Alice Falls into a Black Hole: Entanglement in Noninertial Frames

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Two observers determine the entanglement between two free bosonic modes by each detecting one of the modes and observing the correlations between their measurements. We show that a state which is maximally entangled in an inertial frame becomes less entangled if the observers are relatively accelerated. This phenomenon, which is a consequence of the Unruh effect, shows that entanglement is an observer-dependent quantity in noninertial frames. In the high acceleration limit, our results can be applied to a nonaccelerated observer falling into a black hole while the accelerated one barely escapes. If the observer escapes with infinite acceleration, the state's distillable entanglement vanishes.

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Entanglement is a property of multipartite quantum states that arises from the tensor product structure of the Hilbert space and the superposition principle. It is considered to be a resource for quantum information tasks such as teleportation [1] and has applications in quantum control [2] and quantum simulations [3]. Nonrelativistic bipartite entanglement can be quantified uniquely for pure states by the von Neumann entropy, and for mixed states several measures have been proposed such as entanglement cost, distillable entanglement, and logarithmic negativity [4]. Understanding entanglement in the relativistic framework is crucial from both fundamental and practical perspectives. Relativistic space-time presents naturally a more complete setting for theoretical considerations and many experimental setups require such a treatment. This program is therefore an important and topical one. It is only in this framework that we can understand quantum information tasks involving entanglement between moving observers. A central question in the field of relativistic quantum information is whether entanglement is observer independent. So far, it has been shown that entanglement between inertial moving parties remains constant although the entanglement between some degrees of freedom can be transferred to others [5].

In this Letter we investigate the entanglement between two modes of a noninteracting massless scalar field when one of the observers describing the state is uniformly accelerated. We consider a maximally entangled pure state in an inertial frame and describe its entanglement from a noninertial perspective. Our results imply that only inertial observers in flat space-time agree on the degree of entanglement, whereas noninertial observers see a degradation. While Minkowski coordinates (t, z) are the most suitable to describe the field from an inertial perspective, Rindler coordinates (τ, ξ) are appropriate for discussing the viewpoint of an observer moving with uniform acceleration. Two different sets of Rindler coordinates, which differ from each other by a sign change in the temporal coordinate, are necessary for covering Minkowski space. These sets of coordinates define two Rindler regions that are causally disconnected from each other. A particle undergoing uniform acceleration in a given Rindler region remains constrained to it and has no access to the other Rindler sector. The solutions of the Klein-Gordon equation for a massless scalar field in Minkowski coordinates are related to the solutions of the equation in Rindler coordinates through Bogoliubov transformations. Using these transformations one finds that the ground state of a given mode seen by an inertial observer in Minkowski coordinates corresponds to a two-mode squeezed state in Rindler coordinates [6]. These two modes, respectively, correspond to the field observed in the two distinct Rindler regions. An observer moving with uniform acceleration in one of the regions has no access to field modes in the causally disconnected region. Therefore, the observer must trace over the inaccessible region losing information about the state, which essentially results in the detection of a thermal state. This is known as the Unruh effect [7].

A consequence of this effect is that an entangled pure state seen by inertial observers appears mixed from an accelerated frame. In this case entropy no longer quantifies entanglement. However, it is possible to determine the entanglement of such a state using the logarithmic negativity which is a full entanglement monotone that bounds distillable entanglement from above [8]. In our analysis we use the mutual information [9] to quantify the state's total correlations (classical plus quantum). It is interesting to note that the Schwarzschild space-time very close to the horizon resembles Rindler space in the infinite acceleration limit [10]. Therefore our technique can be applied to study the entanglement between two scalar modes seen by observers near an event horizon. We will see that when two modes of the field are maximally entangled in an inertial frame, the presence of the horizon degrades the entanglement seen by one observer falling and the other escaping the fall into a black hole. The state remains only classically correlated when the acceleration approaches infinity. We prove this by showing that, in the infinite acceleration limit in Rindler space, the logarithmic negativity is zero.

To formalize the above, consider that two modes, k and s, of a free massless scalar field in Minkowski space-time, are maximally entangled from an inertial perspective; i.e., the quantum field is in a state

$$\frac{1}{\sqrt{2}}(|0_s\rangle^{\mathcal{M}}|0_k\rangle^{\mathcal{M}} + |1_s\rangle^{\mathcal{M}}|1_k\rangle^{\mathcal{M}}). \tag{1}$$

The states $|0_j\rangle^{\mathcal{M}}$ and $|1_j\rangle^{\mathcal{M}}$ are the vacuum and single particle excitation states of the mode j in Minkowski space. We assume that Alice has a detector which only detects mode s and Rob has a detector sensitive only to mode k. If Rob undergoes uniform acceleration a, the states corresponding to mode k must be specified in Rindler coordinates in order to describe what Rob sees. Considering only one spatial dimension z, the world lines of uniformly accelerated observers in Minkowski coordinates correspond to hyperbolae, to the left (region I) and right (region II) of the origin, bounded by lightlike asymptotes constituting the Rindler horizon. The Rindler coordinates are defined by

$$\begin{aligned} t &= a^{-1}e^{a\xi}\sinh a\tau, \quad z = a^{-1}e^{a\xi}\cosh a\tau, \quad |z| < t, \\ t &= -a^{-1}e^{a\xi}\sinh a\tau, \quad z = a^{-1}e^{a\xi}\cosh a\tau, \quad |z| > t, \end{aligned} \tag{2}$$

where the hyperbolae correspond to the spacelike coordinates ξ and τ is the proper time, i.e., the length of the hyperbolic world line measured by the Minkowski metric. The Minkowski vacuum state, defined as the absence of any particle excitation in any of the modes

$$|0\rangle^{\mathcal{M}} = \prod_j |0_j\rangle^{\mathcal{M}}, \tag{3}$$

can be expressed in terms of a product of two-mode squeezed states of the Rindler vacuum [6],

$$|0_k\rangle^{\mathcal{M}} \sim \frac{1}{\cosh r}\sum_{n=0}^{\infty}\tanh^n r |n_k\rangle_I |n_k\rangle_{II}, \tag{4}$$

$$\cosh r = (1 - e^{-2\pi\Omega})^{-1/2}, \qquad \Omega = |k|c/a, \tag{5}$$

where $|n_k\rangle_I$ and $|n_k\rangle_{II}$ refer to the mode decomposition in region I and II, respectively, of Rindler space. Each Minkowski mode j has a Rindler mode expansion given by Eq. (4). In our problem, we consider detectors sensitive to a single Minkowski mode s for Alice and k for Rob and we consider that the rest of the modes in the field are in the vacuum. In our analysis we trace over all the modes except for s and k. The result of this trace is a pure state because different modes j and j' do not mix.

Using Eq. (4) and

$$|1_k\rangle^{\mathcal{M}} = \frac{1}{\cosh^2 r}\sum_{n=0}^{\infty}\tanh^n r\sqrt{n+1}|(n+1)_k\rangle_I|n_k\rangle_{II},$$

we can rewrite Eq. (1) in terms of Minkowski modes for Alice and Rindler modes for Rob. Since Rob is causally disconnected from region II, we must trace over the states in this region, which results in a mixed state

$$\rho_{AR} = \frac{1}{2\cosh^2 r}\sum_n (\tanh r)^{2n}\rho_n,$$

$$\begin{aligned} \rho_n = {} & |0n\rangle\langle 0n| + \frac{\sqrt{n+1}}{\cosh r}|0n\rangle\langle 1n+1| + \frac{\sqrt{n+1}}{\cosh r}|1n+1\rangle\langle 0n| \\ & + \frac{(n+1)}{\cosh^2 r}|1n+1\rangle\langle 1n+1|, \end{aligned} \tag{6}$$

where $|nm\rangle = |n_s\rangle^{\mathcal{M}}|m_k\rangle_I$. The partial transpose criterion [11] provides a sufficient criterion for entanglement. If at least one eigenvalue of the partial transpose is negative, then the density matrix is entangled; but a state with positive partial transpose can still be entangled. This type of entanglement is called bound or nondistillable entanglement [8]. We obtain the partial transpose by interchanging Alice's qubits and we find the eigenvalues in the $(n, n+1)$ block to be

$$\lambda_{\pm}^n = \frac{\tanh^{2n} r}{(4\cosh^2 r)}\left[\left(\frac{n}{\sinh^2 r} + \tanh^2 r\right) \pm \sqrt{Z_n}\right],$$

where

$$Z_n = \left(\frac{n}{\sinh^2 r} + \tanh^2 r\right)^2 + \frac{4}{\cosh^2 r}.$$

It is clear that for finite acceleration ($r < \infty$) one eigenvalue is always negative; thus the state is always entangled. Only in the limit $r \to \infty$ could the negative eigenvalue possibly go to zero. To investigate this further, we sum over all the negative eigenvalues and calculate the logarithmic negativity. This entanglement monotone is defined as $N(\rho) = \log_2 ||\rho^T||_1$ where $||\rho^T||_1$ is the trace norm of the density matrix ρ. The result is $N(\rho_{AR}) = \log_2(\frac{1}{2\cosh^2 r} + \Sigma)$ where

$$\Sigma = \sum_{n=0}^{\infty}\frac{\tanh^{2n} r}{2\cosh^2 r}\sqrt{\left(\frac{n}{\sinh^2 r} + \tanh^2 r\right)^2 + \frac{4}{\cosh^2 r}}.$$

For vanishing acceleration ($r = 0$), $N(\rho_{AR}) = 1$ as expected. For finite acceleration the entanglement is degraded (Fig. 1). The limit $r \to \infty$ can be explored by analyzing an upper and lower bound on the negativity constructed by bounding the sum in the above equation by two sums that can be carried out exactly. We find

$$1 \le \Sigma < \frac{2\cosh^2 r + 2\cosh r}{2\cosh^2 r}.$$

Since the bounds converge to 1, the negativity is exactly 0 in the limit. This means that the state has no longer distillable entanglement. We can also estimate the total amount of correlation in the state by calculating the mutual information, defined as $I(\rho_{AR}) = S(\rho_A) + S(\rho_R) -$

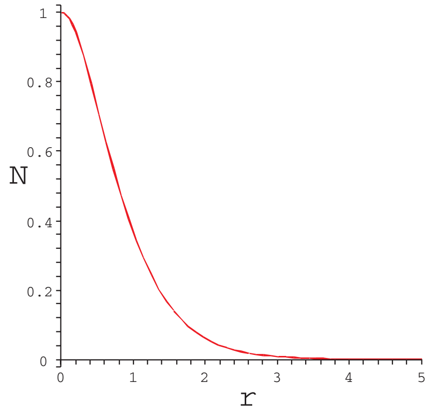

FIG. 1 (color online). The negativity as a function of the acceleration r.

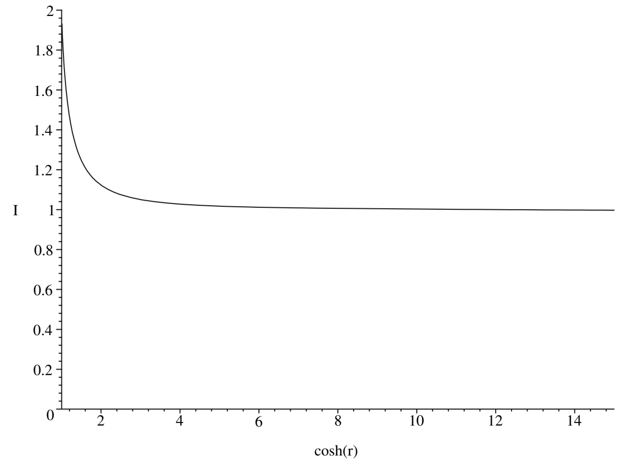

FIG. 2. Mutual information as a function of $\cosh(r)$.

$S(\rho_{AR})$, where $S(\rho) = -\mathrm{Tr}\,[\rho\log_2(\rho)]$ is the entropy of the density matrix ρ. The entropy of the joint state is

$$S(\rho_{AR}) = -\frac{1}{2\cosh^2 r}\sum_{n=0}^{\infty}\tanh^{2n} r \mathrm{L}_n,$$
$$\mathrm{L}_n = \left(1+\frac{n+1}{\cosh^2 r}\right)\log_2\left[\frac{\tanh^{2n} r}{2\cosh^2 r}\left(1+\frac{n+1}{\cosh^2 r}\right)\right]. \tag{7}$$

We obtain Rob's density matrix in region I by tracing over Alice's states; its entropy is

$$S(\rho_{RI}) = -\frac{1}{2\cosh^2 r}\sum_{n=0}^{\infty}\tanh^{2n} r \mathrm{M}_n$$
$$\mathrm{M}_n = \left(1+\frac{n}{\sinh^2 r}\right)\log_2\frac{\tanh^{2n} r}{2\cosh^2 r}\left(1+\frac{n}{\sinh^2 r}\right). \tag{8}$$

Tracing over Rob's states we find Alice's density matrix:

$$\rho_A^{\mathcal{M}} = \frac{1}{2}(|0\rangle\langle 0| + |1\rangle\langle 1|), \tag{9}$$

whose entropy is $S(\rho_A) = 1$. The mutual information is

$$I(N) = 1 - \frac{1}{2}\log_2(\tanh^2 r) - \frac{1}{2\cosh^2 r}\sum_{n=0}^{N}\tanh^{2n} r \mathcal{D}_n,$$
$$\mathcal{D}_n = \left(1+\frac{n}{\sinh^2 r}\right)\log_2\left(1+\frac{n}{\sinh^2 r}\right) - \left(1+\frac{n+1}{\cosh^2 r}\right)\log_2\left(1+\frac{n+1}{\cosh^2 r}\right),$$

which we plot in Fig. 2. For vanishing acceleration, the mutual information is 2. As the acceleration increases, it becomes smaller, converging to unity in the limit of infinite acceleration. Note that a maximally mixed state of maximally entangled states has mutual information equal to one. Since the distillable entanglement in the infinite acceleration limit is zero, we know that in this limit the total correlations consist of classical correlations plus bound entanglement. The entropy of the density matrices for Rob and Alice in region I and Rob in region II are equal $S(\rho_{ARI}) = S(\rho_{RII})$. This is because the state in Eq. (1) is pure, and therefore the entropies of the reduced density matrices of any bipartite division of the system are equal. In the limit of infinite acceleration $S(\rho_{ARI}) = 1$. The modes in region II are maximally entangled with the state in region I. When the bosons are maximally entangled, for vanishing acceleration, there is no distillable entanglement with region II. For finite acceleration, the entanglement between the bosons is degraded as the entanglement with region II grows. In general, entanglement in tripartite pure states cannot be arbitrarily distributed amongst the subsystems [12]. This phenomenon, called entanglement sharing, explains here why the entanglement between the bosons is degraded as acceleration grows.

Our results for the infinite acceleration limit describe the entanglement of the two bosonic modes seen by Alice and Rob in the case that they are extremely close to the horizon of a static black hole. The Schwarzschild space-time describes the geometry of space-time for a spherical nonrotating mass m. Considering only the radial component, the metric is

$$ds^2 = -\left(1-\frac{2m}{R}\right)dT^2 + \left(\frac{1}{1-2m/R}\right)dR^2. \tag{10}$$

The presence of a Schwarzschild black hole corresponds to a region causally cut off from the rest of space-time by an horizon at $R = 2m$. Changing coordinates so that $R - 2m = x^2/8m$, we have $1 - 2m/R = (Ax^2)/[1 + (Ax)^2] \approx (Ax)^2$ near $x = 0$ with $A = 1/4m$. This means that $dR^2 = (Ax)^2$ and thus, very close to the horizon of the black hole at $R \approx 2m$, the Schwarzschild space-time can be approximated by Rindler space

$$ds^2 = -(Ax)^2 dT^2 + dx^2, \tag{11}$$

where the acceleration parameter $a = A^{-1}$. The infinite acceleration limit corresponds to Rob moving on a trajectory arbitrarily close to the Rindler horizon; in the context of a black hole, this is arbitrarily close to the event horizon. Therefore, our analysis can be applied to the case of Alice falling into the black hole while Rob escapes. Each of them measures one of the modes and Rob sends the results of his experiment to Alice. Alice can then compare the results and estimate the entanglement between the modes.

If we considered Alice to be accelerated as well, the density matrix would be mixed to a higher degree, resulting in a higher degradation of entanglement. Only two inertial observers in that space would agree that the state investigated is maximally entangled. This shows that entanglement is an observer-dependent quantity in noninertial frames. The presence of a horizon for the uniformly accelerated observers results in a loss of information producing the degradation in the entanglement. In flat space-time one could prescribe a well-defined notion of entanglement by stating that only inertial observers are good observers of entanglement. This is not a problem in this case since inertial observers have a preferred role in flat space-time. In curved space-time, even two nearby inertial observers are relatively accelerated, due to the geodesic deviation equation. The results of this Letter strongly suggest that in curved space-time not even two inertial observers agree on the degree of entanglement of a given bipartite quantum state of some quantum field. The detailed analysis of entanglement between modes of a quantum field on a curved space-time, however, is more involved, and will be treated elsewhere [13].

With the intention of investigating entanglement between accelerated observers, the state fidelity in a teleportation protocol was studied [14] using relatively accelerated cavities. It was found that the fidelity decreases as the acceleration grows. Since state fidelity in conventional teleportation protocols is related to entanglement, the authors interpret this result as an indication of entanglement degradation. Unfortunately, the mode expansions used in that work correspond to those of free space. Although there is some indication that these results are qualitatively correct, a detailed calculation of the effects of an accelerated cavity still remains to be done.

We have calculated the entanglement between two free modes of a scalar field as seen by an inertial observer detecting one of the modes and a uniformly accelerated observer detecting the second mode. The entanglement which appeared to be maximal in an inertial frame is then degraded by the Unruh effect. In the limit of infinite acceleration, which can be applied to the situation of one of the observers falling into a black hole while the other barely escapes, the distillable entanglement vanishes but the state remains correlated through classical correlations and bound entanglement. The entanglement degradation between the bosons is due to the increase of entanglement with the modes in the causally disconnected Rindler region. The accelerated observer has only partial access to the information and therefore entanglement appears degraded. Similar effects have been noted to have relevance for black hole entropy bounds [15]. A well-defined notion of entanglement in flat space-time can be provided by restricting attention to inertial observers. In curved space-time, however, the notion of entanglement can be expected to become a rather subtle one, as does the notion of particles.

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