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Limits of Quintessence

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We present evidence that the simplest particle-physics scalar-field models of dynamical dark energy can be separated into distinct behaviors based on the acceleration or deceleration of the field as it evolves down its potential towards a zero minimum. We show that these models occupy narrow regions in the phase plane of w and w', the dark energy equation of state and its time derivative in units of the Hubble time. Restricting an energy scale of the dark energy microphysics limits how closely a scalar field can resemble a cosmological constant. These results, indicating a desired measurement resolution of order $\sigma(w') \approx (1 + w)$, define firm targets for observational tests of the physics of dark energy.

Observations and experiments at the close of the 20th century have transformed our understanding of the physics of the Universe. A consistent picture has emerged indicating that nearly three quarters of the cosmos is made of "dark energy"—some sort of gravitationally repulsive material responsible for the accelerated expansion of the Universe [for reviews, see [1–3]]. Proposals for the dark energy include Einstein's cosmological constant (Λ), or a dynamical field such as quintessence. Here we show how scalar-field dynamics separates into distinct behaviors which, through future cosmological measurements, can reveal the nature of the new physics accelerating our universe.

Einstein's cosmological constant (Λ) is attributed to the quantum zero-point energy of the particle-physics vacuum, with a constant energy density ρ, pressure p, and an equation of state $w \equiv p/\rho = -1$. In contrast, quintessence is a proposed time-varying, inhomogeneous field with a spatially averaged equation of state $w > -1$ [4–8]. The simplest physical model consists of a scalar field, slowly rolling in a potential characterized by an extremely low mass. (This is similar to inflation, the period of accelerated expansion in the early Universe, but at an energy scale many orders of magnitude lower.) Since a scalar field evolving in a very shallow potential may be indistinguishable from a Λ, the task of elucidating the physics of dark energy becomes difficult if observations continue to find that w is close to -1, e.g., [9–11]. In this Letter, we examine the likely behavior of scalar fields and characterize them into two distinct classes, based on their evolution in the w-w' phase space. These results should help define targets for observational and experimental tests of the physics of dark energy.

Our approach is a new take on a familiar system, the scalar field. By emphasizing the dynamics, we discover restricted regions of the trajectories of canonical scalar-field models in "position" and "velocity"—the value of the equation-of-state ratio w and its time variation w'.

While there is a myriad of scalar-field models motivated by particle physics beyond the standard model, this treatment allows a broad, model-independent assessment of a quintessence scalar field slowly relaxing in a potential.

The physics is straightforward: the field ϕ will seek to roll towards the minimum of its potential V, according to the Klein-Gordon equation $\ddot{\phi} + 3H\dot{\phi} = -dV/d\phi$. The rate of evolution is driven by the slope of the potential and damped by the cosmic expansion through the Hubble parameter H. The average energy density and pressure are $\rho = \dot{\phi}^2/2 + V$, $p = \dot{\phi}^2/2 - V$ so that a field stuck in a local, nonzero minimum of the potential has $w = -1$. To distinguish from an effective cosmological constant, however, we will only consider cases in which the field is evolving towards a zero minimum.

In perhaps the simplest such scenario, the field has been frozen by Hubble damping at a value displaced from its minimum until recently, when it starts to roll down to the minimum. We call these "thawing" models. At early times the equation-of-state ratio is $w \approx -1$, but grows less negative with time as $w' \equiv \dot{w}/H > 0$. Since the Hubble damping limits the scalar-field acceleration, $\ddot{\phi} < \dot{\phi}/t \approx (3/2)H\dot{\phi}$, then the equation of motion implies such models will lie at $w' < 3(1 + w)$ in the phase plane. The scalar-field dynamics suggest a lower bound, too, due to the fact that dark energy is not entirely dominant today, with a fractional energy density $\Omega_{de} \lesssim 0.8$. Our study of several classes of thawing models, such as a pseudo Nambu-Goldstone boson (PNGB) [6] or polynomial potentials, indicates the bound $w' > (1 + w)$. These simple bounds are valid for $(1 + w) \ll 1$, and so $w \lesssim -0.8$ is a practical limit of applicability.

We have analyzed the following potentials for thawing behavior: concave potentials with $V = M^{4-n}\phi^n$ are ubiquitous, and we have allowed for continuous values of the exponent $n > 0$. The motivation for cases $n < 2$ is not as straightforward, although $n = 1$ has been considered

[12,13]. We restrict attention to power-law potentials with index $n < 6$; for higher powers the field rolls too quickly down the potential, with $w > -0.8$ by today. Also, such potentials can lead to tracking behavior [14] whereby the scalar field evolves with a positive equation of state and there is no cosmic acceleration. Exponential potentials are typical for moduli or dilaton fields, e.g., [15], with $V = M^4 \exp(-\beta\phi/M_P)$ where $M_P \approx 10^{19}$ GeV is the Planck energy. To avoid scaling, which would not provide for the cosmic acceleration, we restrict $\beta < \sqrt{24\pi}$ [16]. PNGBs, like a dark energy axion [e.g., [17,18]], have $V = M^4\cos^2(\phi/2f)$ where f is a symmetry restoration energy scale. We have not included the case $f \ll M_P$ since the field rapidly evolves to $w \to 0$ unless the initial conditions are finely tuned to keep the field balanced upon the top of the potential maxima and maintain $1 + w \ll 1$.

A second scenario consists of a field which was already rolling towards its potential minimum, prior to the onset of acceleration, but which slows down and creeps to a halt as it comes to dominate the Universe. For these "freezing" models, initially $w > -1$ and $w' < 0$. These are essentially tracking models [19], but may be described more generally as vacuumless fields (in the sense that the minimum is attained as $\phi \to \infty$) or runaway potentials characterized by a potential with curvature that slows the field evolution as it rolls down towards the minimum. It follows [20] that there is some value of the field beyond which the evolution is critically damped by the cosmic expansion, whence the field is frozen [but, like a glacier [20], continues to move] and $w \to -1$, $w' \to 0$. The deceleration of the field is limited by the steepness of the potential, roughly $\ddot{\phi} > dV/d\phi$, leading to the lower bound $w' > 3w(1 + w)$. Our investigation of a variety of scalar-field models leads to a less definite upper bound $w' \lesssim 0.2w(1 + w)$ since a redshift $z \sim 1$ but evolving beyond $w' \lesssim w(1 + w)$ by the present. Again, $w \lesssim -0.8$ is a practical limit of applicability of these bounds.

We have analyzed the following tracker potentials for their freezing behavior: $V = M^{4+n}\phi^{-n}$ and $V = M^{4+n}\phi^{-n}\exp(\alpha\phi^2/M_P^2)$ for $n > 0$ [21–25]. The latter has an effective cosmological constant, but has been widely studied and so we consider it nonetheless, provided the field is closer to the origin than the nonzero minimum. Proposed tracker models such as $V = M^4\exp(M_P/\phi)$ do not have a zero minimum, and $V = M^4[\exp(M_P/\phi) - 1]$ does not achieve $w \lesssim -0.8$, $\Omega_{de} \lesssim 0.8$. Other functions have been proposed as tracking potentials, but lack a firm basis in particle theory.

These distinct, physically motivated thawing and freezing behaviors are illustrated in Fig. 1, while several examples of specific models are presented in Fig. 2. It would be quite useful to determine if one of these classes of physics phenomena is responsible for the dark energy accelerating our universe. We see that to distinguish thawing from freezing, a measurement resolution of order $\sigma(w') \approx (1 + w)$ is required.
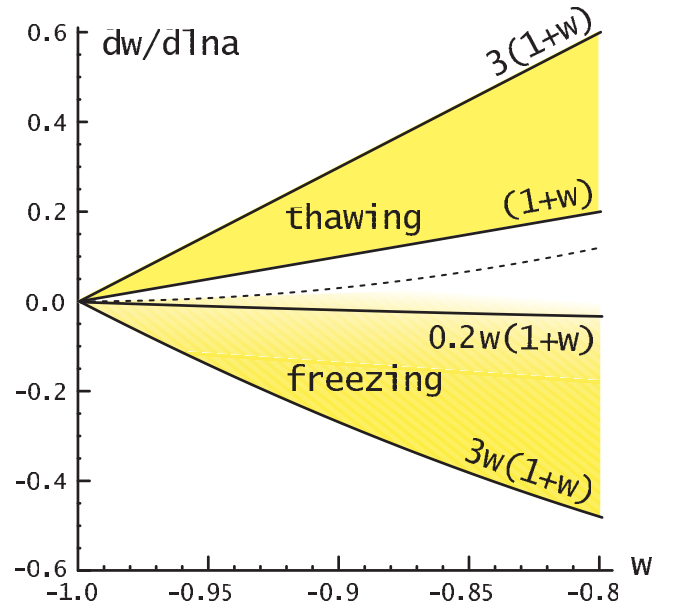

FIG. 1 (color online). The w-w' phase space occupied by thawing and freezing fields is indicated by the shaded regions. No strong constraints on this range of dark energy properties exist at present. The fading at the top of the freezing region indicates the approximate nature of this boundary. Freezing models start above this line, but pass below it by a redshift $z \sim 1$. The short-dashed line shows the boundary between field evolution accelerating and decelerating down the potential. Future cosmological observations will aim to discriminate between these two fundamental scenarios.

Complicated potentials that have a built-in cosmological constant, for which V' vanishes, can violate the inequalities we have established in the w-w' phase plane. If we add a constant potential energy to a quintessence field, the equation of state w is shifted closer to -1. Although the Klein-Gordon equation is insensitive to this addition until the dark energy drives the evolution of H, the magnitude of w' changes. We find that our inequalities continue to hold for freezing models. For thawing fields, w' gets smaller, and the phase-plane trajectory can just cross the line $w' = (1 + w)$ by the present.

The results of this study strictly apply only to canonical scalar-field theories, in particular, with $w \geq -1$. Preliminary investigation indicates that at least some other dark energy models also follow the dynamics discussed here (work in progress). Also note that thawing models would appear to have an averaged, or enforced constant, value of w very close to -1, indicating the importance of looking for dynamics in the form of w'.

The question of the absolute level of deviation of w from -1, i.e., the distinction from Einstein's cosmological constant, is less tractable. Certainly, one can obtain a scalar-field solution, however unrealistic, at any given point in the w-w' phase space. Even for the thawing and freezing
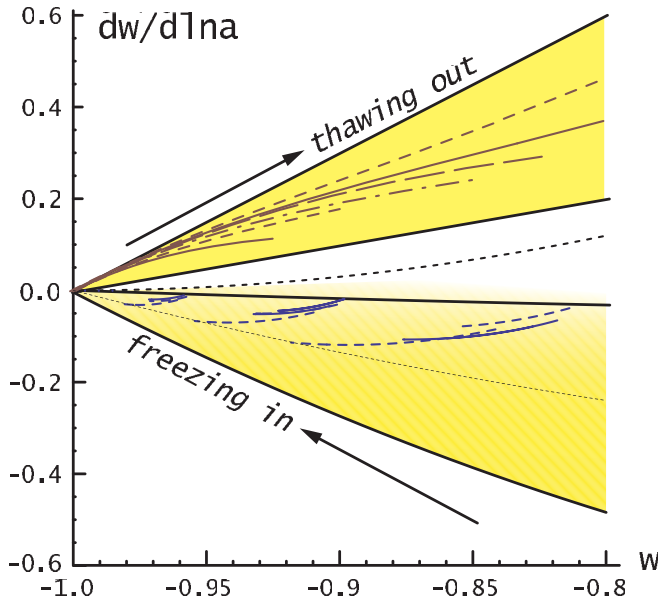
FIG. 2 (color online).   The evolutionary tracks in w-w' phase space are shown for a variety of particle-physics models of scalar fields. The two broad classes are clear: those that initially are frozen and look like a cosmological constant, starting at $w = -1$, $w' = 0$, and then thaw and roll to $w' > 0$, and those that initially roll and then slow to a creep as they come to dominate the Universe. The sample of thawing models shown have potentials $V \propto \phi^n$ for $n = 1, 2, 4$ (short-, dot-, and long-dashed curves) and a PNGB with $V \propto \cos^2(\phi/2f)$ (solid curves). The rightmost point of the tracks corresponds to the present. For variety, the $n = 4$ model has $\Omega_{de} = 0.6$, and the $n = 1$ model ending at $w = -0.8$ has $\Omega_{de} = 0.65$. All other models end with a fractional energy density $\Omega_{de} = 0.7$. The sample of freezing models shown have potentials $V \propto \phi^{-n}$, $\phi^{-n}e^{\alpha\phi^2}$ (solid and dashed curves). The line $w' = 1.5w(1 + w)$ indicated by the light, dotted line is a possible lower bound on the freezing models. The leftmost point of the tracks corresponds to the present; the rightmost point is at $z = 1$. For variety, upper and lower close pairs of curves have $\Omega_{de} = 0.7, 0.8$, respectively. All other models end with a fractional energy density $\Omega_{de} = 0.7$.

models, parameters may be finely tuned to keep $1 + w$ arbitrarily close to zero within the shaded regions of Fig. 1.

If the scalar field is prohibited from attaining values exceeding the Planck scale, lest quantum gravitational effects dominate, then there is a lower bound on $1 + w$. Defining a characteristic scale $E \equiv |V/(dV/d\phi)|$ we demand $E < M_P$. Next, for a field rolling down its potential, we can express the scalar-field equation of motion as

$$w' = -3(1 - w^2) + (1 - w)\frac{M_P}{E}\sqrt{\frac{3}{8\pi}\Omega_{de}(1 + w)}.$$

Taking $\Omega_{de} \approx 0.7$ then thawing models must satisfy $1 + w \gtrsim 0.004$ whereas for freezing models $1 + w \gtrsim 0.01$; this may be the limit of quintessence. These margins correspond to a $\sim 0.2\%$ difference from a Λ cosmology

in distance to redshift $z = 1$. Such an absolute precision goal is clearly extraordinarily challenging.

Note that early universe inflation can similarly approach pure exponential expansion, with its deviations broadly characterized by dynamics into models tilted to prefer large-scale or small-scale power, and important implications in the distinction [26]. The structure we find in the canonical scalar-field phase plane based on simple physical considerations may prove useful, and we have here presented firm targets for a basic test of dark energy. The language is different from inflation for two reasons: the dark energy need not be rolling as slowly as the inflaton, and the dark energy is not totally dominant, unlike the inflaton.

Charting the late-time cosmic evolution—through Type Ia supernovae distances, weak gravitational lensing probes of large-scale structure evolution, distance ratios from baryon acoustic oscillations in galaxy clustering, etc.—is the subject of intense investigations. As a gauge of the requisite resolution, a 1% variation in luminosity distance to redshift $z = 1$ distinguishes between: Λ and $w = -0.95$, $w' = 0$; models which evolve along the top and bottom of the thawing region out to $w = -0.8$; models which evolve along the top and bottom of the freezing region in to $w = -0.95$. The goal of making the fundamental physics distinction between the thawing and freezing regions is challenging but achievable in the next generation of experiments [27–30] if the dynamics is sufficiently apparent. In the case $1 + w \gtrsim 0.05$, dedicated dark energy experiments now being designed, such as the joint dark energy mission, will probe cosmology with sufficient accuracy to be able to decide the issue.

This will probably not be the final word on dark energy [cf. [31]], but if the answer is not consistent with a cosmological constant then the rewards are obvious in discovering new physics beyond our current standard models. It is interesting to note that the fate of the Universe is very different for the case of a thawing field, as the acceleration is temporary, as compared to a freezing field, for which the acceleration continues unabated. If the result lies outside the two phase-space regions categorized here then we may have to look beyond simple explanations, perhaps to even more exotic physics such as a modification of Einstein gravity.

[1] R. R. Caldwell, Phys. World **17**, 37 (2004).
[2] T. Padmanabhan, Phys. Rep. **380**, 235 (2003).
[3] P. J. E. Peebles and B. Ratra, Rev. Mod. Phys. **75**, 559 (2003).
[4] B. Ratra and P. J. E. Peebles, Phys. Rev. D **37**, 3406 (1988).

[5] C. Wetterich, Nucl. Phys. **B302**, 668 (1988).

[6] J. A. Frieman, C. T. Hill, A. Stebbins, and I. Waga, Phys. Rev. Lett. **75**, 2077 (1995).

[7] K. Coble, S. Dodelson, and J. A. Frieman, Phys. Rev. D **55**, 1851 (1997).

[8] R. R. Caldwell, R. Dave, and P. J. Steinhardt, Phys. Rev. Lett. **80**, 1582 (1998).

[9] R. A. Knop *et al.* (The Supernova Cosmology Project Collaboration), Astrophys. J. **598**, 102 (2003).

[10] A. G. Riess *et al.* (Supernova Search Team Collaboration), Astrophys. J. **607**, 665 (2004).

[11] U. Seljak *et al.*, Phys. Rev. D **71**, 103515 (2005).

[12] A. D. Linde, in *Three Hundred Years of Gravitation*, edited by S. W. Hawking and W. Israel (Cambridge University Press, Cambridge, England, 1987), p. 604.

[13] R. Kallosh, J. Kratochvil, A. Linde, E. V. Linder, and M. Shmakova, J. Cosmol. Astropart. Phys. 10 (2003) 015.

[14] A. R. Liddle and R. J. Scherrer, Phys. Rev. D **59**, 023509 (1999).

[15] T. Barreiro, E. J. Copeland, and N. J. Nunes, Phys. Rev. D **61**, 127301 (2000).

[16] P. G. Ferreira and M. Joyce, Phys. Rev. D **58**, 023503 (1998).

[17] J. E. Kim and H. P. Nilles, Phys. Lett. B **553**, 1 (2003).

[18] L. J. Hall, Y. Nomura, and S. J. Oliver, astro-ph/0503706.

[19] I. Zlatev, L. Wang, and P. J. Steinhardt, Phys. Rev. Lett. **82**, 896 (1999).

[20] P. J. Steinhardt, in *Proceedings of the Pritzker Symposium on Status of Inflationary Cosmology*, edited by M. Turner (University of Chicago Press, Chicago, 2000).

[21] P. Binetruy, Phys. Rev. D **60**, 063502 (1999).

[22] P. Brax and J. Martin, Phys. Lett. B **468**, 40 (1999).

[23] A. Masiero, M. Pietroni, and F. Rosati, Phys. Rev. D **61**, 023504 (2000).

[24] E. J. Copeland, N. J. Nunes, and F. Rosati, Phys. Rev. D **62**, 123503 (2000).

[25] A. de la Macorra and C. Stephan-Otto, Phys. Rev. Lett. **87**, 271301 (2001).

[26] W. H. Kinney, E. W. Kolb, A. Melchiorri, and A. Riotto, Phys. Rev. D **69**, 103516 (2004).

[27] G. Aldering *et al.*, astro-ph/0405232.

[28] M. Jarvis, B. Jain, G. Bernstein, and D. Dolney, astro-ph/0502243.

[29] E. V. Linder and R. Miquel, Phys. Rev. D **70**, 123516 (2004).

[30] A. Upadhye, M. Ishak, and P. J. Steinhardt, Phys. Rev. D **72**, 063501 (2005).

[31] I. Maor and R. Brustein, Phys. Rev. D **67**, 103508 (2003).