Observation of a Metallic Superfluid in a Numerical Experiment

E. Smørgrav,[1] E. Babaev,[2,1] J. Smiseth,[1] and A. Sudbø[1]

[1]*Department of Physics, Norwegian University of Science and Technology, N-7491 Trondheim, Norway*
[2]*Laboratory of Atomic and Solid State Physics, Cornell University, Ithaca, New York 14853-2501, USA*

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We report the observation, in Monte Carlo simulations, of a novel type of quantum ordered state: *the metallic superfluid*. The metallic superfluid features Ohmic resistance to counterflows of protons and electrons, while featuring dissipationless coflows of electrons and protons. One of the candidates for a physical realization of this remarkable state of matter is hydrogen or its isotopes under high compression. This adds another potential candidate to the presently known quantum dissipationless states, namely, superconductors, superfluid liquids and vapors, and supersolids.

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At low temperatures, fluids become dominated by the wavelike nature of their constituent particles when the thermal de Broglie wavelength exceeds the interparticle separation. Such quantum fluids usually feature superconductivity or superfluidity, which, however, may be destroyed by topological line defects (vortices) threading the entire system. Vortices may be induced by a magnetic field or rotation [1], or by thermally excited *transverse* phase fluctuations of the macroscopic wave function of superconductors and superfluids [2–4]. In a system of rapidly growing interest, a two-component superconductor [5], vortices yield dramatic physical consequences. Here we report the first observations, in a numerical experiment, of a novel type of quantum fluid originating in aggregate states of vortex matter. Increasing temperature, a composite vortex lattice melts into a composite vortex liquid, whence superconductivity is lost while superfluidity is retained, yielding a metallic (Ohmic) superfluid. At higher temperature, another unusual transition occurs where the composite vortex liquid ‘‘ionizes’’ into a ‘‘plasma’’ of constituent vortices, destroying superfluidity.

Recently, there has been considerable interest in theories of superconductors with several components coupled by a magnetic field, but with no possibility of Josephson tunneling of one component into another. This is predicted to occur in a wide variety of physical systems, most notably in condensed matter such as hydrogenic atoms subjected to extreme pressure [5–10] or effective theories of easy-plane quantum antiferromagnets [11]. Renewed interest in the long sought liquid metallic hydrogen (LMH) is due to recent *ab initio* calculations [12] along with a breakthrough in synthesis of ultrahard artificial diamonds, essential for obtaining the required extreme pressures in anvil cells [13]. These facts, along with a recent measurement of an unusually low melting temperature of dense Na [14], strongly hint at a realization of LMH in the not too distant future. Thus, understanding the superconducting properties of this system is important, since magnetic field experiments can be conducted in high pressure anvil cells, possibly confirming the realization of this novel state of matter in a terrestrial laboratory. It is commonly accepted that LMH is abundant in the interior of Jupiter and Saturn and quite possibly also present in some of the known 200 extrasolar giant planets [15]. In these cases, however, LMH is conjectured to exist in the classical metallic regime at several thousand degrees Kelvin. In contrast, the state projected to exist in Ref. [6] is a ground state quantum fluid, and is the one we focus on in this Letter.

Realization of LMH could well constitute the next milestone in quantum fluids. It is projected to feature Cooper pairs of both electrons and protons at low temperatures [6]. The resulting quantum fluid differs radically from previously known quantum fluids, in that its physical properties cannot be classified exclusively as a superconductor or a superfluid [5]. Remarkably, such a system features both superconductivity and superfluidity, which appear as collective phenomena corresponding to coflows and counterflows of two species of Cooper pairs, with a complicated interplay between them. Thus, LMH has been conjectured to sustain phase transitions connecting a superconducting *and* superfluid state to a metallic state featuring a superfluid mode [5], or to a superconducting state with no superfluid mode.

The transition from a state featuring superconductivity *and* superfluidity to a state where superfluidity is lost and superconductivity is retained has recently been observed in a large-scale Monte Carlo (MC) simulation [10]. However, the remarkable possibility of a transition from a ‘‘composite’’ vortex lattice (superconducting state where vortex matter forms a solid) to a ‘‘composite vortex liquid’’ (metallic superfluid state of the system) along with a subsequent transition from a composite vortex liquid to ‘‘vortex line plasma’’ has thus far not been confirmed. It is the purpose of this Letter to report an observation of the metallic superfluid, not dealt with in simulations previously, in a numerical experiment.

The superconducting phase of LMH is given by the Ginzburg-Landau model with two scalar fields $\Psi_0^{(1)}(\mathbf{r})$ and $\Psi_0^{(2)}(\mathbf{r})$ describing superconducting condensates of protons and electrons, respectively. It is defined via the energy density

$$\mathcal{H} = \sum_{\alpha=1}^{2} \frac{|\mathbf{D}\Psi_0^{(\alpha)}(\mathbf{r})|^2}{2M^{(\alpha)}} + V(\{|\Psi_0^{(\alpha)}(\mathbf{r})|\}) + \frac{1}{2}[\nabla \times \mathbf{A}(\mathbf{r})]^2. \tag{1}$$

Here, $M^{(1)}$ and $M^{(2)}$ are the masses of the condensates, the covariant derivative is given by $\mathbf{D} \equiv \nabla - ie\mathbf{A}(\mathbf{r})$, and $V(\{|\Psi_0^{(\alpha)}(\mathbf{r})|\})$ is the potential term. The particular form of $V(\{|\Psi_0^{(\alpha)}(\mathbf{r})|\})$ is not essential for the large-scale physics we address, but its dependence only on $|\Psi_0^{(\alpha)}|$ reflects the fact that Cooper pairs of electrons cannot be converted into Cooper pairs of protons, and vice versa. For the issues discussed in this Letter, it suffices to work in the phase only approximation $\Psi_0^{(\alpha)}(\mathbf{r}) = |\Psi_0^{(\alpha)}| \exp[i\theta^{(\alpha)}(\mathbf{r})]$, where $|\Psi_0^{(\alpha)}|$ is treated as a constant [10]. In LMH, the two order parameters correspond to electronic and protonic Cooper pairs. Moreover, Eq. (1) may be rewritten as follows [5,7,8,10]:

$$\mathcal{H} = \frac{1}{2\Psi^2}(|\psi^{(1)}|^2\nabla\theta^{(1)} + |\psi^{(2)}|^2\nabla\theta^{(2)} - e\Psi^2\mathbf{A})^2 + \frac{1}{4\Psi^2}|\psi^{(1)}|^2|\psi^{(2)}|^2[\nabla(\theta^{(1)} - \theta^{(2)})]^2 + \frac{1}{2}(\nabla \times \mathbf{A})^2, \tag{2}$$

where $|\psi^{(\alpha)}|^2 = |\Psi_0^{(\alpha)}|^2/M^{(\alpha)}$ and $\Psi^2 \equiv |\psi^{(1)}|^2 + |\psi^{(2)}|^2$. The neutral and charged modes, described by the second and first term in Eq. (2), are explicitly identified.

The topological objects of Eq. (1) are vortices of type 1 and type 2 defined by a 2π winding in $\theta^{(1)}$ and $\theta^{(2)}$, respectively. The interaction potential between these vortices is a superposition of a Coulomb potential and a Yukawa potential, arising out of the neutral and the charged mode, respectively. (For a derivation of the formulas for the vortex interaction, see Ref. [10].) The energy of a vortex associated with $\pm 2\pi$ winding in $\theta^{(1)} - \theta^{(2)}$ is logarithmically divergent. As the neutral mode tends to lock $\nabla\theta^{(1)}$ to $\nabla\theta^{(2)}$, in order to minimize the second term in Eq. (2), the vortices of type 1 and type 2 pair up into a composite vortex for which $\nabla[\theta^{(1)} - \theta^{(2)}] = 0$. Therefore, a composite vortex is an object where a type-1 and a type-2 vortex are cocentered and codirected in space, and the Coulomb part of the pair potential exactly cancels. The screened potentials add, but the associated overall energy is finite [5,8–10]. In the presence of an externally applied magnetic field, the ground state of the system is a lattice of cocentered vortices of type 1 and type 2, a composite vortex lattice.

We have performed MC simulations on Eq. (1) at finite temperature using local Metropolis updating on the fields $\theta^{(1)}(\mathbf{r})$, $\theta^{(2)}(\mathbf{r})$, and $A(\mathbf{r})$. The system size we have used is $L \times L \times L$, with $L = 120$. The coupling constants investigated are $|\psi^{(1)}|^2 = 0.5$, $|\psi^{(2)}|^2 = 1.0$, and $e = 1.0$, and the external magnetic field is $\mathbf{B} = \nabla \times \mathbf{A}(\mathbf{r}) = (0, 0, 2\pi f)$ with $f = 1/20$. Thus, there are 20 plaquettes in the xy plane for each field-induced vortex. The external magnetic field is imposed by splitting $\mathbf{A}(\mathbf{r}) = \mathbf{A}_F(\mathbf{r}) + \mathbf{A}_0(\mathbf{r})$, where $\mathbf{A}_F$ is free to fluctuate subject to periodic boundary conditions and $\mathbf{A}_0 = (0, 2\pi x f, 0)$ is kept fixed. We have chosen the amplitude ratios $|\psi^{(2)}|^2/|\psi^{(1)}|^2 = 2.0$ for numerical convenience. We emphasize that the results in this Letter are dictated by symmetry, and the physical picture we present will thus be representative also for LMH, where $|\psi^{(2)}|^2/|\psi^{(1)}|^2 \sim 10^3$.

To find the lattice ordering of vortices, we compute the planar structure function $S^{(\alpha)}(\mathbf{k}_\perp)$ of the local vorticity $\mathbf{n}^{(\alpha)}(\mathbf{r})$, defined by $\mathbf{\Delta} \times [\mathbf{\Delta}\theta^{(\alpha)} - e\mathbf{A}] = 2\pi\mathbf{n}^{(\alpha)}(\mathbf{r})$, given by $S^{(\alpha)}(\mathbf{k}_\perp) = \langle|\sum_{\mathbf{r}} n_z^{(\alpha)}(\mathbf{r}) e^{i\mathbf{k}_\perp \cdot \mathbf{r}_\perp}|^2\rangle/(fL^3)^2$. Here, Δ_μ is the lattice difference operator, $\Delta_\mu\theta^{(\alpha)} - eA_\mu \in [0, 2\pi\rangle$, $\mathbf{r}$ runs over the possible positions of the vortices, and $\mathbf{k}_\perp$ and $\mathbf{r}_\perp$ are perpendicular to $\mathbf{B}$. If vortices form a lattice, $S^{(\alpha)}(\mathbf{k}_\perp)$ will exhibit a sixfold symmetric Bragg structure and feature a ring structure in the vortex-liquid phase.

The MC results are given in Fig. 1, showing the structure function $S^{(1)}(\mathbf{K})$ (red or light gray) for protonic vortices, and $S^{(2)}(\mathbf{K})$ (blue or dark gray) for electronic vortices, where $\mathbf{K} = (\pi/4, 2\pi/5)$ is a Bragg vector. In the low-temperature regime, both functions are finite, but decrease

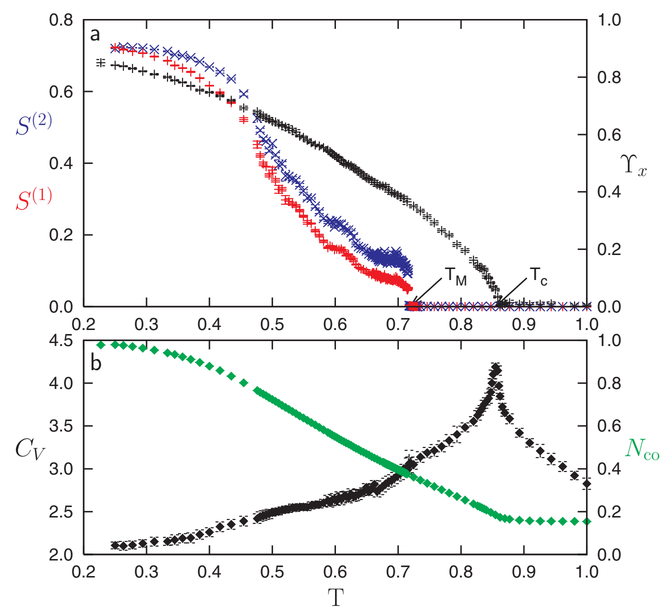

FIG. 1 (color online). Results of numerical experiments on a two-component vortex system, for $|\psi^{(1)}|^2 = 0.5$, $|\psi^{(2)}|^2 = 1.0$, and $e = 1.0$. (a) Structure functions for protonic $S^{(1)}(\mathbf{K})$ (red or light gray) and electronic $S^{(2)}(\mathbf{K})$ (blue or dark gray) vortices, for $\mathbf{K} = (\pi/4, 2\pi/5)$. They drop to zero discontinuously at T_M where the system loses its superconducting properties, but retains superfluidity, as evidenced by a finite helicity modulus Υ_x (black line). Υ_x serves as an order parameter in the vortex-liquid phase, dropping to zero at T_C. (b) N_{co} (green or light gray) is finite across the melting transition, but drops continuously to a small value at T_C, where it has a kink. At T_C, the specific heat C_V has an anomaly.

gradually as the temperature is increased. At T_M, $S^{(1)}(\mathbf{K})$ and $S^{(2)}(\mathbf{K})$ vanish at the same temperature even though the ratio of the bare stiffnesses of the condensates is 2.0. Moreover, both structure functions vanish discontinuously, the hallmark of a first-order melting transition of the composite vortex lattice.

To probe the splitting of the cocentered vortices into constituent vortices, we compute the vortex cocentricity, defined as $N_{\rm co} \equiv N_{\rm co}^{(+)} - N_{\rm co}^{(-)}$, where $N_{\rm co}^{(\pm)} = \sum_{\mathbf{r}} |n_z^{(2)}(\mathbf{r})| \delta_{n_z^{(1)}(\mathbf{r}), \pm n_z^{(2)}(\mathbf{r})} / \sum_{\mathbf{r}} |n_z^{(2)}(\mathbf{r})|$ and $\delta_{i,j}$ is unity if $i = j$ and zero otherwise. Therefore, $N_{\rm co}$ is the fraction of type-2 vortex segments that are cocentered and codirected with type-1 segments. We find that passing through the first-order melting transition renders $N_{\rm co}$ unaffected, whence we conclude that the observed transition is a melting of a composite vortex lattice.

In one-component type-II superconductors, melting of the vortex lattice amounts to a complete destruction of dissipationless currents [4]. To follow the fate of the superfluid mode of the two-component system in the vortex-liquid state, we measure the ordering in $\gamma(\mathbf{r}) \equiv \theta^{(1)}(\mathbf{r}) - \theta^{(2)}(\mathbf{r})$. To probe the global phase coherence in this variable, we consider the helicity modulus Υ_μ, equivalently the superfluid density, given by $\Upsilon_\mu = [\langle c \rangle - \eta \langle s^2 \rangle]/L^3$, where $c = \sum_{\mathbf{r}} \cos(\Delta_\mu \gamma(\mathbf{r}))$, $s = \sum_{\mathbf{r}} \sin(\Delta_\mu \gamma(\mathbf{r}))$, and $\eta = \beta |\psi^{(1)}|^2 |\psi^{(2)}|^2 / 2\Psi^2$. When the composite vortex lattice melts, destroying superconductivity [4], the superfluid density remains unaffected (cf. Fig. 1). Hence, this transition separates a superconducting superfluid from a metallic superfluid state.

Thus, we have identified a transition from a composite vortex lattice into a composite vortex liquid. Increasing the temperature further, we find a phase transition between the composite vortex liquid and the ''ionized vortices'' plasma, as evidenced by the vanishing of the superfluid density Υ_x (cf. Fig. 1). We observe that this transition is accompanied by a pronounced anomaly in the specific heat C_V, indicating that this is a critical phenomenon. This is corroborated by the following physical argument. The liquid state of vortices in an ordinary superconductor is a state where translational symmetry is restored and superconductivity is lost [4]. In the composite vortex-liquid state, every electronic vortex is accompanied by a protonic vortex performing only finite excursions away from the electronic vortex line. Therefore, for every plane slicing an electronic

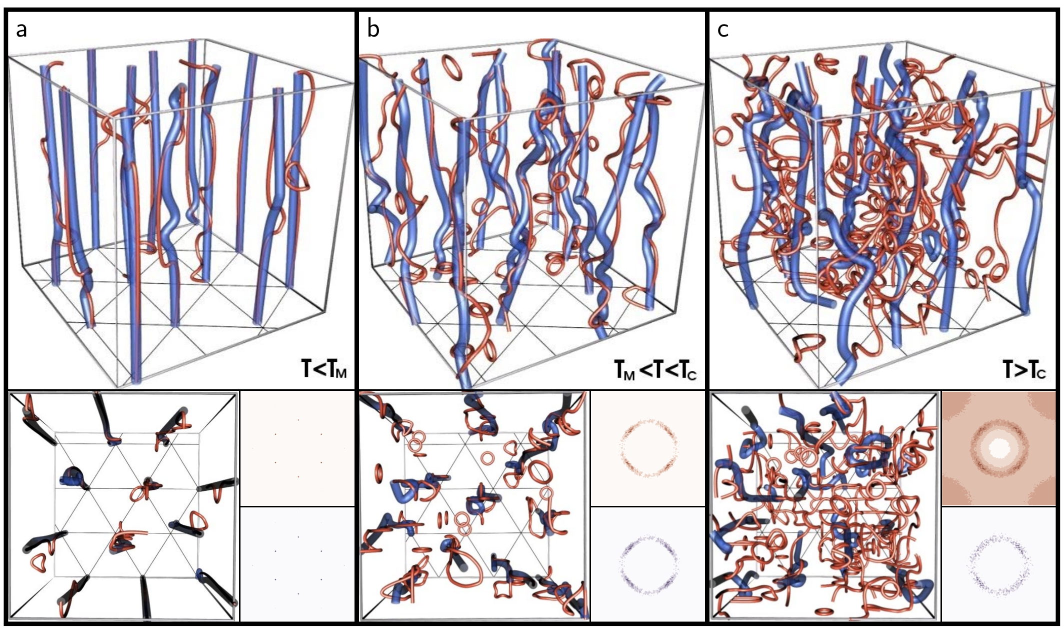

FIG. 2 (color online). Snapshots of the states of vortex matter and momentum space structure functions generated from MC simulations, taken at three different temperatures: $T = 0.50$ ($T < T_M$), $T = 0.72$ ($T_M < T < T_C$), and $T = 0.86$ ($T > T_C$). The snapshots are extracted from a small segment ($15 \times 15 \times 15$) of the vortex system. Each thick frame contains a side view and a top view of the vortices, as well as the protonic structure function $S^{(1)}(\mathbf{k}_\perp)$ (red, small upper box) and the electronic structure function $S^{(2)}(\mathbf{k}_\perp)$ (blue, small lower box). (a) For $T < T_M$ the vortices are arranged in a cocentered lattice. Electronic (thick blue) and protonic (thin red) vortices perform only small excursions from each other. Both structure functions exhibit a sixfold Bragg pattern characteristic of a vortex lattice. (b) For $T_M < T < T_C$ the composite vortex lattice has melted as illustrated by the ring pattern in the structure functions. The electronic and protonic vortices perform stronger excursions from each other but essentially remain cocentered. This is the superfluid metallic phase in which codirected currents of protonic and electronic Cooper pairs can propagate without dissipation. (c) For $T > T_C$ the superfluidity is lost and the electronic and protonic vortices are no longer cocentered. The proliferation of the protonic vortices is reflected in the increase in the uniform background of the protonic structure function.

vortex in a direction perpendicular to the magnetic field, it is possible to identify a finite length closed contour which also encompasses an accompanying codirected protonic vortex. Along such a contour, there is no nontrivial winding in the phase-difference γ. Therefore, melting of the composite vortex lattice into a composite vortex liquid does not restore the broken global $U(1)$ symmetry associated with γ. On the other hand, in the vortex plasma state one cannot find a protonic vortex accompanying every electronic vortex, implying a disordering of γ. Hence, the associated global $U(1)$ symmetry is restored during the "vortex ionization" transition taking place within the vortex liquid, and it is therefore in the 3Dxy universality class.

To gain further insight, we have also extracted vortices and visualized snapshots of configurations of vortex matter in small segments of the system. The results are shown in Fig. 2.

Figure 2(a) shows the side and top views of vortex matter at a small but finite temperature when two species of vortices perform only small excursion from each other. The insets show the structure function in momentum space of the protonic (thin red) and electronic (thick blue) vortices. Parallel and codirected vortices interact with each other as positively and negatively charged strings, and the splitting is a temperature-induced fluctuation. Figure 2(b) shows snapshots of the vortex matter when the system is heated above the temperature of vortex lattice melting. The protonic and electronic structure functions have developed ringlike structures, characteristic of a vortex liquid in both the protonic and electronic sectors. While the vortices perform stronger excursions from each other, these excursions are still limited and one can always identify a red line attached to any given blue line. Thus, cocentricity of protonic and electronic vortex lines is still largely intact. In such a configuration, a dissipationless electrical current cannot propagate in any direction. Quite remarkably, however, codirected currents of protonic and electronic pairs can propagate through this system without dissipation, as evidenced by the measurements of the helicity modulus given in Fig. 1. Figure 2(c) shows a state of vortex matter which occurs above the vortex "ionization" temperature. The cocentricity of vortices is strongly reduced; see Fig. 1.

This is also reflected in the subtle difference between protonic and electronic structure functions in passing from panel (b) to panel (c). While the structure function of the electronic vortices essentially is unaffected by passing through the temperature T_C, the structure function for the protonic vortices is distinctly further isotropized. Namely, the relative increase of the uniform background for the structure function of protonic vortices is a manifestation of the fact that protonic vortices suffer a vortex-loop proliferation transition inside the metallic vortex-liquid phase. We have argued above that this transition belongs to the 3Dxy universality class, i.e., the same universality class as the superfluid-to-normal fluid transition in liquid helium ^{4}He [16].

Concluding, we report the observation, in a numerical experiment, of a novel dissipationless quantum state of matter, namely, the metallic superfluid. Such a state might be realized in hydrogen or deuterium, if those systems were to take up a projected low-temperature liquid metallic state at an extreme pressure. The highest pressure obtained to date appears to be around 320 GPa [17]. However, recent breakthroughs in artificial ultrahard diamond synthesis technology [13] represent significant progress towards achieving extreme pressures in diamond anvil cells. Thus, a metallic superfluid might be the next "superstate" of matter to be realized in the laboratory [18].

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