Inflationary Prediction for Primordial Non-Gaussianity

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We extend the δN formalism so that it gives all of the stochastic properties of the primordial curvature perturbation ζ if the initial field perturbations are Gaussian. The calculation requires only the knowledge of some family of unperturbed universes. A formula is given for the normalization f_{NL} of the bispectrum of ζ, which is the main signal of non-Gaussianity. Examples of the use of the formula are given, and its relation to cosmological perturbation theory is explained.

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Introduction.—The primordial curvature perturbation of the Universe, denoted here by ζ, is already present a few Hubble times before cosmological scales start to enter the horizon [1]. Its time-independent value at that stage seems to set the initial condition for the subsequent evolution of all cosmological perturbations. As a result, observation probes the stochastic properties of ζ, which is found to be almost Gaussian with an almost scale-invariant spectrum.

According to present ideas ζ is supposed to originate from the vacuum fluctuations during inflation of one or more light scalar fields, which on each scale are promoted to classical perturbations around the time of horizon exit. One takes inflation to be almost exponential (quasi–de Sitter spacetime) corresponding to a practically constant Hubble parameter H_*, and the effective masses of the fields to be much less than H_*. This ensures that the fields are almost massless and live in almost unperturbed quasi–de Sitter spacetime, making their perturbations indeed almost Gaussian and scale invariant. This automatically makes ζ almost scale invariant, and can [though not automatically [2,3]] make it also almost Gaussian.

All of this is of intense interest at the present time, because observation over the next few years will rule out most existing scenarios for the generation of ζ, by detecting or bounding the scale dependence and non-Gaussianity of ζ. In this Letter we describe a general procedure for calculating the level of non-Gaussianity, by means of the δN formalism [4,5].

Defining the curvature perturbation.—Perturbations of the observable Universe are defined with respect to an unperturbed reference universe, which is homogeneous and isotropic. Its line element may be written as $ds^2 = -dt^2 + a^2(t)\delta_{ij}dx^i dx^j$ defining the unperturbed scale factor $a(t)$, time t, and the Cartesian spatial coordinates $\mathbf{x}$.

The curvature perturbation is only of interest after the Universe has been smoothed on some scale $(\frac{k}{a})^{-1}$ much bigger than the horizon H^{-1}. To define it, one takes the fixed-t slices of spacetime to have uniform energy density, and the fixed-x world lines to be comoving. The spatial metric is [3,5–7]

$$g_{ij} = a^2(t)e^{2\zeta(t,\mathbf{x})}\gamma_{ij}(t,\mathbf{x}) = \tilde{a}^2(t,\mathbf{x})\gamma_{ij}(t,\mathbf{x}). \quad (1)$$

In this expression, $\gamma_{ij}(t,\mathbf{x})$ has unit determinant, so that a volume of the Universe bounded by fixed spatial coordinates is proportional to the locally defined scale factor $\tilde{a}(t,\mathbf{x})$. In the inflationary scenario the factor γ_{ij} just accounts for the tensor perturbation, but its form is irrelevant here. According to this definition, ζ is the perturbation in $\ln\tilde{a}$.

One can also consider a slicing whose metric has the form in Eq. (1) without the ζ factor, which we call the flat slicing. Starting from any initial flat slice at time t_{in}, let us define the amount of expansion $N(t,\mathbf{x}) \equiv \ln[\frac{\tilde{a}(t)}{a(t_{\mathrm{in}})}]$ to a final slice of uniform energy density. Then [4,5]

$$\zeta(t,\mathbf{x}) = \delta N \equiv N(t,\mathbf{x}) - N_0(t), \quad (2)$$

where $N_0(t) \equiv \ln[\frac{a(t)}{a(t_{\mathrm{in}})}]$ is the unperturbed amount of expansion.

To make use of the above formalism we assume that in the superhorizon regime ($aH \gg k$), the evolution of the Universe at each position (the local evolution), is well approximated by the evolution of some unperturbed universe [5,8,9]. This "separate universe" assumption will presumably be correct on cosmological scales because these scales are so big [9].

By virtue of the separate universe assumption, $N(t,\mathbf{x})$ is the amount of expansion in some unperturbed universe, allowing ζ to be evaluated knowing the evolution of a family of such universes. For a given content of the Universe it can be checked using the gradient expansion [5,6,10,11] method, but we do not wish to assume a specific content.

The separate universe assumption leads also to local energy conservation, so that ζ is conserved as long as the pressure is a unique function of the energy density. This consequence of the separate universe assumption was first recognized in full generality in Refs. [5,10] [see also Ref. [6] for the case of inflation, Ref. [8,9] for the case of linear perturbation theory, and Ref. [11] for a coordinate-free treatment].

The inflationary prediction.—The evolution of the observable Universe, smoothed on the shortest cosmological scale, is supposed to be determined by the values of one or more light scalar fields when that scale first emerges from the quantum regime a few Hubble times after horizon exit. Defined on a flat slicing, each field ϕ_i at this epoch will be of the form $\phi_i(\mathbf{x}) = \phi_i + \delta\phi_i(\mathbf{x})$.

Because quasiexponential inflation is assumed, and only light fields are considered, the perturbations $\delta\phi_i$ generated from the vacuum are almost Gaussian, with an almost flat spectrum [12]

$$\mathcal{P}_{\delta\phi_i} = \left(\frac{H_*}{2\pi}\right)^2. \tag{3}$$

Now we invoke the separate universe assumption, and choose the homogeneous quantities ϕ_i to correspond to the unperturbed Universe. Then Eq. (2) for ζ becomes

$$\zeta(t, \mathbf{x}) = N(\rho(t), \phi_1(\mathbf{x}), \phi_2(\mathbf{x}), \cdots) - N(\rho(t), \phi_1, \phi_2, \cdots). \tag{4}$$

In this expression, the expansion N is evaluated in an unperturbed universe, from an epoch when the fields have assigned values to one when the energy density has an assigned value ρ. This expression [4,5] allows one to propagate forward the stochastic properties of ζ to the epoch when it becomes observable, given those of the initial field perturbations.

Since the observed curvature perturbation is almost Gaussian, it must be given to good accuracy by one or more of the linear terms (we use the notation $N_{,i} \equiv \frac{\partial N}{\partial\phi_i}$ and $N_{,ij} \equiv \frac{\partial^2 N}{\partial\phi_i\partial\phi_j}$)

$$\zeta(t, \mathbf{x}) \simeq \sum_i N_{,i}(t)\delta\phi_i(\mathbf{x}), \tag{5}$$

with the field perturbations being almost Gaussian. In this Letter we include for the first time the quadratic terms

$$\zeta(t, \mathbf{x}) \simeq \sum_i N_{,i}(t)\delta\phi_i + \frac{1}{2}\sum_{ij} N_{,ij}(t)\delta\phi_i\delta\phi_j. \tag{6}$$

They may be entirely responsible for any observed non-Gaussianity if the field perturbations are Gaussian to sufficient accuracy.

The bispectrum.—The stochastic properties of the perturbations are specified through expectation values which, according to the inflationary paradigm, are taken with respect to the time-independent (Heisenberg picture) quantum state of the Universe (to be precise, the quantum state of the Universe before it somehow collapses to give the observed Universe). Focusing on ζ, we consider Fourier components, $\zeta_{\mathbf{k}} \equiv \int d^3k\zeta(t, \mathbf{x})\exp(i\mathbf{k}\cdot\mathbf{x})$.

The stochastic properties of a Gaussian perturbation are specified entirely by the spectrum $\mathcal{P}_\zeta$, defined through $\langle\zeta_{\mathbf{k}}\zeta_{\mathbf{k}'}\rangle \equiv (2\pi)^3 P_\zeta(k)\delta^3(\mathbf{k} + \mathbf{k}')$ and $\mathcal{P}_\zeta(k) \equiv \frac{k^3}{2\pi^2}P_\zeta(k)$. From Eqs. (3) and (5)

$$\mathcal{P}_\zeta = \left(\frac{H_*}{2\pi}\right)^2 \sum_i N_{,i}^2. \tag{7}$$

Non-Gaussianity is defined through higher correlations. We consider only the three-point correlation. [The four-point correlation may give a competitive observational signature and can be calculated in a similar fashion [13,14].] It defines the bispectrum B_ζ through $\langle\zeta_{\mathbf{k}}\zeta_{\mathbf{k}'}\zeta_{\mathbf{k}''}\rangle \equiv (2\pi)^3 B_\zeta(k, k', k'')\delta^3(\mathbf{k} + \mathbf{k}' + \mathbf{k}'')$. Its normalization is specified by a parameter $f_{\rm NL}$ according to [15,16]

$$B_\zeta \equiv -\frac{6}{5} f_{\rm NL}(k, k', k'')[P_\zeta(k)P_\zeta(k') + \text{cyclic}]. \tag{8}$$

[In first-order cosmological perturbation the gauge-invariant gravitational potential Φ during matter domination before horizon entry is $\Phi = -\frac{3}{5}\zeta$, and our definition of $f_{\rm NL}$ coincides with the definition [15] $B_\Phi \equiv 2f_{\rm NL}(k, k', k'')[P_\Phi(k)P_\Phi(k') + \text{cyclic}]$. At second-order these definitions of $f_{\rm NL}$ differ [17].]

We shall take $\mathcal{P}_\zeta$ and $f_{\rm NL}$ to be evaluated when cosmological scales approach the horizon and ζ becomes observable. Observation gives $\mathcal{P}_\zeta = (5\times 10^{-5})^2$, and $|f_{\rm NL}| \lesssim 100$ [18]. Absent a detection, this will eventually come down to roughly $|f_{\rm NL}| \lesssim 1$ [15].

Ignoring any non-Gaussianity of the $\delta\phi_i$, our formula in Eq. (6) makes $f_{\rm NL}$ practically independent of the wave numbers. Indeed, generalizing the result found in Ref. [13], we have calculated

$$-\frac{3}{5} f_{\rm NL} = \frac{\sum_{ij} N_{,i}N_{,j}N_{,ij}}{2[\sum_i N_{,i}^2]^2} + \ln(kL)\frac{\mathcal{P}_\zeta}{2}\frac{\sum_{ijk} N_{,ij}N_{,jk}N_{,ki}}{[\sum_i N_{,i}^2]^3}. \tag{9}$$

In deriving this expression we used the spectrum $(\frac{H_*}{2\pi})^2$ of the field perturbations, and used Eq. (7) to eliminate H_* in favor of $\mathcal{P}_\zeta$. As discussed in Ref. [13], the logarithm can be taken to be of order 1, because it involves the size k^{-1} of a typical scale under consideration, relative to the size L of the region within which the stochastic properties are specified. Except for the logarithm, $f_{\rm NL}$ *is scale independent if the field perturbations are Gaussian.*

If only one $\delta\phi_i$ is relevant, Eq. (6) becomes

$$\zeta(t, \mathbf{x}) = N_{,i}\delta\phi_i + \frac{1}{2}N_{,ii}(\delta\phi_i)^2, \tag{10}$$

and because the first term dominates, Eq. (9) becomes

$$-\frac{3}{5} f_{\rm NL} = \frac{1}{2}\frac{N_{,ii}}{N_{,i}^2}. \tag{11}$$

In this case, $f_{\rm NL}$ may equivalently be defined [15] by writing $\zeta = \zeta_{\rm g} - \frac{3}{5}f_{\rm NL}\zeta_{\rm g}^2$, where $\zeta_{\rm g}$ is Gaussian.

To include the possible non-Gaussianity of the $\delta\phi_i$, we define the bispectra B_{ijk} of the dimensionless field perturbations $(2\pi/H_*)\delta\phi_i$ and their normalization f_{ijk}, in exactly the same way that we defined B_ζ and $f_{\rm NL}$. These bispectra add the following contribution to $f_{\rm NL}$ in Eq. (9)

$$\Delta f_{\rm NL} = \frac{\Sigma_{ijk} N_{,i} N_{,j} N_{,k} f_{ijk}(k, k', k'')}{(\Sigma_i N_{,i}^2)^{3/2}} \mathcal{P}_\zeta^{-1/2}. \qquad (12)$$

The f_{ijk}, generated directly from the vacuum fluctuation, will depend strongly on the wave numbers.

Cosmological perturbation theory.—In the superhorizon regime the nonlinear theory [5] that we have used is a complete description. The basic expression in Eq. (4) is nonperturbative, giving $\zeta(t, \mathbf{x})$ in terms of the initial fields and the expansion of a family of unperturbed universes. The second-order expansion in Eq. (6) is a matter of convenience. As we shall see it seems to be adequate in practice, but Eq. (4) would still be applicable if the expansion converged slowly or not at all.

Cosmological perturbation theory (CPT) is completely different. It is applicable both inside and outside the horizon, being at each instant a power series in the perturbations of the metric and the stress-energy tensor, together with whatever variables are needed to completely specify the latter and close the system of equations. During inflation these variables are the components of the inflaton, while afterwards they may involve oscillating fields and a description of the particle content. First-order CPT is usually adequate and can describe non-Gaussianity at the level $f_{\rm NL} \gg 1$, which has to be generated by the second-order term in Eq. (6). Second-order CPT is generally needed only to handle non-Gaussianity at the level $|f_{\rm NL}| \sim 1$.

Quantized CPT is needed to calculate the stochastic properties of the initial field perturbations $\delta\phi_i$, which are the input for our calculation. The slow-roll spectrum in Eq. (3) comes from the first-order calculation. The bispectrum is a second-order effect and has, in the context of slow-roll inflation, been calculated in Refs. [16,19]. It is shown elsewhere [20] that $|\Delta f_{NL}| \ll 1$ in this case. Higher correlators have not been calculated yet and would give an additional contribution to Eq. (9) which presumably is also negligible. Exotic non-slow-roll models [21] can make $|\Delta f_{NL}| \gg 1$, but from now on we set $\Delta f_{NL} = 0$.

In the regime $aH \gg k$, perturbation theory must be compatible with Eq. (6). In particular, the nonlocal terms, present at second order for a generic perturbation, must be absent for ζ [see also Ref. [22]]. Finally, CPT is needed to evolve the perturbations after horizon entry, but that is not our concern here. In the following, we apply our formalism to calculate $f_{\rm NL}$ in various cases and compare it with the CPT result where that is known.

A two-component inflation model.—As a first use of Eq. (9) we consider the two-component inflation model of Kadota and Stewart [23], estimating for the first time the non-Gaussianity which it predicts. The model works with a complex field Φ, which is supposed to be a modulus with a point of enhanced symmetry at the origin. Writing $\Phi \equiv |\Phi| e^{i\theta}$, the tree-level potential has a maximum at $\Phi = 0$ and depends on both $|\Phi|$ and θ. A one-loop correction turns the maximum into a crater and inflation occurs while Φ is rolling away from the rim of the crater. The curvature perturbation is supposed to be constant after the end of slow-roll inflation. For $\theta \ll \theta_c$, with θ_c being a parameter of the model, it is found that $N \propto |\frac{\theta_c}{\theta}|$. Through the first term of Eq. (9) $f_{\rm NL} \simeq |\frac{\theta}{\theta_c}|$ which is too small ever to be observed.

The curvaton model.—In the curvaton model [24] [see also Ref. [25]] the curvature perturbation ζ grows, from a negligible value in an initially radiation dominated epoch, due to the oscillations of a light field σ (the curvaton) around the minimum of its quadratic potential $V_\sigma(t, \mathbf{x}) = \frac{1}{2} m_\sigma^2 \sigma^2(t, \mathbf{x})$, where m_σ is the curvaton effective mass. Because of the oscillations, the initially negligible curvaton energy density redshifts as $\rho_\sigma(t, \mathbf{x}) \approx \frac{1}{2} m_\sigma^2 \sigma_a^2(t, \mathbf{x}) \propto a^{-3}(t, \mathbf{x})$, where σ_a represents the amplitude of the oscillations. Meanwhile the radiation energy density ρ_r redshifts as a^{-4}. Soon after the curvaton decay, the standard Hot Big Bang is recovered and ζ is assumed to be conserved until horizon reentry.

To calculate $f_{\rm NL}$ using Eq. (9) we first realize that σ_* (the value of σ a few Hubble times after horizon exit) is the only relevant quantity since the curvature perturbation produced by the inflaton, and imprinted in the radiation fluid during the reheating process, is supposed to be negligible. Thus, Eq. (11) applies. Second, we can redefine N as the number of e folds from the beginning of the sinusoidal oscillations to the curvaton decay. This is because the number of e folds from the end of inflation to the beginning of the oscillations is completely unperturbed as the radiation energy density dominates during that time. Thus, N is now a function of three variables

$$N(\rho_{\rm dec}, \rho_{\rm osc}, \sigma_*) = \frac{1}{3} \ln\left(\frac{\rho_{\sigma_{\rm osc}}}{\rho_{\sigma_{\rm dec}}}\right) = \frac{1}{3} \ln\left[\frac{\frac{1}{2} m_\sigma^2 [g(\sigma_*)]^2}{\rho_{\sigma_{\rm dec}}}\right], \qquad (13)$$

where $g \equiv \sigma_{\rm osc}$ is the amplitude at the beginning of the sinusoidal oscillations, as a function of σ_*. Here the curvaton energy density just before the curvaton decay $\rho_{\sigma_{\rm dec}}$ is expressed in terms of the total energy density $\rho_{\rm dec}$ at that time, the total energy density at the beginning of the sinusoidal oscillations $\rho_{\rm osc}$, and g by $\rho_{\sigma_{\rm dec}} = \frac{1}{2} m_\sigma^2 [g(\sigma_*)]^2 (\frac{\rho_{\rm dec} - \rho_{\sigma_{\rm dec}}}{\rho_{\rm osc}})^{3/4}$. After evaluating $\frac{\partial}{\partial \sigma_*} = g' \frac{\partial}{\partial g}$, at fixed $\rho_{\rm dec}$ and $\rho_{\rm osc}$, we obtain

$$N_{,\sigma_*} = \frac{2}{3} r \frac{g'}{g}, \qquad (14)$$

where $r \equiv \frac{3\rho_{\sigma_{\rm dec}}}{3\rho_{\sigma_{\rm dec}} + 4\rho_{r_{\rm dec}}}$ being $\rho_{r_{\rm dec}}$ the radiation energy density just before the curvaton decay, so that

$$\mathcal{P}_\zeta = \frac{H_*}{2\pi} N_{,\sigma_*} = \frac{H_* r}{3\pi} \frac{g'}{g}, \qquad (15)$$

in agreement with first-order cosmological perturbation theory [2]. Differentiating again we find from Eq. (11)

$$f_{\rm NL} = -\frac{5}{6}\frac{N_{,\sigma_*\sigma_*}}{N_{,\sigma_*}^2} = \frac{5}{3} + \frac{5}{6}r - \frac{5}{4r}\left(1 + \frac{gg''}{g'^2}\right), \quad (16)$$

which nicely agrees with the already calculated $f_{\rm NL}$ using first- and second-order perturbation theory [see Refs. [2,3,26]].

Another two-component model.—Finally we consider the two-component inflation model of Refs. [3,27]. For at least some number N of e folds after cosmological scales leave the horizon, the potential is $V = V_0(1 + \frac{1}{2}\eta_\phi \frac{\phi^2}{m_P^2} + \frac{1}{2}\eta_\sigma \frac{\sigma^2}{m_P^2})$, with the first term dominating, η_ϕ and η_σ being the slow-roll η parameters, and m_P being the reduced Planck mass. The idea is to use Eq. (9) to calculate the non-Gaussianity after the N e folds which, barring cancellations, will place a lower limit on the observed non-Gaussianity.

The slow-roll equations give the field values $\phi(N)$ and $\sigma(N)$, in terms of those obtaining just after horizon exit; $\phi(N) = \phi\exp(-N\eta_\phi)$ and $\sigma(N) = \sigma\exp(-N\eta_\sigma)$. This gives $V(N, \phi, \sigma)$ and allows us to calculate the derivatives of N with respect to ϕ and σ at fixed V. Focusing on the case $\sigma = 0$ considered in Ref. [27], we find

$$\zeta = \frac{\delta\phi}{\eta_\phi\phi} - \frac{\eta_\phi}{2}\left(\frac{\delta\phi}{\eta_\phi\phi}\right)^2 + \frac{\eta_\sigma}{2}e^{2N(\eta_\phi - \eta_\sigma)}\left(\frac{\delta\sigma}{\eta_\phi\phi}\right)^2, \quad (17)$$

in agreement with the second-order pertubation calculation of Ref. [28]. If the observed ζ has a non-Gaussian part ζ_σ equal to the last term of Eq. (17) and a Gaussian part generated mostly *after* inflation, one can obtain $|f_{\rm NL}| > 1$ by choosing $\eta_\phi \geq 0.26$, $\eta_\sigma = \frac{\eta_\phi}{2}$, $N = 70$, and $\zeta_\sigma = 10^{-2}\zeta$.

This model was studied originally [3,27] using a second-order perturbation expression for the *time derivative* of $\varepsilon H m_P^2 \zeta_\sigma$, with ε being the ε slow-roll parameter. This expression disagrees with ours [29] through the appearance of non-local terms, though the order of magnitude is similar [30].

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[1] Standard material concerning observation and the early Universe are given, for instance, in D. H. Lyth and A. Riotto, Phys. Rep. **314**, 1 (1999), and A. R. Liddle and D. H. Lyth, *Cosmological Inflation and Large-Scale Structure* (Cambridge University Press, Cambridge, England, 2000).

[2] D. H. Lyth, C. Ungarelli, and D. Wands, Phys. Rev. D **67**, 023503 (2003).

[3] D. H. Lyth and Y. Rodríguez, Phys. Rev. D **71**, 123508 (2005).

[4] M. Sasaki and E. D. Stewart, Prog. Theor. Phys. **95**, 71 (1996); A. A. Starobinsky, Phys. Lett. **117B**, 175 (1982); A. A. Starobinsky, Pis'ma Zh. Eksp. Teor. Fiz. **42**, 124 (1985) [JETP Lett. **42**, 152 (1985)].

[5] D. H. Lyth, K. A. Malik, and M. Sasaki, J. Cosmol. Astropart. Phys. 05 (2005) 004.

[6] D. S. Salopek and J. R. Bond, Phys. Rev. D **42**, 3936 (1990).

[7] P. Creminelli and M. Zaldarriaga, J. Cosmol. Astropart. Phys. 10 (2004) 006.

[8] D. Wands, K. A. Malik, D. H. Lyth, and A. R. Liddle, Phys. Rev. D **62**, 043527 (2000).

[9] D. H. Lyth and D. Wands, Phys. Rev. D **68**, 103515 (2003).

[10] G. I. Rigopoulos and E. P. S. Shellard, Phys. Rev. D **68**, 123518 (2003).

[11] D. Langlois and F. Vernizzi, Phys. Rev. Lett. **95**, 091303 (2005).

[12] T. S. Bunch and P. C. W. Davies, Proc. R. Soc. Lond. A **360**, 117 (1978).

[13] L. Boubekeur and D. H. Lyth, astro-ph/0504046.

[14] T. Okamoto and W. Hu, Phys. Rev. D **66**, 063008 (2002); W. Hu, Phys. Rev. D **64**, 083005 (2001).

[15] E. Komatsu and D. N. Spergel, Phys. Rev. D **63**, 063002 (2001).

[16] J. Maldacena, J. High Energy Phys. 05 (2003) 013.

[17] N. Bartolo, S. Matarrese, and A. Riotto, J. Cosmol. Astropart. Phys. 08 (2005) 010.

[18] E. Komatsu *et al.*, Astrophys. J. Suppl. Ser. **148**, 119 (2003).

[19] D. Seery and J. E. Lidsey, astro-ph/0506056.

[20] D. H. Lyth and I. Zaballa, astro-ph/0507608.

[21] N. Arkani-Hamed, P. Creminelli, S. Mukohyama, and M. Zaldarriaga, J. Cosmol. Astropart. Phys. 04 (2004) 001; E. Silverstein and D. Tong, Phys. Rev. D **70**, 103505 (2004); D. Seery and J. E. Lidsey, J. Cosmol. Astropart. Phys. 06 (2005) 003.

[22] G. I. Rigopoulos, E. P. S. Shellard, and B. J. W. van Tent, astro-ph/0506704; G. I. Rigopoulos, E. P. S. Shellard, and B. J. W. van Tent (to be published).

[23] K. Kadota and E. D. Stewart, J. High Energy Phys. 12, (2003) 008; K. Kadota and E. D. Stewart, J. High Energy Phys. 07 (2003) 013.

[24] D. H. Lyth and D. Wands, Phys. Lett. B **524**, 5 (2002); T. Moroi and T. Takahashi, Phys. Lett. B **522**, 215 (2001); **539**, 303(E) (2002).

[25] S. Mollerach, Phys. Rev. D **42**, 313 (1990); A. Linde and V. Mukhanov, Phys. Rev. D **56**, R535 (1997); K. Enqvist and M. S. Sloth, Nucl. Phys. **B626**, 395 (2002).

[26] D. H. Lyth, Phys. Lett. B **579**, 239 (2004); N. Bartolo, S. Matarrese, and A. Riotto, Phys. Rev. D **69**, 043503 (2004).

[27] K. Enqvist and A. Väihkönen, J. Cosmol. Astropart. Phys. 09 (2004) 006.

[28] K. A. Malik, astro-ph/0506532.

[29] D. H. Lyth and Y. Rodríguez (to be published).

[30] A. Väihkönen, astro-ph/0506304.