Tunneling Spectroscopy of Two-Level Systems Inside a Josephson Junction

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We consider a two-level system (TLS) with energy level separation $\hbar\Omega_0$ inside a Josephson junction. The junction is shunted by a resistor R and is voltage V biased. If the TLS modulates the Josephson energy and/or is optically active, it is Rabi driven by the Josephson oscillations in the running phase regime near the resonance $2eV = \hbar\Omega_0$. The Rabi oscillations, in turn, translate into oscillations of current and voltage that can be detected in noise measurements. This effect provides an option to fully characterize the TLS inside Josephson junction and to find the TLS's contribution to the decoherence when the junction is used as a qubit.

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While the microscopic mechanism of Josephson coupling and electrodynamics of superconductor-insulator-superconductor Josephson junctions (JJ) are well understood, the microscopic origin of dissipation in these junctions at low temperatures remains largely unknown. As temperature decreases and quasiparticles freeze out, low lying degrees of freedom inside the insulating layer that interact with the phase difference cause dissipation. The problem to characterize them is important for any low temperature applications of JJ, but it is especially relevant for the use of JJ as qubits for quantum computing where long decoherence time is needed. In fact, even small-area Josephson junctions, as solid-state mesoscopic systems, have many low-energy degrees of freedom inside the amorphous insulating layer. Generally, those are phonons and two-level systems (TLS), that is, systems where only transitions between the ground and a single excited state are possible at any given frequency.

Previously it was shown that optical phonons inside intrinsic Josephson junctions in cuprate layered superconductors cause anomalies in the dc current-voltage characteristics. Namely, peaks in the tunneling current at the voltages corresponding to the phonon frequencies, $V = \hbar\omega_{ph}/2e$, were observed and mechanisms of their coupling with the junction phase difference were identified; see Refs. [1–4]. The role of phonon radiation on the qubit decoherence was discussed by Ioffe *et al.* [5]. In this Letter we focus on the effects of TLS, which may dominate at low frequencies [6]. We propose a method to identify TLS inside JJ and to distinguish their role in dissipation and decoherence from that of phonons. This may help to design junctions with minimal decoherence due to TLS.

Microscopically, a TLS can be an ion or electron having two possible positions inside potential wells with tunneling between them. These degrees of freedom interact with the junction phase difference if they modulate the Josephson energy and/or if they are optically active. In the latter case they couple to the phase difference $\phi(t)$ via the electric field inside the junction, $\mathcal{E} = \dot{\phi}/d$, where d is the thickness of the insulating layer in the junction [the phase is in the units of magnetic flux quantum $\Phi_0 = h/(2e)$].

Here we discuss both mechanisms of coupling between the phase and a TLS and propose a method to distinguish between them. Similar to phonons, a TLS can cause anomalies in the dc I-V characteristics. But what is more, a TLS can precess at its Rabi frequency Ω_R when the Josephson oscillation frequency matches the level splitting, $\hbar\omega_J \equiv 2$ eV $= \hbar\Omega_0$. Coherent Rabi oscillations between the ground and excited states of the TLS are possible when the drive strength (the Rabi frequency) exceeds the intrinsic relaxation rate of the TLS. For a TLS inside JJ Ω_R is proportional to the coupling of TLS and the junction. Thus measurement of the low frequency noise spectrum can be used to determine this coupling.

The system we consider is a voltage-biased pointlike Josephson junction with the normal-state resistance R_N (which could be used as a qubit), shunted by a small resistor, $R \ll R_N$ (Fig. 1). This ensures that in the running phase regime, $V > I_c R$, voltage on the junction ($\approx V$) can be small compared with the superconducting gap Δ. We also assume that $2eV \gg k_B T$. In this regime the Cooper

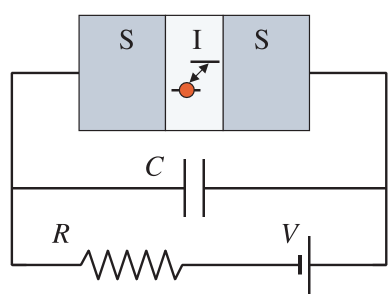

FIG. 1 (color online). Model. The system is a Josephson junction with an embedded TLS. The TLS can be modeled as a charged particle that can tunnel between two nearby positions in the insulator. It modulates the Josephson coupling and/or interacts with the Josephson oscillation via dipole moment. When Josephson oscillations resonantly drive TLS, its Rabi oscillations can be detected on the resistor R.

pairs tunnel incoherently and give rise to the shot noise [7–10].

First, as in Refs. [6,11], we consider a junction with the Josephson energy modified by the interaction with TLS, described by the pseudospin-1/2 operator $\mathbf{S}$

$$H_J = -E_J(1 + \mathbf{j} \cdot \mathbf{S})\cos(2\pi\phi/\Phi_0), \tag{1}$$

where $E_J = \Phi_0 I_c/2\pi$, I_c is the Josephson critical current without a TL system. Coupling constants $\mathbf{j} = (j_x, j_y, j_z)$ characterize the modulation of the Josephson critical current by the TLS. The Hamiltonian of the voltage-biased system reads [9,12]

$$H = \frac{q^2}{2C} + H_{\rm R}(Vt - \phi) + H_J(\phi) - \hbar\Omega_0 S_z, \tag{2}$$

where $H_{\rm R}(\xi)$ is the Hamiltonian of the resistor on which the phase ξ drops, and q is the charge on the capacitor C, q being conjugate to ϕ. After the transformation $\tilde{\phi} = \phi - Vt$ and $\tilde{H} = H - Vq$, we obtain

$$\tilde{H} = \frac{\tilde{q}^2}{2C} + H_{\rm R}(-\tilde{\phi}) + H_J(\tilde{\phi} + Vt) - \hbar\Omega_0 S_z, \tag{3}$$

where $\tilde{q} \equiv q - CV$. The Josephson current operator is

$$I_{\rm J} \equiv I_c(1 + \mathbf{j} \cdot \mathbf{S})\sin(\omega_{\rm J} t + 2\pi\tilde{\phi}/\Phi_0). \tag{4}$$

Our goal is to determine the width of the Rabi peak and its magnitude relative to the noise background. The origin of both peak width and the background is the Johnson-Nyquist (JN) noise. Because of the nonlinearity of the Josephson coupling, at any given frequency, the voltage fluctuation spectrum on the shunt has three main contributions: JN noise of the resistor itself (first two terms in $\tilde{H}$), shot noise of Cooper pairs tunneling incoherently through the JJ (the third term $\tilde{H}$, for $\mathbf{j} = 0$), and a resonant contribution induced by the Rabi nutation of the spin. For frequencies such that $\hbar\omega \ll 2eV$ and in the running phase regime, $V \gg I_c R$, the *symmetrized* voltage autocorrelator $S_V(\omega)$ is given by the sum of JN noise and a contribution proportional to the *symmetrized* current autocorrelator, $S_I(\omega)$,

$$S_V \approx S_V^{\rm JN} + \frac{R^2}{Y(\omega)} S_I, \quad S_V^{\rm JN}(\omega) = \frac{\hbar\omega R}{Y(\omega)}\coth\frac{\hbar\omega}{2k_{\rm B}T}, \tag{5}$$

where $Y(\omega) \equiv 1 + C^2R^2\omega^2$. In Eq. (5) the equilibrium correlator $S_V^{\rm JN}$ is due to the first two terms of Hamiltonian (3), while the next (S_I) term accounts for the Josephson coupling. Equation (5) can be obtained either from an exact relation between the phase and current Green's functions or from the quasiclassical Langevin equation. In typical experiments, $RC\omega \ll 1$ for all relevant frequencies ω, and, therefore, $Y(\omega) \approx 1$. We, however, keep the factor $Y(\omega)$ to be able to discuss the regime of a junction shunted by a big capacitor.

Next, we calculate the correlator of the Josephson current operators. We are particularly interested in a near-resonance situation, $\Omega_0 \approx \omega_{\rm J}$. As we can see from Eq. (3), the spin is subject to an ac driving at frequency $\omega_{\rm J}$, "broadened" by the fluctuating phase $\tilde{\phi}(t)$. Thus it is convenient to transform to the frame rotating with the angular velocity $\omega_{\rm J} + (2\pi/\Phi_0)(d/dt)\tilde{\phi}$. Formally this amounts to performing canonical transformation $H' = U\tilde{H}U^{-1} + i\hbar\dot{U}U^{-1}$, with $U = \exp[2\pi i(\tilde{\phi} + Vt)S_z/\Phi_0]$. Without loss of generality we take $\mathbf{j} = (j_\perp, 0, j_\parallel)$. The result is

$$H' = \frac{\tilde{q}^2}{2C} + H_{\rm R}(-\tilde{\phi}) - \hbar(\Omega_0 - \omega_{\rm J})S_z - \frac{2e\tilde{q}S_z}{C} - E_{\rm J}(1 + j_\parallel S_z)\cos(\omega_{\rm J} t + 2\pi\tilde{\phi}/\Phi_0) - \hbar\Omega_{\rm R} S_x, \tag{6}$$

where

$$\Omega_{\rm R} \equiv j_\perp I_c/(4e)$$

is the Rabi frequency of the spin. The counterrotating term [$\propto \exp\pm 4\pi i(\tilde{\phi} + Vt)/\Phi_0$] can be shown to be not important. The resonance is reached when $\omega_{\rm J} = \Omega_0$. Then the spin rotates around the x axis (of the rotating frame) with the Rabi frequency $\Omega_{\rm R}$, but its dynamics is affected by the noise due to the charge and phase fluctuations.

The operator of the charge on the capacitor is transformed in the rotating frame as

$$q' = U\tilde{q}U^{-1} = \tilde{q} - 2eS_z. \tag{7}$$

The Josephson current operator transforms as

$$I_{\rm J}' = I_c(1 + j_\parallel S_z)\sin(\omega_{\rm J} t + 2\pi\tilde{\phi}/\Phi_0) - \frac{j_\perp I_c}{2} S_y, \tag{8}$$

which shows that dynamics of the spin translates into dynamics of the current.

For $|\mathbf{j}| \ll 1$, we may neglect the spin's contribution to the average current. However, we may not neglect the spin contribution to the current noise near the Rabi frequency $\omega \approx \Omega_{\rm R}$. Indeed, as we shall see, the spin-dependent part of $I_{\rm J}$ gives rise to a peaked contribution to the correlator $S_I(\omega)$ at $\omega = \omega_{\rm R}$. We calculate $S_I(\omega)$ in the rotating frame using Eqs. (6) and (8). The smooth part of $S_I(\omega)$ is given by the shot noise of Cooper pairs. For $VY(\omega) \gg I_c R$, $\omega \ll \omega_{\rm J}$, and $R \ll R_Q \equiv h/4e^2$, at high voltages $2eV \gg k_{\rm B}T$ we obtain [13]

$$S_I^{\rm shot}(\Omega_{\rm R}) \approx S_I^{\rm shot}(0) \approx \frac{eRI_c^2}{VY(\omega_{\rm J})}. \tag{9}$$

In addition, there exists a peak-shaped contribution to $S_I(\omega)$ near $\omega \approx \Omega_{\rm R}$. Using $\langle\sin 2\pi(\tilde{\phi} + Vt)/\Phi_0\rangle = RI_c/[2VY(\omega_{\rm J})] \equiv I_{av}/I_c$ we take into account the $j_\parallel$ term by defining $S_* \equiv j_\perp S_y + (I_{av}/I_c)j_\parallel S_z$. Then

$$S_I^{\rm spin}(t, t') = \frac{I_c^2}{8}\langle\{S_*(t)S_*(t')\}_+\rangle. \tag{10}$$

To calculate this correlator, we use Hamiltonian (6). At resonance the effective magnetic field $\hbar\Omega_{\rm R}$ is directed along the x axis. Assuming the Rabi oscillations are underdamped (to be checked for self-consistency), we obtain

$$S_I^{\rm spin} = \frac{j_{\rm eff}^2 I_c^2}{16}\left[\frac{\Gamma_2}{(\omega - \Omega_{\rm R})^2 + \Gamma_2^2} + (\omega \to -\omega)\right], \tag{11}$$

where $j_{\rm eff}^2 \equiv j_\perp^2 + (I_{av}/I_c)^2 j_\parallel^2$.

The spin dephasing rate Γ_2 originates from the fourth and the fifth terms in the Hamiltonian (6), which contain S_z. Thus they contribute to Γ_2 through the longitudinal relaxation rate Γ_1, with $\Gamma_2 = (1/2)\Gamma_1$. The fourth term (voltage noise) consists of two (uncorrelated to the lowest order) contributions: the equilibrium JN noise, $S_V^{\rm JN}(\Omega_{\rm R})$, which gives $\Gamma_2^{\rm JN} = (e/\hbar)^2 S_V^{\rm JN}(\Omega_{\rm R})$, and shot noise $S_V^{\rm shot}(\Omega_{\rm R})$ resulting in $\Gamma_2^{\rm shot} = (e/\hbar)^2 S_V^{\rm shot}(\Omega_{\rm R})$. Finally the fifth term in Eq. (6) contributes the rate $\Gamma_2^{\parallel} = (j_\parallel/4e)^2 \times S_I^{\rm shot}(\Omega_{\rm R})$. This follows from $\langle\langle \sin\varphi(t)\sin\varphi(t')\rangle\rangle = \langle\langle \cos\varphi(t)\cos\varphi(t')\rangle\rangle$, where $\varphi(t) \equiv 2\pi[\tilde\phi(t) + Vt]/\Phi_0$. One can check that all three rates are to be added "incoherently"; that is, the noise cross terms vanish. Thus, accounting for intrinsic TLS dephasing rate Γ_0,

$$\Gamma_2 = \Gamma_2^{\rm JN} + \Gamma_2^{\rm shot} + \Gamma_2^{\parallel} + \Gamma_0. \tag{12}$$

For the "signal," i.e., the height of the voltage peak, we obtain

$$S_V^{\rm peak}(\Omega_{\rm R}) = \frac{j_{\rm eff}^2 R^2 I_c^2}{16\Gamma_2 Y(\Omega_{\rm R})}, \tag{13}$$

while the "noise," i.e., the background, is given by

$$S_V^{\rm bg} = \frac{R^2}{Y(\Omega_{\rm R})}\left(S_I^{\rm shot} + \frac{\hbar\Omega_{\rm R}}{R}\coth\frac{\hbar\Omega_{\rm R}}{2k_{\rm B}T}\right). \tag{14}$$

Finally, we obtain the signal-to-noise ratio

$$\mathcal{R} = \frac{Y(\Omega_{\rm R})A_\parallel\left[\coth\frac{\hbar\Omega_{\rm R}}{2k_{\rm B}T} + \frac{eR^2I_c^2}{\hbar\Omega_{\rm R}VY(\omega_{\rm J})}\right]^{-1}}{\left[\coth\frac{\hbar\Omega_{\rm R}}{2k_{\rm B}T} + \frac{eR^2I_c^2B_\parallel}{\hbar\Omega_{\rm R}VY(\omega_{\rm J})} + \frac{2R_QY(\Omega_{\rm R})\Gamma_0}{\pi R\Omega_{\rm R}}\right]}, \tag{15}$$

where $A_\parallel \equiv 1 + [\frac{j_\parallel}{j_\perp}\frac{RI_c}{VY(\omega_{\rm J})}]^2$ and $B_\parallel \equiv 1 + Y(\Omega_{\rm R})(\frac{j_\parallel}{2\pi}\frac{R_Q}{R})^2$.

For purely transverse coupling, $j_\parallel = 0$, the essential physics is the following: we illuminate the spin with the "magnetic" field $2\hbar\Omega_{\rm R}\cos(\omega_{\rm J}t + 2\pi\tilde\phi/\Phi_0)$. This field can be thought of as having a sharp peak (a line) near $\omega = \omega_{\rm J}$. The width of this Josephson line is given by the total voltage noise at zero frequency, $\hbar\Delta\omega = \pi S_V(0)/R_{\rm Q} = (\pi/R_{\rm Q})[S_V^{\rm JN}(\omega = 0) + R^2 S_I^{\rm shot}(\omega = 0)]$. This relation between the width of the Josephson line and the total voltage noise was obtained in Refs. [14,15]. The Rabi oscillation produced by this "line" are, in turn, also broadened by the same amount $\Gamma_2 = \Delta\omega/2$ (in addition to the intrinsic broadening Γ_0). Finally, the spin's (broadened) Rabi precession leads to broadened oscillations of the Josephson current and voltage at $\Omega_{\rm R}$ on top of the background of the JN and shot noise.

It is also important to note that the Rabi oscillations of the pseudospin correspond to exactly one Cooper pair going back and forth across the junction. This can be seen from Eq. (7), or from the fact that for the Rabi oscillations to occur exactly one "Josephson photon" with the energy $\hbar\omega_{\rm J}$ must be absorbed and reemitted by the spin; i.e., exactly one Cooper pair must go through the voltage drop V.

To get a feeling for the relevant numbers, we take the data obtained by Simmonds *et al.* [6], where the two lowest levels of a junction (phase qubit) in the superconducting (phase-nonrunning) regime were driven resonantly. The level splitting $\omega_{01}(I)$ was varied by the bias current I in the frequency interval 8.6–9.1 GHz. At some values of $\omega_{01}(I)$ appreciable splittings (avoided level crossings) were observed. This was suggested to originate from TLS with $\Omega_0 \approx \omega_{01}(I)$, and the interaction with the phase difference of the type (1). The splitting is caused by the $j_\perp$ term, while the $j_\parallel$ term is inessential as long as $j_\parallel E_{\rm J} \ll \hbar\Omega_0$. Thus, the strongest impurity had $j_\perp \approx 6.5 \times 10^{-5}$, while we have an upper bound for the strength of the longitudinal coupling $j_\parallel < 10^{-3}$. This gives $\Omega_{\rm R} \approx 2\pi \times 200$ MHz (the splitting of 25 MHz in [6] is due to the reduction factors corresponding to the zero-point motion of the phase degree of freedom in the potential well). In Ref. [6] the critical current is $I_c \approx 10\ \mu$A, the normal resistance of the junction is $R_N \sim 30\ \Omega$, while $C \sim 1$ pF.

Using these parameters, we estimate the signal strength and the signal-to-noise ratio for the Rabi oscillations of TLS in the running phase regime, $\mathcal{R}$. For the temperature we assume $T \approx 10$ mK, or $k_{\rm B}T/\hbar = 2\pi \times 200$ MHz. The minimum voltage is given by I_cR. We assume the shunting resistance of order $R \sim 0.1\ \Omega \ll R_N$. Shunts of this magnitude have been used in [16]. Hence, we have $\omega_{\rm J} > (2e/\hbar)I_cR \approx 2\pi \times 0.5$ GHz. From above $\omega_{\rm J}$ is restricted by the gap, which gives $\omega_{\rm J} < (2e/\hbar)I_cR_N \approx 2\pi \times 150$ GHz. Thus we can take $\omega_{\rm J} \sim 2\pi \times 10$ GHz to be in resonance with the observed TLS. For $j_\parallel = 0$ and assuming $\Gamma_0 = 0$ the Rabi linewidth is dominated by the Johnson-Nyquist noise, $\Gamma_2 \approx 2\pi \times 5$ kHz. We obtain the signal-to-noise ratio $\mathcal{R} \approx 0.25$. If Γ_0 dominates (from experiment, $\Gamma_0 < 2\pi \times 25$ MHz), the signal-to-noise ratio $\mathcal{R}$ will be reduced. For the maximally allowed $j_\parallel = 10^{-3}$ the ratio $\mathcal{R}$ does not change considerably. For the integrated signal (signal amplitude) we obtain

$$\left[\int_{\Omega_{\rm R}} \frac{d\omega}{2\pi} S_V^{\rm peak}(\omega)\right]^{1/2} \approx \frac{j_{\rm eff}RI_c}{4\sqrt{2Y(\Omega_{\rm R})}} \approx 10^{-2}\ {\rm nV}.$$

We also note that for the above introduced parameters we have $1/(CR) \sim 10^{13}s^{-1}$. Thus $Y(\Omega_{\rm R}) \approx Y(\omega_{\rm J}) \approx 1$. One has to increase C by at least 3 orders of magnitude in order to start having $Y(\Omega_{\rm R}) > 1$. For such parameters we obtain $\mathcal{R} \gg 1$, which is in contrast with the limitation $\mathcal{R} \le 4$ found for the measurements of the peak in the current noise at the frequency Ω_0 [17–19] using a normal-state tunnel junction. In that case the voltage $V \gg \hbar\Omega_0/e$ (broadband) is applied. It *incoherently* excites the TLS but also introduces the relaxation due to dissipation necessary for measurement procedure. The relaxation is determined by the noise at frequency Ω_0, and the signal is measured on the background of the noise at the same frequency. As a result, $\mathcal{R}$ is a universal number. In the case considered here, spin is excited at (high) frequency Ω_0, but the signal is observed at low frequency, due to nonlinearity of the coupled spin and Josephson junction system. However, in the regime with large signal-to-noise $\mathcal{R}$ the single Cooper pair mainly charges and discharges the capacitor, barely going through the resistor; thus, the integrated signal in this regime is reduced.

Now let us consider the mechanism where spin couples to the junction via the electric field. The Hamiltonian is

$$H = \frac{q^2}{2C} + H_R(Vt - \phi) - E_J \cos\frac{2\pi\phi}{\Phi_0} - \hbar\Omega_0 S_z - \frac{Q_{TL} q}{C} S_x,$$

where Q_{TL} is the effective charge of the TL system, given by $Q_{TL} = d_{TL}/L$, where d_{TL} is the TL's dipole moment, while L is the junction's width. For simplicity we assumed purely transverse coupling. Remarkably, the splitting observed in Ref. [6] could also be explained by this model with $Q_{TL} \sim e$. In this mechanism the spin is coupled to the variable q, which in zeroth order in tunneling (E_J) does not oscillate and has only the Johnson-Nyquist noise spectrum. At $V > RI_c$ this variable acquires an oscillating part due to the Josephson oscillations of the current. Thus the Rabi driving becomes possible. The width of the Rabi line is again determined by the full width of the Josephson line $\Gamma_2^{JN} + \Gamma_2^{shot}$. From the integral (weight) of the Josephson peak in the $S_q(\omega)$ correlator we obtain the Rabi frequency

$$\Omega_R = RI_c Q_{TL}/(2\hbar\sqrt{Y(\omega_J)}).$$

Then, analysis similar to the one presented above again gives Eq. (15) for the signal-to-noise ratio (with $A_\parallel = B_\parallel = 1$ as we assumed purely transverse coupling). For $Q_{TL} \sim e$, we obtain a similar to the previous mechanism Ω_R and a similar value of $\mathcal{R}$. For the integrated signal we obtain

$$\left[\int_{\Omega_R} \frac{d\omega}{2\pi} S_V^{peak}(\omega)\right]^{1/2} \sim \frac{Q_{TL} R^2 I_c}{e R_Q} \approx 10^{-2}\ \text{nV}.$$

Note that the Rabi frequency Ω_R and the integrated signal depend differently on R in two coupling mechanisms. This may allow one to distinguish between the two, while they are undistinguishable in measurements of type [6].

In this Letter we discussed what happens when the Josephson oscillations are in resonance with one TLS. Let us mention another interesting possibility to manipulate the system. By changing the applied voltage slowly ($\Gamma_1\Omega_R < \dot{\omega}_J < \Omega_R^2$), one can create the regime of the "adiabatic passage" when $\omega_J(t)$ passes slowly via Ω_0 and exactly one additional Cooper pair is transferred through the junction. Varying $\omega_J(t)$ in a wide enough interval, one can flip many TLS, thus creating a measurable current in addition to the dc value (i.e., $\dot{\omega}_J < \Gamma_1\Omega_R$).

In conclusion, we propose that the measurements of the low frequency voltage noise in a Josephson junction in the dissipative (running phase) regime may be used to characterize the TLS inside the junctions, i.e., energy splitting Ω_0, coupling strength $j_\perp$ from the Rabi frequency, and intrinsic dephasing rate Γ_0 from the height of the voltage peak, Eq. (13). We predict a peak at the Rabi frequency when a TLS is resonantly driven by the Josephson oscillations, $\omega_J = \Omega_0$, with the Rabi frequency proportional to the interaction strength between the TLS and the Josephson phase. The peak intensity (signal-to-noise ratio) can be controlled by the shunt resistor and capacitor. Observation of the Rabi oscillations will allow one to unambiguously distinguish the TLS origin of the splittings from phonons and the macroscopic tunneling mechanism proposed by Johnson *et al.* [20], neither of which lead to the Rabi oscillations.

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