Negative Refraction and Focusing of Circularly Polarized Waves in Optically Active Media

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Analysis indicates that certain types of optically active media are capable of producing negative refraction and focusing of circularly polarized waves. It is established that a slab of such material acts just as Veselago's hypothetical left-handed media lens, providing subwavelength resolution as Sir Pendry's ideal lens, but for circularly polarized waves.

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There is a long tradition of optical activity (or rotation of the polarization vector as the wave advances) in physics and chemistry, possibly since the time of Pasteur [1]. Chirality, or handedness, satisfies reciprocity, and has been exploited in the optical regime for a long time. Artificial chirality at lower frequencies (rf microwave) was first induced and experimentally verified by Lindeman [2,3] over 80 years ago, which he accomplished with a random collection of insulated coils of certain handedness. Bi-isotropic media, analytically proposed by Tellegen [4] over half a century ago, can be nonreciprocal, and is not as popular, possibly because it has not been found in nature. Both chiral and bi-isotropic media are isotropic, and they become polarized when placed in a magnetic field, and magnetized when placed in an electric field. Bi-isotropic media is characterized by two Cherenkov radiation cones [5] and, under special circumstances [6], can produce dipole fields with magnetic lines, which are open spirals that go from pole to pole, or Cherenkov radiation with spiraling magnetic field lines.

Here we refer to "left-handed" media (LHM) as the media exhibiting negative refraction, i.e., Veselago's original concept [7]. We do not use the "left-handed" term in the context of chirality. LHM possesses simultaneous negative permeability μ, and permittivity ε, resulting in phase velocity and energy flowing in opposite directions. Because of the scarcity of experimental materials, controversy surrounds the properties of LHM [8–16]. LHM prototypes are typically a periodic array of wires and split ring resonators, of the type first proposed by Sir Pendry [17].

Veselago used Snell's law in a planar LHM slab geometry, and showed that focusing both inside and outside of the slab occurs. More recently, Sir Pendry *et al.* [9,10] claimed that LHM can amplify evanescent modes allowing a complete reconstruction of a point source to a perfect point image. Here we show that a planar slab of chiral or bi-isotropic material can act in the same fashion as Veselago's hypothetical LHM slab, but for circularly polarized (CP) waves.

The constitutive relations of bi-isotropic media are [5,6,18]

$$\bar{D} = \varepsilon\bar{E} + \gamma\bar{H}, \qquad \bar{B} = \mu\bar{H} + \beta\bar{E}. \tag{1}$$

Such a medium is lossless if ε and μ are real, and provided $\gamma = \beta^*$. The medium becomes reciprocal if $\gamma = -\beta$. The bi-isotropic medium reduces to a chiral reciprocal medium (three parameters) for $\gamma = -\beta = j\chi$, where χ is a real number (lossless material). For a $e^{j\omega t}$ time convention (suppressed throughout), we have plane wave solutions of the form

$$\bar{E} = \bar{e}e^{-j\bar{\nu}\cdot\bar{x}}, \qquad \bar{H} = \bar{h}e^{-j\bar{\nu}\cdot\bar{x}} \tag{2}$$

for $\bar{h}$ and $\bar{e}$ orthogonal to the direction of propagation $\hat{\nu}$. Wave components possess different wave number and impedance depending on their helicity. They are given by

$$\nu_\pm = \omega\left[\sqrt{\mu\varepsilon - \left(\frac{\gamma+\beta}{2}\right)^2} \pm j\left(\frac{\gamma-\beta}{2}\right)\right], \tag{3}$$

$$\eta_\pm = \left[\sqrt{\frac{\mu}{\varepsilon} - \left(\frac{\gamma+\beta}{2\varepsilon}\right)^2} \mp j\left(\frac{\gamma+\beta}{2\varepsilon}\right)\right]. \tag{4}$$

The $(+)$ sign refers to right circularly polarized (RCP) waves and the $(-)$ sign to left circularly polarization (LCP). The vectors $\bar{h}$ and $\bar{e}$ can be expressed as [5]

$$\bar{e}_\pm = \mp j\eta_\pm\bar{h}_\pm, \qquad \bar{h}_\pm = \frac{1}{\sqrt{2}}(\hat{s} \mp j\hat{u}), \tag{5}$$

where $[\hat{\nu}\,\hat{s}\,\hat{u}]$ form a right-handed triad. For unit amplitude $\bar{h}_\pm$, we have the power law

$$\bar{e}_\pm \times \bar{h}^*_\pm = \eta_\pm\hat{\nu}. \tag{6}$$

And the modes are power orthogonal since

$$\bar{e}_\pm \times \bar{h}^*_\mp = 0. \tag{7}$$

Here we are interested in backward wave type propagation. We first focus our attention on $(+)$ RCP. One way we can obtain isotropic backward $(+)$ waves is by enforcing

$$\mathrm{Re}\{\nu_+\} < 0, \tag{8}$$

while simultaneously demanding ordinary LCP waves, i.e.,

$$\mathrm{Re}\{\nu_-\} > 0. \tag{9}$$

On the other hand, impedance match to the external impedance η_e is accomplished via

$$\eta_+ = \eta_e. \tag{10}$$

Conditions (9) and (10) are based on our experience with ordinary materials, but as the analysis shows, they help obtain a clean focal point in bi-isotropic media. To create ideal conditions for pure RCP, we could eliminate all propagating RCP waves by enforcing existence of evanescent ($-$) fields only. This is accomplished via

$$\mathrm{Re}\{\eta_-\} = 0, \tag{11}$$

which is analogous to the impedance of a waveguide below cutoff.

From (5) we see that (10) and (11) cannot be satisfied simultaneously unless the material is not reciprocal. This is because for reciprocal materials $\gamma = -\beta$, which implies $\eta_+ = \eta_-$. Since (8)–(11) are four scalar conditions, it appears that four scalar unknowns are sufficient to satisfy them. As we have eight scalar unknowns in four complex variables ($\varepsilon, \mu, \gamma, \beta$), bi-isotropic media provide us with plenty of parameters for a proper design.

Condition (11) is desirable, but not mandatory. The same polarization rejection can be achieved, if needed, by means of mature technologies. For instance, we can use a filter at the front end of the material, in the manner of a frequency selective surface. Assuming that this is the case, we can relax (11), and use reciprocal materials. Use of $\gamma = -\beta = j\chi$, for χ complex, results in

$$\nu_\pm = \omega[\sqrt{\mu\varepsilon} \mp \chi], \qquad \eta_\pm = \sqrt{\frac{\mu}{\varepsilon}} = \eta_e. \tag{12}$$

For the particular case of free space surroundings, and an effective index of -1 for the RCP ($+$) waves, so as to meet the conditions for "perfect focusing," we have

$$\begin{aligned} \mu/\mu_0 &= \varepsilon/\varepsilon_0, \qquad \chi = \sqrt{\mu_0\varepsilon_0}\left(1 + \frac{\varepsilon}{\varepsilon_0}\right), \\ \nu_+ &= -\omega\sqrt{\mu_0\varepsilon_0}, \qquad \nu_- = \omega\sqrt{\mu_0\varepsilon_0}\left(1 + \frac{2\varepsilon}{\varepsilon_0}\right). \end{aligned} \tag{13}$$

Since the index of the ($-$) wave is complex depending on ε, we can make the ($-$) wave rapidly attenuate in the material by controlling ε (subject to $\mu/\mu_0 = \varepsilon/\varepsilon_0$).

So far we have demonstrated that we can selectively make one of the CP waves exhibit backward wave behavior. We have shown that this is possible with bi-isotropic media, and even chiral media. We show next that we can, indeed, obtain focusing in bi-isotropic media under the above stated conditions (8)–(10), and imposing an effective index of -1 for the RCP waves.

For simplicity, consider the 2D geometry shown in Fig. 1, where a point source radiating pure RCP waves (this can be accomplished by a properly designed helical antenna), is located at a distance "d" from a bi-isotropic half-space. Previous analyses of a bi-isotropic–chiral interface have considered only linearly polarized incident plane waves and not point sources [19–21].

The field of an RCP line source of strength I_+ can be obtained by setting the phasing constant to zero in Ref. [5], in Eq. (16), and after some algebra can be rewritten as

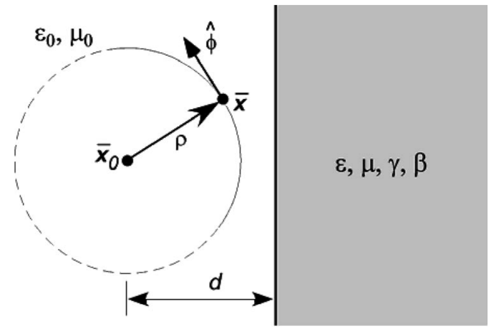

FIG. 1. Geometry of RCP line source in front of a bi-isotropic half-space.

$$\bar{H}^{\,\mathrm{inc}} = -\frac{j}{4} I_+\{k_0\hat{z} - \hat{z} \times \nabla\} H_0^{(2)}(k_0\rho). \tag{14}$$

As the Hankel function is representable as a sum over plane waves (Ref. [22], p. 624), we have

$$H_0^{(2)}(k\rho) = \frac{1}{\pi}\int_C e^{-j\bar{k}(\theta)\cdot(\bar{x}-\bar{x}_0)} d\theta \tag{15}$$

for θ the angle formed by $\bar{k}(\theta)$ and $\bar{x} - \bar{x}_0$, and where $|\bar{k}(\theta)| = k_0$. Furthermore, $\bar{x}_0$ represents the source location, and the contour C runs from $-j\infty$ just to the left of the imaginary axis, to $j\infty$ just to the right of the imaginary axis. In view of (14) and (15) we can express the inhomogeneous RCP field as a bundle of RCP plane waves,

$$\bar{H}^{\,\mathrm{inc}} = \frac{k_0}{2\sqrt{2}\pi} I_+ \int_C e^{-j\bar{k}(\theta)\cdot\bar{x}} \left\{\frac{\hat{s}(\theta) - j\hat{z}}{\sqrt{2}}\right\} e^{j\bar{k}(\theta)\cdot\bar{x}_0} d\theta, \tag{16}$$

where $\hat{s}(\theta) = \hat{z} \times \hat{k}(\theta)$, and $[\hat{k}(\theta)\hat{s}(\theta)\hat{z}]$ form a right-handed triad. Since we do have some flexibility in deforming the contour C, it is possible to make it independent of the individual choice of $\bar{x} - \bar{x}_0$. Accordingly, we define the angle θ, as measured with respect to the y axis.

For an incident RCP plane wave components of (16), we expect two reflected waves (RCP, LCP), and two transmitted waves (RCP, LCP). Figure 2 sketches the five CP beams, and defines the incident and reflection wave numbers $\bar{k}_1(\theta)$ and $\bar{k}_2(\theta)$, respectively, as well as the transmission wave numbers $\hat{\nu}_+(\theta)$ and $\hat{\nu}_-(\theta)$, and the corresponding $\hat{s}_\pm(\theta)$ vectors.

From (2), (4), and (16), and upon ignoring for the time being the phase term $e^{j\bar{k}(\theta)\cdot\bar{x}_0}$, we can write the plane wave fields as follows. On the free space side, the total fields are

$$\begin{aligned} \bar{H}^{\mathrm{TOTAL}}_{\mathrm{FreeSpace}} &= \frac{(\hat{s} - j\hat{z})}{\sqrt{2}} e^{-j\bar{k}_1\cdot\bar{x}} \\ &\quad + e^{-j\bar{k}_2\cdot\bar{x}}\left\{\frac{(\hat{s}_2 - j\hat{z})}{\sqrt{2}} R_+ + \frac{(\hat{s}_2 + j\hat{z})}{\sqrt{2}} R_-\right\}, \\ \bar{E}^{\mathrm{TOTAL}}_{\mathrm{FreeSpace}} &= -j\eta_0 \frac{(\hat{s} - j\hat{z})}{\sqrt{2}} e^{-j\bar{k}_1\cdot\bar{x}} \\ &\quad - j\eta_0 e^{-j\bar{k}_2\cdot\bar{x}}\left\{\frac{(\hat{s}_2 - j\hat{z})}{\sqrt{2}} R_+ - \frac{(\hat{s}_2 + j\hat{z})}{\sqrt{2}} R_-\right\}, \end{aligned} \tag{17}$$

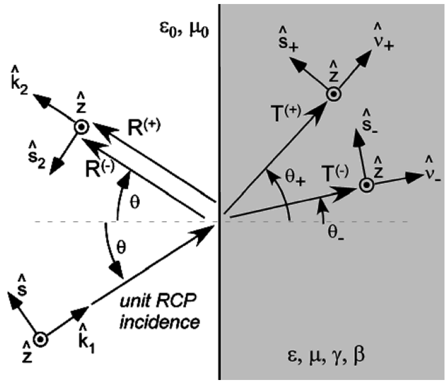

FIG. 2. Circularly polarized plane wave transmission and reflection on a bi-isotropic half-space.

where the first term of the equations represent the incident RCP plane wave, and the remaining terms the reflected RCP and LCP fields. On the bi-isotropic side we have

$$\bar{H}^{\mathrm{TOTAL}}_{\mathrm{Biisotropic}} = \frac{(\hat{s}_+ - j\hat{z})}{\sqrt{2}} e^{-j\bar{\nu}_+\cdot\bar{x}} T_+ + \frac{(\hat{s}_- + j\hat{z})}{\sqrt{2}} e^{-j\bar{\nu}_-\cdot\bar{x}} T_-,$$
$$\bar{E}^{\mathrm{TOTAL}}_{\mathrm{Biisotropic}} = -j\eta_+ \frac{(\hat{s}_+ - j\hat{z})}{\sqrt{2}} e^{-j\bar{\nu}_+\cdot\bar{x}} T_+ + j\eta_- \frac{(\hat{s}_- + j\hat{z})}{\sqrt{2}} e^{-j\bar{\nu}_-\cdot\bar{x}} T_-. \tag{18}$$

Enforcing the continuity of the tangential ($\hat{x}$ and $\hat{z}$) components of the electric and magnetic fields at the $y = 0$ interface yields four equations for the four unknowns $R_\pm$ and $T_\pm$, the strengths of the reflected and transmitted RCP/LCP waves, respectively. We do not need the generality of the full bi-isotropic solution. Enforcement of the impedance matching condition $\eta_+ = \eta_0$ results in

$$R_- = T_- = 0, \qquad 1 + R_+ = T_+,$$
$$T_+ = \frac{2\cos\theta}{\cos\theta + \cos\theta_+}. \tag{19}$$

Hence, even though an RCP impedance match eliminates the LCP fields, there is still a reflected RCP field. This is counterintuitive, and does not occur with ordinary isotropic materials, for which Snell's law dictates that $\theta = \theta_+$. The coefficient R_+ can be zero if $T_+ = 1$, which can occur if either $\theta = \theta_+$ or $\theta = -\theta_+$. The latter is consistent with an index of -1, and will be adopted. Hence, the condition of an RCP impedance match, coupled with an RCP index of -1, results in conditions favorable for "perfect focusing" of RCP waves. Under these conditions, the total transmitted fields can be written in terms of a potential $U(\bar{x})$ as

$$\bar{H} = \frac{jk_0}{4\pi} I_+ (\hat{z} \times \nabla - \hat{z}) U(\bar{x}),$$
$$U(\bar{x}) = \int_C e^{-j\bar{\nu}_+(\theta)\cdot\bar{x}} e^{j\bar{k}(\theta)\cdot\bar{x}_0} d\theta. \tag{20}$$

The square magnitude of $U(\bar{x})$ was calculated for $d = \lambda_0$, in the range $x \in [-\lambda_0, \lambda_0]$, $y \in [0, 3\lambda_0]$, and is presented in Fig. 3. The expected focal point for an effective index of -1 is observed at $\bar{x} = (0, \lambda_0)$. This constitutes confirmation of negative refraction and focusing of CP waves in bi-isotropic media. In view of an effective index of -1, ray tracing indicates that a slab of bi-isotropic–chiral media can be used as a planar lens in the manner of Veselago's LHM, but for CP waves. This is shown in the inset of Fig. 3, which introduces the internal $F1$ and external F focal points.

Recently verified experimentally [23], $|\chi/\sqrt{\mu\varepsilon}|$ can easily achieve a value of 1/2 (obtained after a proper change from the Drude-Born-Fedorov notation of [23]) for a random ensemble of helices (chiral case), in a case where the geometry was not optimized in any way. In view of Eq. (12), this is half of the χ needed to reverse the sign of the wave number and achieve negative refraction. Because of this, we feel a practical realization may be possible with a random array of helices. A single resonance model for a helix results in polarizabilities

$$\alpha_{EE} = \frac{Ch^2}{1 - (f/f_0)^2 - if/fQ},$$
$$\alpha_{HH} = \frac{C(2\pi f \mu S)^2}{1 - (f/f_0)^2 - if/fQ}, \tag{21}$$
$$\alpha_{EH} = \frac{-ihC2\pi f\mu S}{1 - (f/f_0)^2 - if/fQ}, \qquad \alpha_{HE} = -\alpha_{EH},$$

where S is the area of the helix loop, C the capacitance of the helix, h the length of the dipole part of the helix, f_0 the resonant frequency, and Q the helix quality factor. The small helices can be loaded if necessary, and embedded

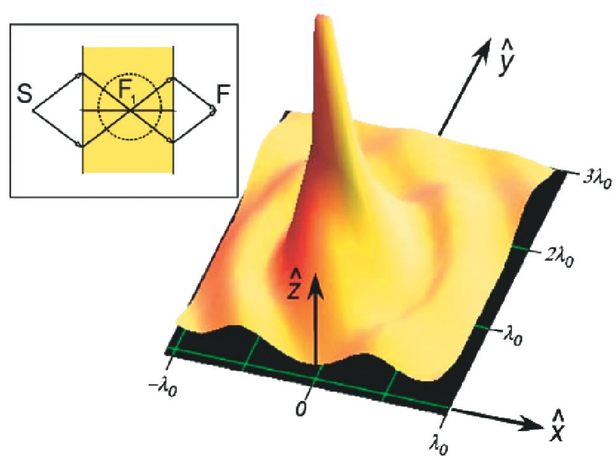

FIG. 3 (color). Plot of $|U(\bar{x})|^2$ in the bi-isotropic medium for $x \in [-\lambda_0, \lambda_0]$, $y \in [0, 3\lambda_0]$, due to an RCP line source located in the free space side at $\bar{x}_0 = (0, -\lambda_0)$. Peak transmission occurs at $\bar{x} = (0, \lambda_0)$, the expected focal point for an effective index of -1. The figure is confirmation of negative refraction, enhancement of evanescent fields, and subwavelength focusing of CP waves in bi-isotropic media. Inset: Ray optics description of the flat bi-isotropic–chiral lens for CP waves (S = source, $F/F1$ the external/internal focal regions).

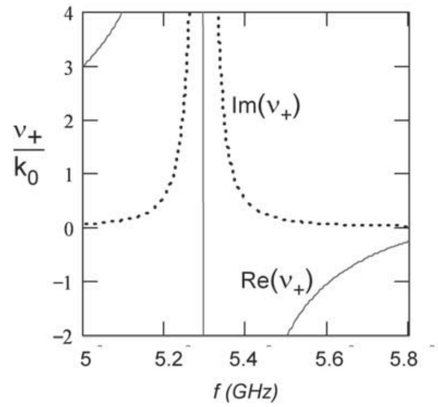

FIG. 4. Example of Maxwell-Garnett estimate for a random helix composite in low density foam.

randomly in a very low index foam (at a density N per cubic meter), and spheres can be formed of the resulting material, which can then be packed with volume fraction ϕ, and analyzed via the well known Maxwell-Garnett mixing formula, which has been recently found to be very successful for such a random array of helices [24]. We have used this approximation to verify that ν_+ can become -1. For instance, for $f_0 = 5.5$ GHz, $C = 8.4$ pF, $Q = 300$, a loop radius of 2 mm, $h = 2$ mm, $N = 2.4 \times 10^5$, and $\phi = 0.7$ (densely packed), we find that with minimal losses, $\nu_+ \approx -k_0$ at 5.6 GHz. This is depicted in Fig. 4. It is important that the helices be identical; otherwise, the strong resonance will weaken, reducing the optical rotatory power, and smearing out the negative refraction effect.

A related publication by Sir Pendry [25] appeared during the review of this Letter. While the goal is the same as here, it uses resonant electric dipoles in a background isotropic chiral medium. The dipole resonance splits the otherwise chiral transverse bands and results in negative refraction for one of the CP waves. Unlike the present proposal, the chiral medium is assumed frequency independent in the range of operation. The proposed chiral element in [25] is an interesting variant of the Swiss roll (and unlike here, it is stacked in a 3D log pile to achieve isotropy), and is essentially a helix made out of wide strips. In view of the present development, we feel this element alone, if properly designed, could be capable of producing sufficient optical rotatory power to achieve negative refraction.

Unlike LHM materials, which do not occur in nature, chiral materials occur naturally, and there is a plethora of them. We can then speculate on reasonable grounds that there may be available out there an inexpensive substance, with extremely low losses, either natural or man-made, which satisfies the above conditions (RCP/LCP impedance match, coupled with a corresponding RCP/LCP index of -1 in some frequency range (perhaps optical or IR), resulting in counterintuitive refractive and focusing properties. A film of such substance will be a natural focusing lens for CP waves. Such a desirable substance is yet to be identified or synthesized. Among the candidates we have the poly-L-lactic acid, a synthesized polymer with a reported huge $7.2°/\mu$m rotatory power in the visible range [26]. Simple estimates reveal that if the polymer's strong rotatory power persists into the THz band, negative refraction is expected.

In summary, we have established theoretically that a slab of certain chiral or bi-isotropic material can act in the same fashion as Veselago's hypothetical LHM focusing slab, with subwavelength resolution properties as Sir Pendry's ideal lens, but for CP waves. The result broadens the horizon of possibilities for achieving a "superlens," and is also an opportunity to exploit the wealth of information available on chiral media and optically active substances.

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