Sequential Generation of Entangled Multiqubit States

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We consider the deterministic generation of entangled multiqubit states by the sequential coupling of an ancillary system to initially uncorrelated qubits. We characterize all achievable states in terms of classes of matrix-product states and give a recipe for the generation on demand of any multiqubit state. The proposed methods are suitable for any sequential generation scheme, though we focus on streams of single-photon time-bin qubits emitted by an atom coupled to an optical cavity. We show, in particular, how to generate familiar quantum information states such as W, Greenberger-Horne-Zeilinger, and cluster states within such a framework.

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Entangled multiqubit states are a valuable resource for the implementation of quantum computation and quantum communication protocols, like distributed quantum computing [1], quantum cryptography [2] or quantum secret sharing [3]. Using photonic degrees of freedom as qubits, say, polarization states or time bins of energy eigenstates, has the advantage that photons propagate safely over long distances. Consequently, photonic devices are the most promising systems for quantum communication tasks. For this purpose, a lot of effort has been made in recent years to develop efficient and deterministic single-photon sources [4–7].

Photonic multiqubit states can be generated by letting a source emit photonic qubits in a sequential manner [8]. If we do not initialize the source after each step, the created qubits will be, in general, entangled. Moreover, if we allow for specific operations inside the source before each photon emission, we will be able to create different multiqubit states at the output. In fact, this is a particular instance of a general sequential generation scheme, where an ancillary system is coupled in turn to a number of initially uncorrelated qubits.

It is the purpose of this Letter to provide a complete characterization of all multipartite quantum states achievable within a sequential generation scheme. It turns out that the classes of states attainable with increasing resources are exactly given by the hierarchy of so-called matrix-product states (MPS) [9,10]. These states typically appear in the theory of one-dimensional spin systems [11], as they are the variational set over which density matrix renormalization group techniques are carried out [12]. Thus, our analysis stresses the importance of MPS, since we show that they naturally appear in a completely different and relevant physical context. Moreover, particular instances of low-dimensional MPS, like cluster states [13] or Greenberger-Horne-Zeilinger (GHZ) states [14], are a valuable resource in quantum information [15]. Conversely, we will provide a recipe for the generation on demand of any multiqubit state within a sequential generation scheme. Because of the general validity of these results, we will first state and prove them without referring to any particular physical system. This will be then applicable to all sequential setups, like streams of photonic qubits emitted either by a cavity QED (CQED) source [4,5] or by a quantum dot coupled to a microcavity [6,7].

In the second part, we will focus on the physical implementation of these ideas within the realm of CQED. The role of the ancillary system will be performed by a D-level atom coupled to a single mode of an optical cavity. The sequentially generated qubits will be time-bin qubits $|0\rangle$ and $|1\rangle$, describing the absence and presence of a photon emitted from the cavity in a certain time interval (see Fig. 1).

We will concentrate on setups where all intermediate operations are arbitrary unitaries and the ancilla decouples in the last step. The latter enables us to generate pure entangled states in a deterministic manner without the need of measurements. Let $\mathcal{H}_{\mathcal{A}} \simeq \mathbb{C}^D$ and $\mathcal{H}_{\mathcal{B}} \simeq \mathbb{C}^2$ be the Hilbert spaces characterizing a D-dimensional ancillary system and a single qubit (e.g., a time-bin qubit), respectively. In every step of the sequential generation of a multiqubit state, we consider a unitary time evolution of the joint system $\mathcal{H}_{\mathcal{A}} \otimes \mathcal{H}_{\mathcal{B}}$. Assuming that each qubit is initially in the state $|0\rangle$ (i.e., the time bin is empty), we disregard the qubit at the input and write the evolution in the form of an isometry V: $\mathcal{H}_{\mathcal{A}} \to \mathcal{H}_{\mathcal{A}} \otimes \mathcal{H}_{\mathcal{B}}$.

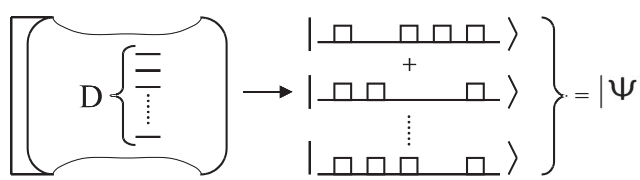

FIG. 1. A trapped D-level atom is coupled to a cavity qubit, determined by the energy eigenstates $|0\rangle$ and $|1\rangle$. After arbitrary bipartite source-qubit operations, photonic time bins are sequentially and coherently emitted at the cavity output, creating a desired entangled multiqubit stream.

Expressing the latter in terms of a basis $V = \sum_{i,\alpha,\beta} V^i_{\alpha,\beta}|\alpha, i\rangle\langle\beta|$, the isometry condition reads $\sum_{i=0}^{1} V^{i\dagger} V^i = 1$, where each V^i is a $D \times D$ matrix. We choose a basis where $\{|\alpha\rangle, |\beta\rangle\}$ are any of the D ancillary levels. If we apply successively n, not necessarily identical, operations of this form to an initial state $|\varphi_I\rangle \in \mathcal{H}_\mathcal{A}$, we obtain the state $|\Psi\rangle = V_{[n]} \ldots V_{[2]} V_{[1]} |\varphi_I\rangle$, where indices in squared brackets represent the steps in the generation sequence. The n generated qubits are in general entangled with the ancilla as well as among themselves. Assuming that in the last step the ancilla decouples from the system, such that $|\Psi\rangle = |\varphi_F\rangle \otimes |\psi\rangle$, we are left with the n-qubit state

$$|\psi\rangle = \sum_{i_1 \ldots i_n = 0}^{1} \langle\varphi_F|V^{i_n}_{[n]} \ldots V^{i_1}_{[1]}|\varphi_I\rangle|i_n, \ldots, i_1\rangle. \quad (1)$$

States of this form are called MPS [9,10], and play a crucial role in the theory of one-dimensional spin systems. Equation (1) shows that all sequentially generated multiqubit states, arising from a D-dimensional ancillary system $\mathcal{H}_\mathcal{A}$, are instances of MPS with $D \times D$ matrices V^i and open boundary conditions specified by $|\varphi_I\rangle$ and $|\varphi_F\rangle$. We will now prove that the converse is also true, i.e., that every MPS of the form

$$|\tilde{\psi}\rangle = \langle\tilde{\varphi}_F|\tilde{V}_{[n]} \ldots \tilde{V}_{[1]}|\tilde{\varphi}_I\rangle, \quad (2)$$

with arbitrary maps $\tilde{V}_{[k]}$: $\mathcal{H}_\mathcal{A} \to \mathcal{H}_\mathcal{A} \otimes \mathcal{H}_\mathcal{B}$, can be generated by isometries of the same dimension $2D \times D$, and such that the ancillary system decouples in the last step. Since every state has a MPS representation [16], this is at the same time a general recipe for its sequential generation. The idea of the proof is an explicit construction of all involved isometries by subsequent application of singular value decompositions (SVD). We start by writing $\langle\tilde{\varphi}_F|\tilde{V}_{[n]} = V'_{[n]} M_{[n]}$, where the 2×2 matrix $V'_{[n]}$ is the left unitary in the SVD and $M_{[n]}$ is the remaining part. The recipe for constructing the isometries is the induction

$$(M_{[k]} \otimes 1_2)\tilde{V}_{[k-1]} = V'_{[k-1]} M_{[k-1]}, \quad (3)$$

where the isometry $V'_{[k-1]}$ is constructed from the SVD of the left-hand side, and $M_{[k-1]}$ is always chosen to be the remaining part. After n applications of Eq. (3) in Eq. (2), from left to right, we set $|\varphi_I\rangle = M_{[1]}|\tilde{\varphi}_I\rangle$, producing

$$|\tilde{\psi}\rangle = V'_{[n]} \ldots V'_{[1]}|\varphi_I\rangle. \quad (4)$$

Exploiting the fact that $\tilde{V}_{[k-1]}$ has dimension $2D \times D$, it can be shown, through simple rank considerations in Eq. (3), that $V'_{[n-k]}$ has dimension $2\min[D, 2^k] \times \min[D, 2^{k+1}]$. In particular, every $V'_{[k]}$ could be embedded into an isometry $V_{[k]}$ of dimension $2D \times D$. Physically, this just means we would have redundant ancillary levels that we need not use. Finally, decoupling the ancilla in the last step is guaranteed by the fact that, after the application of $V_{[n-1]}$, merely two levels of $\mathcal{H}_\mathcal{A}$ are still occupied, and can be mapped entirely onto the system $\mathcal{H}_\mathcal{B}$. This is precisely the action of $V_{[n]}$ through its embedded unitary $V'_{[n]}$.

This proves the equivalence of two sets of n-qubit states, which are described either as D-dimensional MPS with open boundary conditions, or as states that are generated sequentially and isometrically via a D-dimensional ancillary system which decouples in the last step. Note that, in order to produce a generic n-qubit state, the dimension of the source is given by $D = 2^n$. For a particular n-qubit state, the minimal dimensionality of the ancillary system is optimized through the procedure described above.

Motivated by current CQED setups, we will now provide a third equivalent characterization, namely, a set of multiqubit states that are sequentially generated by a source consisting of a $2D$-level atom. In contrast to the first sequential scheme, the latter will not require arbitrary isometries. Consider an atomic system with D states $|a_i\rangle$ and D states $|b_i\rangle$, so that $\mathcal{H}_\mathcal{A} = \mathcal{H}_a \oplus \mathcal{H}_b \simeq \mathbb{C}^D \otimes \mathbb{C}^2$. That is, we will write $|\varphi\rangle|1\rangle$ for a superposition of $|a_i\rangle$ states, whereas $|\varphi\rangle|0\rangle$ corresponds to a superposition of $|b_i\rangle$ states. Since the last qubit marks the atomic level, whether it belongs to the $|a_i\rangle$ or to the $|b_i\rangle$ subspace, we will refer to it as the tag qubit and write $\mathcal{H}_\mathcal{A} = \mathcal{H}_{\mathcal{A}'} \otimes \mathcal{H}_\mathcal{T}$. Now consider the atomic transitions from each $|a_i\rangle$ state to its respective $|b_i\rangle$ state accompanied by the generation of a photon in a certain time bin. This is described by a unitary evolution of the form

$$\begin{aligned} T\colon |\varphi\rangle_{\mathcal{A}'}|1\rangle_\mathcal{T}|0\rangle_\mathcal{B} &\mapsto |\varphi\rangle_{\mathcal{A}'}|0\rangle_\mathcal{T}|1\rangle_\mathcal{B}, \\ |\varphi\rangle_{\mathcal{A}'}|0\rangle_\mathcal{T}|0\rangle_\mathcal{B} &\mapsto |\varphi\rangle_{\mathcal{A}'}|0\rangle_\mathcal{T}|0\rangle_\mathcal{B}. \end{aligned} \quad (5)$$

Hence, T effectively interchanges the tag qubit with the time-bin qubit. If, additionally, arbitrary atomic unitaries $U_\mathcal{A}$ are allowed at any time, we can exploit the swap caused by T in order to generate the operation

$$V|\varphi\rangle = \langle 0|_\mathcal{T} T(U_\mathcal{A}(|\varphi\rangle_{\mathcal{A}'}|0\rangle_\mathcal{T})|0\rangle_\mathcal{B}), \quad (6)$$

which is the most general isometry V: $\mathcal{H}_{\mathcal{A}'} \to \mathcal{H}_{\mathcal{A}'} \otimes \mathcal{H}_\mathcal{B}$. Therefore, the so generated n-qubit states include all possible states arising from subsequent applications of $(2D \times D)$-dimensional isometries. On the other hand, they are a subset of the MPS in Eq. (2) with arbitrary $(2D \times D)$-dimensional maps, assuming that the atom decouples at the end. Hence, these three sets are all equivalent.

Now, we show how these results can be applied in the realm of CQED, where an atom is trapped inside a high-Q optical cavity, and we aim at generating multiphoton entangled states. A laser may excite the atom, producing subsequently a photon in the cavity mode, which, after some time, is emitted outside the cavity (Fig. 1). We consider two different scenarios, corresponding to the two families of states considered above. First, we may have fast and complete access to the atom-cavity system. In consequence, after the implementation of the desired isometry in each step, we should wait until the photon leaks out of the cavity before starting the next step. In this case, according to the analysis above, we will be able to produce arbitrary D-dimensional MPS with D equal to the number of involved atomic levels. Second, we may have a $2D$-level atom (D $|a_i\rangle$ levels and D $|b_i\rangle$ levels) and two kind of operations: (i) fast arbitrary operations which allow us to pairwise manipulate all atomic levels; (ii) an operation which maps each $|a_i\rangle$ state to its corresponding $|b_i\rangle$ state while creating a single cavity photon, allowing a tailored output. Here, we will also be able to produce arbitrary D-dimensional MPS.

In the following, we will illustrate the above statements with a specific example which is based on present CQED experiments [5]. We consider a three-level atom coupled to a single cavity mode in the strong-coupling regime. An external laser field drives the transition from level $|a\rangle$ to the upper level $|u\rangle$ with coupling strength Ω_0, and the cavity mode drives the transition between $|u\rangle$ and level $|b\rangle$ with coupling strength g, in a typical Λ configuration [see Fig. 2(a)]. We choose the detunings Δ, with $|\Delta| \gg \{g, \Omega_0\}$, and assume that the cavity decay rate κ is smaller than any other frequency in the problem, so that we can ignore cavity damping during the atom-cavity manipulations. By eliminating level $|u\rangle$, we remain with an effective $D = 2$ atomic system plus the cavity mode. We will show how, by controlling the laser frequency and intensity, it is possible to generate arbitrary 2-dimensional MPS. Note that, by allowing the manipulation of D effective atomic levels, it is straightforward to extend these results to the generation of D-dimensional MPS.

According to the results above, we just have to show that we can implement any isometry $V\colon \mathcal{H}_\mathcal{A} \to \mathcal{H}_\mathcal{A} \otimes \mathcal{H}_\mathcal{B}$. In fact, we will explain how to implement arbitrary operations on the atomic qubit and the $\sqrt{\text{SWAP}}$ operation on the atom-cavity system, which suffice to generate any isometry V (since they give rise to a universal set of gates for the two qubit system [17]). The atomic qubit can be manipulated at will using Raman lasers, as done with trapped ions [18,19]. In order to implement the $\sqrt{\text{SWAP}}$, we notice that the atom-cavity coupling is described in terms of the Jaynes-Cummings model [see Fig. 2(b)], where the coupling constant Ω_0 is controlled by the laser. Thus, application of laser pulses with the appropriate duration and phase [19,20] will implement the unitary operation $U = e^{-iG}$, where generator $G = (|a, 0\rangle\langle b, 1| + \text{H.c.})\pi/4$, which corresponds to the desired $\sqrt{\text{SWAP}}$ operation. In order to generalize this scheme to an arbitrary D-level system, we notice that we can view the atom as a set of M qubits (with $D \leq 2^M$). Thus, if we are able to perform arbitrary atomic operations, together with the $\sqrt{\text{SWAP}}$ operation on two specific atomic levels as explained above, we can then implement a universal set of gates and, in consequence, any arbitrary isometry required for the first scenario.

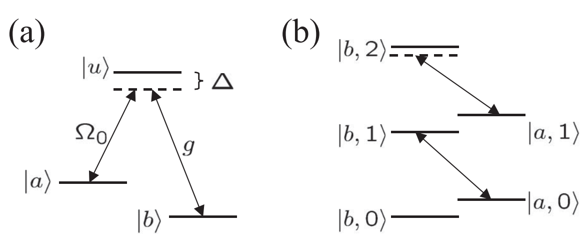

FIG. 2. (a) Atomic level structure: levels $|a\rangle$, ($|b\rangle$), and $|u\rangle$ are coupled by a laser (cavity mode) off resonance. (b) After adiabatic elimination of the upper state $|u\rangle$, we are left with an effective Jaynes-Cummings interaction, coupling states $|a, n\rangle$ and $|b, n+1\rangle$. The ac Stark shift introduced in the energy difference of these levels, ng^2/Δ, and the Jaynes-Cummings coupling, $\sqrt{n+1}\Omega_0 g/2\Delta$, depend on the photon number, n.

In the rest of the Letter, we will use another setup which is closely related to current experiments [5] and optimizes our second method for MPS generation. In this frame, we will show how to generate familiar multiqubit states like W [21], GHZ [14], and cluster states [13], which are all MPS with $D = 2$ [15].

For the purpose, we consider a particular example of the second scenario, where an atom with three levels $\{|a\rangle, |b_1\rangle, |b_2\rangle\}$ is trapped inside an optical cavity. With the help of a laser beam, state $|a\rangle$ is mapped to state $|b_1\rangle$, and a photon is generated, whereas the other states remain unchanged. This process is described by the map

$$M_{\mathcal{AB}}\colon |a\rangle \mapsto |b_1\rangle|1\rangle, \qquad |b_j\rangle \mapsto |b_j\rangle|0\rangle \tag{7}$$

with $j = 1, 2$, and can be realized with the techniques used in [5]. After the application of $M_{\mathcal{AB}}$, an arbitrary operation is applied to the atom, which can be performed by using Raman lasers. The photonic states that are generated after several applications are those MPS where the isometries are given by $V_{[i]} = M_{\mathcal{AB}} U_{\mathcal{A}}^{[i]}$, with $i = 1, \ldots, n$, $U_{\mathcal{A}}^{[i]}$ being arbitrary unitary atomic operators.

For example, to generate a W-type state of the form

$$\begin{aligned}|\psi_W\rangle = {} & e^{i\Phi_1}\sin\Theta_1|0\ldots01\rangle + \cos\Theta_1 e^{i\Phi_2}\sin\Theta_2|0\ldots010\rangle \\ & + \cdots \\ & + \cos\Theta_1\ldots\cos\Theta_{n-2}e^{i\Phi_{n-1}}\sin\Theta_{n-1}|010\ldots0\rangle \\ & + \cos\Theta_1\ldots\cos\Theta_{n-1}|10\ldots0\rangle,\end{aligned} \tag{8}$$

we choose the initial atomic state $|\varphi_I\rangle = |b_2\rangle$ and operations $U_{\mathcal{A}}^{[i]} = U_{ab_2}^{b_1}(\Phi_i, \Theta_i)$, with $i = 1, \ldots, n-1$, where

$$\begin{aligned}U_{kl}^m(\Phi_i, \Theta_i) = {} & \cos\Theta_i|k\rangle\langle k| + \cos\Theta_i|l\rangle\langle l| + e^{i\Phi_i}\sin\Theta_i|k\rangle \\ & \times\langle l| - e^{-i\Phi_i}\sin\Theta_i|l\rangle\langle k| + |m\rangle\langle m|,\end{aligned} \tag{9}$$

and $\{k, l, m\} = \{a, b_1, b_2\}$. To decouple the atom from the photon state, we choose the last atomic operation $U_{\mathcal{A}}^{[n]} = U_{ab_2}^{b_1}(0, \pi/2)$ and, after the last map $M_{\mathcal{AB}}$, the decoupled atom will be in state $|b_1\rangle$.

To produce a GHZ-type state in similar way, we choose $|\varphi_I\rangle = |a\rangle$, $U_{\mathcal{A}}^{[1]} = U_{ab_2}^{b_1}(\Phi_1, \Theta_1)$, $U_{\mathcal{A}}^{[i]} = U_{ab_1}^{b_2}(0, \pi/2)$, with $i = 2, \ldots, n-1$, and $U_{\mathcal{A}}^{[n]} = U_{b_1 b_2}^{a}(0, \pi/2)U_{ab_1}^{b_2}(0, \pi/2)$.

For generating cluster states, we choose $|\varphi_I\rangle = |b_2\rangle$, $U^{[i]}_{\mathcal{A}} = U^{b_1}_{ab_2}(\Phi_i, \Theta_i)U^{b_2}_{ab_1}(0, \pi/2)$, with $i = 1, \ldots, n-1$, and $U^{[n]}_{\mathcal{A}} = U^{b_2}_{ab_1}(\Phi_n, \Theta_n)U^{a}_{b_1 b_2}(0, \pi/2)U^{b_2}_{ab_1}(0, \pi/2)$, obtaining

$$|\psi\rangle = \bigotimes_{i=1}^{n}(O^0_{i-1}|0\rangle_i + O^1_{i-1}|1\rangle_i), \tag{10}$$

where $O^0_{i-1} = \cos\Theta_i|0\rangle_{i-1}\langle 0| - e^{-i\Phi_i}\sin\Theta_i|1\rangle_{i-1}\langle 1|$ and $O^1_{i-1} = e^{i\Phi_i}\sin\Theta_i|0\rangle_{i-1}\langle 0| + \cos\Theta_i|1\rangle_{i-1}\langle 1|$, with $i = 2, \ldots, n-1$. Operators O^0_{i-1} and O^1_{i-1} act on the nearest-neighbor qubit $i-1$ under the assumption $O^0_0 \equiv \cos\Theta_1$ and $O^1_0 \equiv e^{i\Phi_1}\sin\Theta_1$. If one chooses $\Phi_i = 0$ and $\Theta_i = \pi/4$ this leads to the cluster states defined by

$$|\psi_{\rm cl}\rangle = \frac{1}{2^{n/2}}\bigotimes_{i=1}^{n}(\sigma^z_{i-1}|0\rangle_i + |1\rangle_i), \quad \text{with } \sigma^z_0 \equiv 1. \tag{11}$$

The formalism presented here is also valid for other types of single-photon sources, in the context of CQED or quantum dots. For example, it could be extended to characterize the polarization-entangled multiqubit photon states generated by an analogous CQED photon source [8]. In this case two cavity modes with orthogonal polarizations, defining the qubit, are coupled to the atom in each generation step. In fact, the presented ideas and proofs apply to any multiqudit state with $\mathcal{H}_{\mathcal{B}} \simeq \mathbb{C}^d$ that is generated sequentially by a D-dimensional source.

In a wider scope, we have established a formalism describing a general sequential quantum factory, where the source is able to perform arbitrary unitary source-qudit operations before each qudit leaves. Apart from the multiphoton states, the present formalism applies also to other physical scenarios: (a) to coherent microwave CQED experiments [22], where atoms sequentially cross a cavity, and thus the outcoming atoms end up in a MPS with the dimensions given by the effective number of states used in the cavity mode; (b) a light pulse crossing several atomic ensembles [23], where the latter will be left in a matrix-product Gaussian state; (c) trapped ion experiments where each ion interacts sequentially with a collective mode of the motion [18,19,24]. Note also that one can include dissipation in the present formalism, by replacing MPS by matrix-product density operators [10,25]. This description applies, for example, to the micromaser setup [26] and other realistic scenarios.

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