Neutron-Rich Nuclei and Neutron Stars: A New Accurately Calibrated Interaction for the Study of Neutron-Rich Matter

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An accurately calibrated relativistic parametrization is introduced to compute the ground state properties of finite nuclei, their linear response, and the structure of neutron stars. While similar in spirit to the successful NL3 parameter set, it produces an equation of state that is considerably softer—both for symmetric nuclear matter and for the symmetry energy. This softening appears to be required for an accurate description of several collective modes having different neutron-to-proton ratios. Among the predictions of this model are a symmetric nuclear-matter incompressibility of $K = 230$ MeV and a neutron skin thickness in ^{208}Pb of $R_n - R_p = 0.21$ fm. The impact of such a softening on various neutron-star properties is also examined.

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The quest for the equation of state (EOS) of neutron-rich matter—which is likely to lead to the discovery of exotic phases of matter—is an exciting problem that permeates over many areas of physics. While the search for novel phenomena has long been at the forefront of science, learning about "neutron-rich nuclei in heaven and earth" has experienced a recent revitalization due to remarkable advances in both terrestrial experiments and space observations. These developments, coupled to the promise of new *facilities for the future of science*, guarantee continuing discoveries for many years to come. Figuring prominently among the facilities for the future is the Rare Isotope Accelerator (RIA), a facility that by defining the limits of nuclear existence, will constrain the EOS at large neutron-proton asymmetries. In addition, new telescopes operating at a variety of wavelengths have turned neutron stars from theoretical curiosities into powerful diagnostic tools. For some recent excellent reviews on the relevance of the EOS on a variety of phenomena, such as the dynamics of heavy-ion collisions, the structure of neutron stars, and the simulation of core-collapse supernova, see Refs. [1–4] and references contained therein.

Our aim in this Letter is to construct an accurately calibrated parameter set that, while constrained only by the ground-state properties and the linear response of a variety of nuclei, may still be used to predict some neutron-star observables. Such a successful paradigm is the relativistic *NL*3 parameter set of Lalazissis, Konig, and Ring [5]. The *NL*3 parametrization has been used with enormous success in the description of a variety of ground-state properties of spherical, deformed, and exotic nuclei. For some special cases, it has also been used successfully to compute the linear response of the mean-field ground state. In the particular case of the giant monopole resonance (GMR) in ^{208}Pb—the so-called *breathing mode*—the predicted distribution of strength is in close agreement with the experimental data [6]. Thus, it has come as a surprise that to reproduce the GMR in ^{208}Pb, accurately fit nonrelativistic and relativistic models predict compressional moduli in symmetric nuclear matter (K) that differ by about 25%. Indeed, while nonrelativistic models predict $K \simeq 220$–235 MeV [7–9], relativistic models argue for a significantly larger value $K \simeq 250$–270 MeV [5,10,11].

In an earlier work, the density dependence of the symmetry energy, which at present is poorly known, has been proposed as the culprit for the above discrepancy [12]. (Note that the symmetry energy equals, to an excellent approximation, the difference between the energy of pure neutron matter and that of symmetric nuclear matter.) Since first proposed, other groups have tested this assertion reaching similar conclusions [11,13–15]. In particular, in Refs. [12,13] it has been argued that a good description of the breathing mode in ^{208}Pb may be obtained using a large value of K—provided one compensates with an appropriately stiff symmetry energy, namely, one that rises rapidly with baryon density. Thus, it was suggested that ^{90}Zr, a nucleus with both a well-developed breathing mode and a small neutron-proton asymmetry, should be used (rather than ^{208}Pb) to constrain K. With the advent of unprecedented experimental accuracy in the determination of the breathing mode in ^{90}Zr [6], it now appears that the *NL*3 interaction overestimates the value of K [14]. Moreover, the alluded correlation between K and the density dependence of the symmetry energy serves as a telltale of a further problem with most relativistic parametrizations: an underestimation of the frequency of oscillations of neutrons against protons—the so-called isovector giant dipole resonance (IVGDR)—in ^{208}Pb [11,13].

In this Letter we introduce a new, accurately calibrated relativistic parametrization that simultaneously describes the GMR in ^{90}Zr and ^{208}Pb, and the IVGDR in ^{208}Pb, without compromising the success in reproducing ground-state observables. To do so, however, two additional coupling constants must be introduced. Without these additional coupling constants one cannot describe the various modes without seriously compromising the quality of the fit

TABLE I. Model parameters used in the calculations. The parameter κ and the inverse scalar range m_s are given in MeV. The nucleon, omega, and rho masses are kept fixed at $M = 939$ MeV, $m_\omega = 782.5$ MeV, and $m_\rho = 763$ MeV, respectively.

Model	m_s	g_s^2	g_v^2	g_ρ^2	κ	λ	ζ	Λ_v
*NL*3	508.1940	104.3871	165.5854	79.6000	3.8599	−0.0159	0.0000	0.0000
FSUGold	491.5000	112.1996	204.5469	138.4701	1.4203	+0.0238	0.0600	0.0300

[11,16]. The effective field theoretical model is based on an interacting Lagrangian that provides an accurate description of finite nuclei and a Lorentz covariant extrapolation for the equation of state of dense matter. It has the following form [17–19]:

$$\mathcal{L}_{\text{int}} = \bar{\psi}\left[g_s\phi - \left(g_v V_\mu + \frac{g_\rho}{2}\tau\cdot\mathbf{b}_\mu + \frac{e}{2}(1+\tau_3)A_\mu\right)\gamma^\mu\right]\psi - \frac{\kappa}{3!}(g_s\phi)^3 - \frac{\lambda}{4!}(g_s\phi)^4 + \frac{\zeta}{4!}(g_v^2 V_\mu V^\mu)^2 + \Lambda_v(g_\rho^2\mathbf{b}_\mu\cdot\mathbf{b}^\mu)(g_v^2 V_\mu V^\mu). \tag{1}$$

The Lagrangian density includes Yukawa couplings of the nucleon field to various meson fields. It includes an isoscalar-scalar ϕ meson field and three vector fields: an isoscalar V^μ, an isovector $\mathbf{b}^\mu$, and the photon A^μ. In addition to the Yukawa couplings, the Lagrangian is supplemented by four nonlinear meson interactions. The inclusion of isoscalar meson self-interactions (via κ, λ, and ζ) are used to soften the equation of state of symmetric nuclear matter, while the mixed isoscalar-isovector coupling (Λ_v) modifies the density dependence of the symmetry energy. While power counting suggests that other local meson terms may be equally important [17], their phenomenological impact has been documented to be small [17–19], so they will not be considered any further in this study.

Although in Refs. [18,19] it has been proven that the addition of the isoscalar-isovector coupling (Λ_v) is important for the softening of the symmetry energy, no attempt was made to optimize the various parameter sets. Following standard practices [5,14], we use binding energies and charge radii for a variety of magic nuclei—computed in a relativistic mean-field approximation—to produce an accurately calibrated set through a chi-square minimization procedure. We dubbed this set "FSUGold" and list the various mass parameters and coupling constants in Table I. Further details about the calibration procedure will be presented in a forthcoming publication. In Table II a comparison between the very successful *NL*3 parametrization [5], FSUGold, and, (when available) experimental data is provided. While the agreement with experiment (at the 1% level or better) is satisfactory—and this agreement extends all over the periodic table [20]—a question immediately arises: given the success of *NL*3, why the need for another effective interaction having two additional parameters? The answer to this question is provided below.

As alluded earlier, and argued in Refs. [12,13], the success of the *NL*3 set in reproducing the breathing mode in ^{208}Pb is accidental; it results from a combination of both a stiff equation of state for symmetric nuclear matter and a stiff symmetry energy. If true, this implies that *NL*3 should overestimate the location of the breathing mode in ^{90}Zr—a nucleus with a well-developed GMR strength but rather insensitive to the symmetry energy. Similarly, the energy of the IVGDR in ^{208}Pb, an observable sensitive to the density dependence of the symmetry energy, should be underestimated by *NL*3 (note that a stiff symmetry energy predicts a small symmetry energy at the low densities of relevance to the IVGDR). In Table III relativistic random phase approximation (RPA) results for the GMR (centroids) in ^{208}Pb and ^{90}Zr, and the IVGDR (peak energy) in ^{208}Pb are reported. These small-amplitude modes represent the linear response of the mean-field ground state to a variety of probes [6,21]. Note that the FSUGold (*NL*3) parameter set predicts a compression modulus for symmetric nuclear matter of $K = 230(271)$ MeV and a neutron skin in ^{208}Pb of $R_n - R_p = 0.21(0.28)$ fm. The good agreement between FSUGold and experiment is due to the addition of the

TABLE II. Experimental data for the binding energy per nucleon and the charge radii for the magic nuclei used in the least square fitting procedure. In addition, predictions are displayed for the neutron skin of these nuclei.

Nucleus	Observable	Experiment	*NL*3	FSUGold
^{40}Ca	B/A (MeV)	8.55	8.54	8.54
	R_{ch} (fm)	3.45	3.46	3.42
	$R_n - R_p$ (fm)	···	−0.05	−0.05
^{48}Ca	B/A (MeV)	8.67	8.64	8.58
	R_{ch} (fm)	3.45	3.46	3.45
	$R_n - R_p$ (fm)	···	0.23	0.20
^{90}Zr	B/A (MeV)	8.71	8.69	8.68
	R_{ch} (fm)	4.26	4.26	4.25
	$R_n - R_p$ (fm)	···	0.11	0.09
^{116}Sn	B/A (MeV)	8.52	8.48	8.50
	R_{ch} (fm)	4.63	4.60	4.60
	$R_n - R_p$ (fm)	···	0.17	0.13
^{132}Sn	B/A (MeV)	8.36	8.37	8.34
	R_{ch} (fm)	···	4.70	4.71
	$R_n - R_p$ (fm)	···	0.35	0.27
^{208}Pb	B/A (MeV)	7.87	7.88	7.89
	R_{ch} (fm)	5.50	5.51	5.52
	$R_n - R_p$ (fm)	···	0.28	0.21

TABLE III. Centroid energies for the breathing mode in ^{208}Pb and ^{90}Zr, and the peak energy for the IVGDR in ^{208}Pb. Experimental data are extracted from Refs. [6,21].

Nucleus	Observable	Experiment	$NL3$	FSUGold
^{208}Pb	GMR (MeV)	14.17 ± 0.28	14.32	14.04
^{90}Zr	GMR (MeV)	17.89 ± 0.20	18.62	17.98
^{208}Pb	IVGDR (MeV)	13.30 ± 0.10	12.70	13.07

two extra parameters (ζ to reduce the value of K and $\Lambda_{\rm v}$ to soften the symmetry energy). However, it seems that an additional softening of the symmetry energy could further improve the agreement with experiment. With the present parametrization this could not be achieved without compromising the quality of the fit. Thus, our prediction of $R_n - R_p = 0.21$ fm could be regarded as an upper bound. This smaller value for the neutron skin in ^{208}Pb, generated from the softer symmetry energy, is significant as it brings covariant meson-baryon models closer to nonrelativistic predictions based on Skyrme parametrizations [22]. We note that the Parity Radius Experiment (PREX) at the Jefferson Laboratory is scheduled to measure the neutron radius of ^{208}Pb accurately (to within 0.05 fm) and model independently via parity-violating electron scattering [23,24]. This experiment should provide a unique observational constraint on the density dependence of the symmetry energy.

Having constructed a new accurately calibrated parameter set, we now examine predictions for a few neutron-star properties. The structure of spherical neutron stars in hydrostatic equilibrium is solely determined by the EOS of neutron-rich matter in beta equilibrium. For the uniform liquid phase we assume an EOS for matter in beta equilibrium that is composed of neutrons, protons, electrons, and muons. Further, we assume that this description remains valid in the high-density interior of the star. Thus, transitions to exotic phases are not considered here.

However, at the lower densities of the inner crust the uniform system becomes unstable against density fluctuations. In this nonuniform region the system may consist of a variety of complex structures, collectively known as *nuclear pasta* [25,26]. While microscopic calculations of the nuclear pasta are becoming available [27–29], it is premature to incorporate them in our calculation. Hence, following the procedure adopted in Ref. [30], a simple polytropic equation of state is used to interpolate from the outer crust [31] to the uniform liquid.

Results for the transition density from uniform to nonuniform neutron-rich matter are displayed in Table IV. These results are consistent with the inverse correlation between the neutron-skin and the transition density found in Ref. [18]. This correlation suggests that models with a stiff equation of state (like $NL3$) predict a low transition density, as it is energetically unfavorable to separate nuclear matter into regions of high and low densities. We now present results for a few neutron-star observables that

TABLE IV. Predictions for a few neutron-star observables. The various quantities are as follows: ρ_c is the transition density from nonuniform to uniform neutron-rich matter, R is the radius of a 1.4 solar-mass neutron star, $M_{\rm max}$ is the limiting mass, $\rho_{\rm Urca}$ is the threshold density for the direct Urca process, $M_{\rm Urca}$ is the minimum mass neutron star that may cool down by the direct Urca process, and $\Delta M_{\rm Urca}$ is the mass fraction of a 1.4 solar-mass neutron star that supports enhanced cooling by the direct Urca process.

Observable	$NL3$	FSUGold
ρ_c (fm^{-3})	0.052	0.076
R (km)	15.05	12.66
$M_{\rm max}(M_\odot)$	2.78	1.72
$\rho_{\rm Urca}$ (fm^{-3})	0.21	0.47
$M_{\rm Urca}(M_\odot)$	0.84	1.30
$\Delta M_{\rm Urca}$	0.38	0.06

depend critically on the equation of state [3,32], namely, masses, radii, and composition. Table IV includes predictions for the radius of a ‘‘canonical’’ 1.4 solar-mass neutron star alongside the maximum mass that the EOS can support against gravitational collapse; beyond this value the star collapses into a black hole. These results were obtained by numerically integrating the Tolman-Oppenheimer-Volkoff equations. The considerable smaller radius predicted by the FSUGold model originates in its softer symmetry energy. The stiffer symmetry energy of the $NL3$ set does not tolerate large central densities and produces stars with large radii. Note that the same physics that pushes neutrons out against surface tension in the nucleus of ^{208}Pb is also responsible for pushing neutrons out in a neutron star [18,19]. Further, while the sizable reduction in the limiting mass of FSUGold relative to $NL3$ is also due to the softening of the EOS, it is the softening induced by the (isoscalar) quartic vector-meson coupling ζ—rather than the softening of the symmetry energy controlled by $\Lambda_{\rm v}$—that is responsible for this effect [17].

We conclude with a comment on the enhanced cooling of neutrons stars. Recent observations by the Chandra and XMM-Newton observatories suggest that some neutron stars may cool rapidly, suggesting perhaps the need for some exotic component, such as condensates or color superconductors. Here we explore a more conservative alternative, namely, that of enhanced cooling of neutron stars by means of neutrino emission from nucleons in a mechanism known as the direct Urca process [32–35]. This mechanism is not exotic as it only relies on protons, neutrons, electrons, and muons—standard constituents of dense matter. However, it requires a large proton fraction Y_p for the momentum to be conserved in the above reactions. As a large proton fraction requires a stiff symmetry energy, it is interesting to determine if the newly proposed EOS is able to support such a large proton fraction. Note that in order for the direct Urca process to operate, the proton fraction must exceed $Y_p = 0.111$ for the low-density (muonless) case, and $Y_p = 0.148$ for the high-density case (with equal number of electrons and muons). In Table IV we list the threshold density (ρ_{Urca}) and minimum mass (M_{Urca}) required for the onset of the direct Urca process. We note that in spite of its softer symmetry energy, FSUGold predicts that a 1.4 solar-mass neutron star may cool down by the direct Urca process. For completeness, the mass fraction that supports enhanced cooling in such a neutron star is listed as ΔM_{Urca}.

In conclusion, a new accurately calibrated relativistic model (FSUGold) has been fitted to the binding energies and charge radii of a variety of magic nuclei. Symmetric nuclear matter saturates at a Fermi momentum of $k_{\mathrm{F}} = 1.30\ \mathrm{fm}^{-1}$ with a binding energy per nucleon of $B/A = -16.30$ MeV. Relative to the $NL3$ set, used here as a successful paradigm, FSUGold contains two additional parameters whose main virtue is the softening of both the EOS of symmetric matter and the symmetry energy. These two parameters are essential for reproducing a few nuclear collective modes. Specifically, the breathing mode in ^{90}Zr is sensitive to the softening of symmetric matter, the isovector giant dipole resonance in ^{208}Pb to the softening of the symmetry energy, and the breathing mode in ^{208}Pb to both. Incorporating these constraints yields a nuclear-matter incompressibility of $K = 230$ MeV and a neutron skin thickness in ^{208}Pb of $R_n - R_p = 0.21$ fm. While the description of the various collective modes imposes additional constraints on the EOS at densities around saturation density, the high-density component of the EOS remains largely unconstrained. We made no attempts to constrain the EOS at the supranuclear densities of relevance to neutron-star physics. Rather, we simply explored the consequences of the new parametrization on a variety of neutron star observables. In particular, we found a limiting mass of $M_{\max} = 1.72 M_{\odot}$, a radius of $R = 12.66$ km for a $M = 1.4 M_{\odot}$ neutron star, and no direct Urca cooling in neutrons stars with masses below $M = 1.3 M_{\odot}$. While the consequences of these results will be fully explored in a forthcoming publication, it is interesting to note that recent observations of pulsar-white dwarf binaries at the Arecibo observatory suggest a pulsar mass for PSRJ0751 + 1807 of $M = 2.1^{+0.4}_{-0.5} M_{\odot}$ at a 95% confidence level [36]. If this observation could be refined, not only would it rule out the high-density behavior of this (and many other) EOS, but it could provide us with a precious boost in our quest for the equation of state.

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