Whispering-Gallery-Mode Resonances: A New Way to Accelerate Charged Particles

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Looking for future high energy accelerators we point at a very strong interaction between relativistic electrons and powerful electromagnetic fields existing in the vicinity of a dielectric cylinder in conditions of resonantly excited whispering gallery modes (WGM). A particular example of the WGM resonance, corresponding to angular index $n = 22$, shows that the accelerating fields are almost 100 times stronger than these in the incident wave. That yields an acceleration rate of about 5 GeV/m with the incident microwave radiation beam of the wavelength $\lambda = 1$ cm and a moderately high intensity of $P = 1\ \mathrm{MW/cm^2}$.

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The quest for future high energy particle accelerators is dominated by various concepts of laser accelerators due to the extreme fields found in focused short laser pulses. The recent reports on plasma wakefield accelerators [1–3], demonstrating an energy gain equivalent to ~100 GeV/m in a few millimeters, renew the hope for cheap tabletop particle accelerators.

Other proposed and investigated projects of laser accelerators are related to various radiation sources working in the inverse way as radiation absorbers. They include: Inverse Free Electron Laser (IFEL) [4], Inverse Cherenkov effect [5], and Inverse Smith-Purcell effect (called also a diffraction grating accelerator) [6,7]. All these accelerating systems can yield an energy gain surpassing conventional high energy accelerators. Their performance is much closer to conventional accelerators as was illustrated for IFEL accelerators in [8]. However, all those impressive new laser accelerators are limited to very short distances, due to the difficulties in keeping synchronous acceleration for optical frequency radiation and difficulties in the optical guiding of strongly focused very high energy laser beams.

Therefore, the high energy accelerators, TESLA project at DESY and Next Linear Collider (NLC) at Stanford, employ conventional schemes of accelerators operating with radiation from 1 to 10 GHz, and use structures resembling that developed for SLAC. Those projects aim to reach 500 GeV. This will be possible by developing the conventional model, limited by electric discharges. The length of those planned structures would reach 30 km.

Looking for a more efficient mechanism of high energy accelerators, we look in more detail at the scattering of an electromagnetic wave by a dielectric cylinder and its influence on an electron passing nearby. This system could be viewed as a single rod diffraction grating accelerator.

The scattering of a plane wave by a dielectric cylinder, analyzed for the first time by Lord Rayleigh, has been described in many textbooks on optics, e.g., [9]. The following discussion employs the solutions presented in [10–12] that emphasize the role of the total field in determining the radiation intensity pattern and properties of the spontaneous radiation in the vicinity of the cylinder. Similarly, the forces acting on a moving charge require the knowledge of the total field.

Let us consider a plane monochromatic electromagnetic wave propagating in the x direction and scattered by a dielectric cylinder (with dielectric constant ϵ), of radius a placed at the origin of the coordinate system and extended along the z axes. The wave is polarized in such way that its magnetic field remains parallel to the z axis, while the electric field remains in the x, y plane. Throughout this Letter, distances are measured in units of wavelength λ, time in wave periods, so $\omega = k = 2\pi$. The strength of the electric field is measured in the peak amplitude $E_0 = 2.746\ [\mathrm{MV/m}]\sqrt{P[\mathrm{MW/cm^2}]}$ and the energy in $\mathcal{E}_U = 2.746\ [\mathrm{MeV}]\sqrt{P[\mathrm{MW/cm^2}]}$ is determined by the time averaged power density of the incident wave P.

For this field polarization, the total electromagnetic field satisfying Maxwell's equations together with the boundary conditions can be derived from the B_z component written as,

$$B_z(x, y, t) = e^{-i\omega t}\begin{cases} \sum_{n=-\infty}^{\infty} a_n J_n(n_e kr) e^{in\phi}, & r < a, \\ \exp(ikx) + \sum_{n=-\infty}^{\infty} b_n H_n(kr) e^{in\phi}, & r > a, \end{cases} \tag{1}$$

where $\{r, \phi\}$ are cylindrical coordinates of $\{x, y\}$, with

$$a_n = \frac{i^n}{W_n} n_e[J_n'(ka)H_n(ka) - J_n(ka)H_n'(ka)] \quad \text{and} \quad b_n = \frac{i^n}{W_n}[n_e J_n(n_e ka)J_n'(ka) - J_n'(n_e ka)J_n(ka)],$$

where $W_n = J_n'(n_e ka)H_n(ka) - n_e J_n(n_e ka)H_n'(ka)$, while J_n and H_n denote Bessel and Hankel functions, respectively, and $n_e = \sqrt{\epsilon}$. The components of the electric field are $\{E_x, E_y, E_z\} = \frac{i}{\epsilon k}\{\frac{\partial}{\partial y}B_z, -\frac{\partial}{\partial x}B_z, 0\}$.

Figure 1(a) illustrates typical nonresonant scattering and focusing properties of the dielectric cylinder as well as the diffraction interference pattern of the waves. The scattering can reach resonant conditions, Fig. 1(b), when the radiation circulating inside the cylinder due to internal reflections from its internal surface matches the incident radiation according to propagation directions as well as their phases so the scattering can be strongly modified and resonantly enhanced. These resonances can be identified with Rayleigh's whispering gallery modes (WGM) originally discovered for sound waves. In a more formal interpretation of the WGM resonance, a particular coefficient W_m has a very small value and in consequence the corresponding internal fields, determined by a_m, become very large, thus enhancing the external field in the vicinity of the scatterer. Different WGM resonances can be specified by the value of the angular index m.

Highly selective WGM resonances have been proposed in high-Q microlaser cavities, and for experiments involving nonlinear optical effects and strong coupling between electromagnetic field and radiating atomic sources in the context of cavity quantum electrodynamics [13,14].

The motion of a high energy charge passing near a scatterer can be approximated by uniform one and parametrized by the particle velocity direction α, (i.e., $\vec{\beta} = \{\cos\alpha, \sin\alpha, 0\}$), the impact parameter ρ and time passage at the closest distance to the scattering cylinder t_0. Thus the particle trajectory is

$$\vec{R}(t) = \{-\rho\sin\alpha, \rho\cos\alpha, 0\} + \vec{\beta}(t - t_0).$$

The energy change is determined by the electric field component along the direction of velocity. Figures 2(b) and 2(c) show examples of those forces acting on a particle as it passes the scattering cylinder in nonresonant and resonant conditions. Far away from the cylinder, the force oscillates harmonically, while its behavior becomes more complicated in the closest neighborhood of the cylinder and the estimation of the net effect requires a more detailed computation.

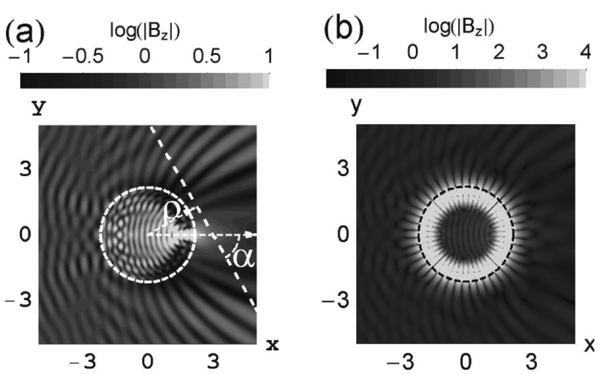

FIG. 1. Field patterns represented by $|B_z(x, y)|$. (a) In nonresonant, (b) in resonant scattering conditions. A dielectric cylinder has the dielectric constant $\epsilon = 4.$, while its radius, in units of the wavelength λ, is in (a) $a = 2.17$ and in (b) $a = 2.162\,260\,47$—corresponding to $m_{\rm res} = 22$ WGM resonance.

To find the net energy change disregarding its oscillations, occurring when the particle is crossing the wave, the longitudinal force should be integrated over an integer multiple of period of particle motion, $T_0 = 1/(1 - \beta_x)$, and including the time of closest encounter t_0. The value of energy change depends on the particle's initial position. All possible energy changes can be derived from a complex longitudinal force integral

$$\Delta E^C = \int_{-T}^{-T+mT_0} e\vec{\beta} \cdot E(\vec{R}(t), t)dt, \tag{2}$$

where the limits of this integral include a passage time. Thus, for a given incident wave, ΔE^C is a function of α, ρ, and t_0. However, for all values of t_0, ΔE^C differs by a phase factor only. Therefore, the maximum possible energy transfer is given by the absolute value $|\Delta E^C|$ not dependent of t_0, while the argument of ΔE^C specifies the resonant accelerating phase $\phi_{\rm acc}$ and the optimal moment $kt_0 = -\phi_{\rm acc}$ for the particle energy gain.

Figure 2(a) shows the energy gain for electrons having phase $\phi_{\rm acc}$ and passing in WGM-resonant as well as non-

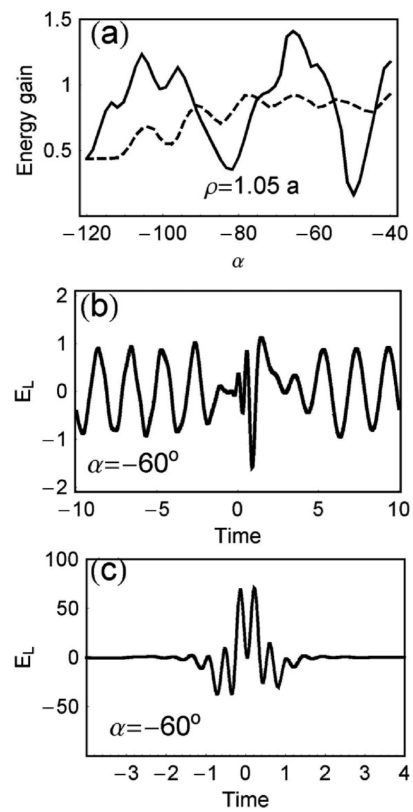

FIG. 2. (a) Energy gain for an electron moving in the x, y plane in nonresonant (dashed line) and WGM-resonant [$n_{\rm res} = 22$ (solid line)] conditions as a function of α for $\rho = 1.05a$. (b) and (c) The in-phase electric field parallel to the electron velocity for an electron passing near the scatterer with $\alpha = -60°$, $\rho = 1.05a$ as a function of time, (b) in nonresonant conditions, (c) in the WGM-22 resonant conditions. See the text for description of energy and electric field units. Note the great change in scale from 2(b) and 2(c).

resonant conditions near the dielectric scatterer as a function of the particle's velocity direction α. It may appear surprising that, although the fields in the WGM resonance [shown in Fig. 1(b)] are much stronger than in the nonresonant case [Fig. 1(a)], the corresponding energy gains take very similar values. However, the strong fields accompanying the WGM resonance cause very strong particle accelerations and decelerations so that both momentum changes almost cancel and the net energy change is rather minor, Fig. 2(c). This force variation exhibits a lack of synchronism between the electron motion and the field space-time variation.

The above situation resembles in the microscale an effect known in nonaccelerating configurations of a diffraction grating accelerator when the accelerated particles and the incident electromagnetic wave lie in a symmetry plane of the grating, which was recognized by Lawson [15]. Palmer has overcome this obstacle noticing that the energy transfer is possible when either the electrons move at skew angles to the grating groves or the incident light impacts at skew angles [6].

The present discussion shows that some synchronism between the oscillating wave and moving electron is necessary for an efficient energy exchange. The oscillating field acting on the moving electron shown in Fig. 2(c) indicates that such synchronization does not occur for a chosen incident wave and electrons moving in the x, y plane. Therefore, to find a better synchronization consider the electron moving at a skew angle, ψ, with respect to this plane. Its velocity is $\vec{\beta} = \{\cos\psi\cos\alpha, \cos\psi\sin\alpha, \sin\psi\}$. Modifying formula Eq. (2) according to the assumed velocity of the electron we get the energy gain factor dependent on the angle ψ. This ψ dependence is shown in Fig. 3(a) for several impact parameters ρ. This figure shows that the accelerating electric force exists only in the closest proximity of the scattering cylinder when the particle impact parameter ρ only slightly exceeds the cylinder radius a.

The energy gain is maximal for $\psi \approx 50°$ and exceeds by 2 orders of magnitude the gain for an electron moving in the x, y plane. The force acting on the electron is shown in Fig. 3(b). This force, despite modulations, never changes its sign. The electric field, therefore, acts synchronously on the moving electron which leads to a significant gain of its energy.

In the example discussed above, we took the WGM resonance corresponding to $m = 22$ of the cylindrical index. A more detailed analysis shows that the maximum resonant energy gain is an exponential function of m and, for example, for $m = 36$ resonance the energy gain is about 4 orders of magnitude higher than in a nonresonant case. The higher resonances, however, become very narrow and therefore harder to obtain and control.

The large energy gain predicted for electrons penetrating the region of WGM resonance excitation is caused by the electromagnetic field accumulated inside the dielectric cylinder. WGM resonances are very narrow and thus long electromagnetic pulses acting over an appropriate interval are required for their excitation and formation of the strong near-wave resonant field. The possibility of a gradual resonant excitation by lower intensity radiation is a very attractive property discussed, e.g., in [16].

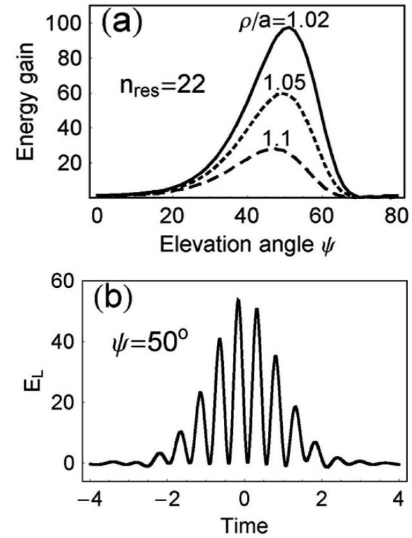

FIG. 3. (a) WGM-22 resonant energy gains for electrons passing near the scatterer as a function of the elevation angle ψ ($v_z = \beta\sin\psi$). (b) The in-phase electric field parallel to the electron velocity experienced by an electron crossing the x, y plane at $\psi = 50°$ with $\alpha = -60°$, $\rho = 1.05a$, in the WGM-22 resonance, as a function of time.

In addition to the energy gain, electrons passing near the scatterer would experience transverse deflections, also resonantly enhanced at the WGM conditions. We must consider momenta changes projected on two directions: $\vec{n}_1 = \{-\sin\alpha, \cos\alpha, 0\}$ and $\vec{n}_2 = \{\cos\alpha\sin\psi, \sin\alpha\sin\psi, -\cos\psi\}$. A net momentum gain along $\vec{n}_1$ is given by:

$$\Delta P_1 = \int_{-T}^{-T+mT_0} \vec{n}_1 \cdot e(\vec{E}(\vec{R}(t), t) + \vec{\beta} \times \vec{B}(\vec{R}(t), t))dt. \tag{3}$$

The change of electron momentum depends on the initial t_0 and can be expressed as: $\Re e(\Delta P_1) = A_1 \cos(kt_0 + \phi_p)$, where the complex ΔP_1 evaluated at $t_0 = 0$ determines the momentum transfer phase ϕ_p and the amplitude of momentum transfer A_1. For the electrons corresponding the maximum energy gain ϕ_p and $\phi_{\rm acc}$ differ by $\approx\pi/2$ and the net transverse momentum change in the direction $\vec{n}_1$ vanishes [$\Re e(\Delta P_1) = A_1 \cos(\phi_p - \phi_{\rm acc}) \approx 0$].

The corresponding momentum gain along $\vec{n}_2$ depends on the electric field only. Thus, $c\Delta P_2 = \Delta E^C \tan\psi$, and particle acceleration is intrinsically connected with its deflection. A particle deflection in a single WGM field passage is $\delta\psi = \frac{c\Delta P_2}{E}$. To be consistent with the assumed straightforward particle trajectory, its energy should be $E \gg 1$ GeV for the estimated $\Delta P_2 \approx 0.2$ GeV$/c$. Although such deflections would have an adverse effect in an operating accelerator, this effect, together with the electron beam energy spread, would be the simplest identifier of the strong interaction of the electrons with the resonant field, and thus might have an experimental significance at an early stage of tests.

WGM resonances proved their usefulness in low-power devices in microelectronics and micro-optics [13,14]. The proposed application of WGM in high energy accelerators, the latter being necessarily high power devices, raises a question whether such an extension is possible. There is a danger that the strong resonant fields existing inside the scattering cylinder may lead to an electrical breakup causing damage. However, the very strong fields existing inside the dielectric do not carry an equally strong electromagnetic field flux density (mean value of the Poynting vector). The electric and magnetic fields in the near-wave radiation zone perform idle oscillations. In that zone, the electromagnetic flux is oscillating with the wave frequency and therefore is not able to cause significant accumulation of momentum by the slow electrons responsible for the electrical discharge and breakup. These very important properties are crucial for the performance of the proposed accelerator, and must be verified experimentally.

In order to implement the described strong acceleration mechanism in any useful high energy particle accelerator one must consider a multistage acceleration processes. The individual accelerating scatterers must be arranged and excited in such a way that the particle motion is not distorted transversally. What is more, along their trajectories the particles must bunch inside stable accelerating buckets, as they do in every accelerator. In a multicylinder system one can expect that placing the scattering cylinders and exciting the radiation waves at alternating angles ψ and $-\psi$ and proper accelerating phases $\phi_{\rm acc}$, the particle energy increases continuously while its deflections cancel out.

Studying our proposed acceleration mechanism we have assumed ideal conditions of a perfect lossless infinite cylinder scatterer in a vacuum, excited by a perfect monofrequency plane wave. Considering a similar accelerating mechanism in real accelerators one must include waveguiding of the electromagnetic wave in a waveguide and find WGM resonances, including the resonant frequencies, in waveguiding environments. Besides the WGM resonances in an infinite perfect cylinder, it is worthwhile to investigate other WGM-resonant scatterers shaped in the forms of spheres, disks, barrels, and tori. Considering an accelerating unit composed of many resonantly excited cylinders one must take into account their mutual interaction and the shift of the resonant frequencies.

The cylinder considered here is a basic element which should be used to build the accelerating unit cell. Let us assume that a proposed LINAC operates in the 30 GHz frequency range. As the radius of the scatterer (the cylinder) is equal to about 2.16λ, where λ is the wavelength, the size of such a basic element is about 4 cm , which means that one can assemble 20 per meter. Thus we can achieve an energy gain of the order of 5 GeV/m, given the intensity of the incident wave 1 MW/cm^2.

We emphasize that, in this scheme, the accelerating electrons move in a vacuum while the formation of the strong electromagnetic accelerating fields does not involve waves of extreme power which would be destructive for an accelerator construction. It is also important that the above enhanced acceleration mechanism can occur in the microwave frequency range, familiar to conventional accelerator operation. Enhanced changes of transverse momentum of electrons in conditions of WGM resonance are also interesting. Particle deflection is obtained in a very short distance, just a few λ, and may reach $A_P/\gamma mc$. This is much more particle deflection, in shorter distance, than achieved using typical magnets. This may lead to the construction of more efficient bending units, particle focusing lenses, x-ray light sources, wigglers, etc. All those challenging applications of extremely strong fields appearing in the WGM resonances require further intensive analyses.

In summary, we have indicated a very efficient acceleration of charged particles by huge electromagnetic fields in the vicinity of a scattering dielectric cylinder in conditions of WGM resonance. It is indicated that strong electromagnetic fields as described can be used not only as a driving accelerating force, but also as a strong deflecting field for very high energy relativistic particles. Practical design of the linac composed of multiple accelerating units and resonant excitation of accelerating field in such setup is an object of future study.

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