U(1)_{\text{PQ}} \times U(1)_A$ to the diagonal subgroup. We assume that the mass of at least one quark flavor in (8) is $\lesssim \Lambda$, so that a condensate forms, $\langle qq^c \rangle \approx \Lambda^3 e^{i\eta'/\Lambda}$, generating the potential (6) with ϕ' becoming the hidden sector η' and $f' = \Lambda$.

The additional explicit symmetry breaking necessary for a naturally light quintessence field, $\tilde{V}$ in (7), is automatic: It is the gauge anomaly that breaks $U(1)_A$, giving the η' a mass of order Λ. Since this explicit symmetry breaking comes from an anomaly and involves the scale Λ, unlike dimensionless symmetry breaking parameters, it does not lead to further radiative instabilities of the mass of the dark energy field. The axion field ϕ is the dark energy field, and obtains a mass from the potential (6) with $\mu^4 \approx m_q \Lambda^3$. Since Λ and m_q are both close to $\tilde{m}$, the scale μ is given by the seesaw $\mu \approx \tilde{m} \approx v^2/M$, solving the "Dark energy: Why now?" problem. The double seesaw

$$m_\phi \approx \frac{\mu^2}{f}, \qquad \mu \approx \frac{v^2}{M}, \tag{9}$$

then leads to the desired result (2) for a seesaw cosmology solution of the "Quintessence: Why now?" problem.

It is straightforward to write a complete set of interactions for the above hidden sector. As an example, consider the supersymmetric interaction Lagrangian

$$\mathcal{L}_{\text{int}} = \int d^2\theta \Big(X(S\bar{S} - f^2) + \frac{Z}{M} W^\alpha W_\alpha \Big) + \int d^4\theta \Big(\frac{Z^\dagger}{M} \frac{S}{M} Q Q^c + \frac{Z^\dagger Z}{M^2} (Q^\dagger Q + Q^{c\dagger} Q^c) \Big), \tag{10}$$

where all coupling constants, color, and flavor indices have been omitted. The chiral superfield Z is the spurion for supersymmetry breaking with $F_Z/M = \tilde{m} \approx v^2/M$. The interactions of (10) possess $U(1)_{\text{PQ}} \times U(1)_B \times U(1)_R$ symmetry, where $U(1)_B$ is the baryon symmetry acting on Q and Q^c and $U(1)_R$ the R symmetry under which $\tilde{q}$ and $\tilde{q}^c$ are neutral. We assume that $U(1)_R$ is explicitly

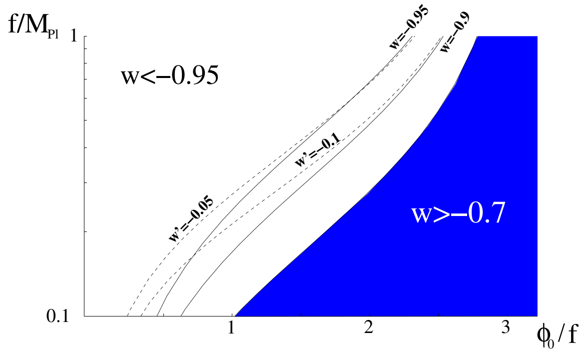

FIG. 1 (color online). Contours for w and w'.

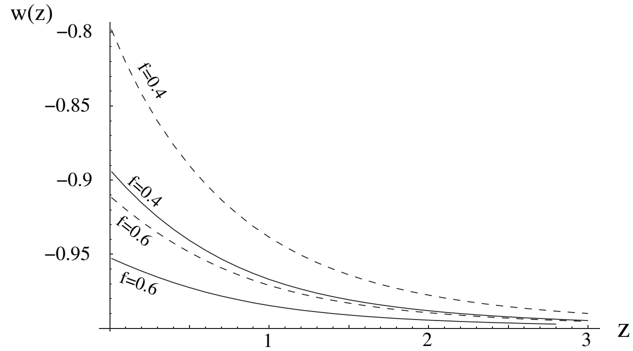

FIG. 2. $w(z)$ for four choices of $(f/M_{\rm Pl},\ \phi_0/f)$. The solid lines are $\phi_0/f = 0.6\pi$ while the dashed lines have $\phi_0/f = 0.7\pi$. The lines are labeled by their value of $f/M_{\rm Pl}$.

broken elsewhere in the theory, and $U(1)_B$ plays no role in our analysis. The two relevant symmetries are then $U(1)_{\rm PQ}$ and the axial $U(1)_A$ symmetry on Q and Q^c. These are $U(1)_\phi$ and $U(1)_\chi$, respectively, and the two explicit symmetry breakings of (7) are provided by the gauge anomaly and by the interaction $Z^\dagger SQQ^c$, respectively. The chiral field X drives the spontaneous breaking of $U(1)_{\rm PQ}$ symmetry, giving $\langle S\rangle = fe^{i\phi/f}$. All hidden sector superpartners obtain a mass of order $\tilde{m}$ through interactions with Z. On inserting F_Z and $\langle S\rangle$ into the interaction $Z^\dagger SQQ^c$, a supersymmetric mass term for the quarks is generated, which includes the desired interaction of (8). We do not include a phase for the pseudo-Goldstone boson of $U(1)_R$ because it acquires a sufficiently large mass from elsewhere. We assume that the flat direction associated with the real parts of S and $\bar{S}$ can be sufficiently lifted.

There are many alternative models. For example, the hidden sector could be a copy of the supersymmetric standard model coupled to a Planck scale axion.

Equation of state predictions.—The class of quintessence theories we have introduced, having a radiatively stable potential resulting from a shift symmetry and a solution to the "Dark energy: Why now?" problem via $\mu \approx v^2/M$, leads to a potential of the form $V = \mu^4\cos(\phi/f)$, with $f \approx M$. Thus the dark energy and its cosmological evolution is described by three parameters: μ^4, f, and ϕ_0. We choose to determine μ^4 from the observed size of $\rho_{\rm DE}$, and display predictions in the $(f/M_{\rm Pl},\ \phi_0/f)$ plane, where $M_{\rm Pl} = 1.2\times 10^{19}$ GeV. In Fig. 1 contours are drawn for $w = -0.7$, -0.9, and -0.95 and also for $w' = -0.1$ and -0.05, where $w' = dw/dz|_{z=0}$. A sizable region of allowed parameter space having deviations from $w = -1$ will be probed by future experiments [13]. In Fig. 2 the redshift dependence of the equation of state parameter, $w(z)$, is shown for four representative values of $(f/M_{\rm Pl},\ \phi_0/f)$. Recent evolution can be quite rapid and is determined by the cosine form of the potential. Our predictions for $w(z)$, for example the curves of Fig. 2, can be compared with analyses of current data, for example with Figs. 10 and 11 of [14], which include an analysis of the Ly-α forest data from the Sloan Digital Sky Survey. The lower three curves of Fig. 2 lie well within the region allowed at 1σ, while the top curve lies just outside the 1σ region but well within the 2σ region.

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