Collective Oscillations of Strongly Correlated One-Dimensional Bosons on a Lattice

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We study the dipole oscillations of strongly correlated 1D bosons, in the hard-core limit, on a lattice, by an exact numerical approach. We show that far from the regime where a Mott insulator appears in the system, damping is always present and increases for larger initial displacements of the trap, causing dramatic changes in the momentum distribution, n_k. When a Mott insulator sets in the middle of the trap, the center of mass barely moves after an initial displacement, and n_k remains very similar to the one in the ground state. We also study changes introduced by the damping in the natural orbital occupations, and the revival of the center-of-mass oscillations after long times.

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Experimental realization of one-dimensional (1D) atomic gases [1,2] has opened the possibility of studying equilibrium and nonequilibrium properties of systems in which quantum fluctuations play a fundamental role. For example, the 1D superfluid–Mott-insulator transition [3] was observed experimentally by Stöferle *et al.* [4]. In addition, 1D geometries have allowed the observation of the strongly correlated hard-core boson limit with [5] and without [6] a lattice along the 1D tubes.

Recent experiments studying transport properties of trapped 1D Bose gases [4,7] have reported a large damping of the center-of-mass (c.m.) oscillation when an axial lattice is present in the system. Additionally, an overdamping was observed in which the c.m. remains displaced from the middle of the trap [4,7]. In Ref. [4] this effect was related to the presence of an incompressible Mott-insulating phase, while Fertig *et al.* [7] reported being far from this regime. For weak interactions, and small trap displacements, the damping has been argued (within mean-field approximations) to relate to the existence of noncondensate fractions, which cause a "dissipative" behavior of the c.m. oscillations [8,9]. For large trap displacements, dynamical instabilities [10,11] can also produce damping.

For strongly correlated 1D bosons on a lattice, the mean-field description assuming Bose-Einstein condensation (BEC) breaks down. For hard-core bosons (HCB), the largest eigenvalues of the one-particle density matrix (OPDM) [also called natural orbitals (NOs)] scale proportional to $\sqrt{N_b}$ (N_b is the number of bosons). There is no BEC. However, there is a finite superfluid fraction ρ_s, which for the periodic case (no trap) can be easily calculated $\rho_s^{1D} = \sin(\pi N_b/N)/(\pi N_b/N)$, where N is the total number of lattice sites (see, e.g., Ref. [12]). In two dimensions (2D) HCB exhibit BEC at zero temperature. However, the 2D superfluid fraction is smaller, at any density, than in 1D, though larger than the 2D condensate fraction [13]. These results exemplify the nontrivial differences between BEC and superfluidity. It is unclear how much the lack of a true BEC is central to understanding transport properties in 1D.

The presence of the trap in the experiments with ultracold quantum gases generates an inhomogeneous density profile [3,14], and modifies the single particle spectrum of the periodic lattice [15]. This could, in general, change the behavior of the superfluid and condensate fractions. However, for the 1D HCB, the power-law decay of the OPDM ($\rho_x \sim 1/\sqrt{x}$), and the occupation of the lowest NO ($\lambda \sim \sqrt{N_b}$), have been shown to be the same as in the periodic system [14].

In this work we study the dynamics of the c.m. oscillations of HCB confined in 1D harmonic traps with an underlying lattice, by an exact numerical method. Our purpose is to examine the damping of such oscillations, and its consequences in the momentum distribution function (n_k) and NO occupation. We show that far from the Mott-insulating (MI) regime in the middle of the trap, the damping of the c.m. oscillations grows with initial displacement and in the large damping regime, causes dramatic changes in n_k. On the contrary, when the MI regime appears in the middle of the trap, even for small initial displacements the c.m. barely moves from its initial position, and n_k remains very similar to the one in the ground state.

The HCB Hamiltonian can be written as

$$H = -t\sum_i (b_i^\dagger b_{i+1} + \text{H.c.}) + V_2 \sum_i x_i^2 n_i, \qquad (1)$$

with the addition of the on-site constraints $b_i^{\dagger 2} = b_i^2 = 0$, $\{b_i, b_i^\dagger\} = 1$. These constraints on the creation ($b_i^\dagger$) and annihilation (b_i) operators avoid double or higher occupancy. In Eq. (1), the hopping parameter is denoted by t, and the last term describes a harmonic confining potential with curvature V_2. $n_i = b_i^\dagger b_i$ is the particle number operator. The nonequilibrium dynamics of the system, when at time $\tau = 0$ the trap is displaced a distance x_0, is obtained by means of the exact approach introduced in Ref. [16]. It is based on the Jordan-Wigner transformation, and allows us to obtain the HCB one-particle Green's function from which all the quantities analyzed in this work are calculated. In addition to being exact, this method allows studying relatively large system sizes.

Previous studies have shown that the key parameter that controls the thermodynamic behavior of this model is the characteristic density $\tilde{\rho} = N_b\sqrt{V_2/t}$ [14]. As $\tilde{\rho} \to 0$, one recovers the continuum limit. On the other hand, as $\tilde{\rho}$ increases beyond a critical value ($\tilde{\rho}_c \sim 2.6$–2.7) a MI region builds up in the middle of the trap, where the density is pinned to unity with no quantum fluctuations.

In Figs. 1(a)–1(c) we show the c.m. oscillations of a trapped HCB system for $\tilde{\rho} = 1 < \tilde{\rho}_c$, far from the regime with a MI in the middle of the trap. At time $\tau = 0$ the maximum density in the trap is $n = 0.48$, and the cloud radius is $R_0 \sim 145a$, with a the lattice constant. Figure 1(a) shows that even for a very small displacement, 7% of the cloud radius, a damping of the oscillations occurs. For this displacement the maximum momentum of the c.m. of the system is $\sim\pi/30a$, very far from the momentum at which the classical modulation instability occurs, which for HCB is $\pi/2a$ (see, e.g., Ref. [17]). Since HCB can be mapped into noninteracting fermions, one can immediately see that damping occurs for any initial density and trap displacement. In the fermion language, the damping is caused by a dephasing of the particles, which arises due to their dispersion in the lattice-harmonic trap system [15,18]. As the initial displacement of the trap is increased [Fig. 1(b)], the damping also increases. For large displacements, Bragg scattering starts to occur producing a shift of the c.m. oscillations [Fig. 1(c)]. A large damping of the c.m. oscillations can also be observed. In Fig. 1(c) the maximum momentum of the c.m. is $\sim\pi/4a$, still smaller than $\pi/2a$.

We next analyze the consequences of the damping of the c.m. motion in another physical observable, the momentum distribution function $n_k = (a/\zeta)\sum_{jl} e^{-ik(j-l)}\rho_{jl}$, with $\zeta = (V_2/t)^{-1/2}$ [14], and $\rho_{jl} = \langle b_j^\dagger b_l \rangle$ the OPDM. In Figs. 1(d)–1(f) we show how the maximum value of n_k (n_k^m) evolves as a function of time. In Fig. 1(d) one can see that even for small damping, the oscillations of n_k^m are accompanied by an overall decrease of its mean value, which reaches almost half of the original height for $\tau = 4000\hbar/t$. The changes in n_k^m become more dramatic with increasing x_0. In the large damping regime [Fig. 1(f)], one can see that n_k^m reduces almost to its minimum value in the first oscillation of the c.m. The reduction of n_k^m is accompanied by a large increment in the full width at half maximum w, as shown in Fig. 2. These results are in contrast with the ones for the equivalent noninteracting fermions [15], where after the damping of the c.m. oscillations, n_k^m and w are almost the same as the ones at $\tau = 0$. (Contrary to the density, the evolution of n_k for fermions and bosons is very different.)

At this point it is relevant to know the behavior of the "condensate" occupation during the oscillations of the c.m. For these systems it is important not to confuse n_k^m with the condensate occupation. The condensate can be defined as the largest eigenvalue of the OPDM (the lowest NO) [19,20], and since for HCB its occupation λ_0 scales proportionally to $\sqrt{N_b}$ [14,21], we prefer to call it a quasi-condensate ($\lambda_0 \to \infty$ for $N_b \to \infty$, but $\lambda_0/N_b \to 0$). One particular example in which the differences between n_k^m and λ_0 are extreme was reported by one of us in Ref. [22]. During the expansion of the HCB gas, after turning the harmonic trap off, n_k^m decreases while λ_0 increases.

In Fig. 1(g)–1(i) we show the time evolution of the three lowest NO occupations (the highest occupied ones). It can be seen that indeed the behavior of λ_0 is different from the one of n_k^m. At short times, and for low damping rates [Fig. 1(g)], the lowest NO *occupation* does not change

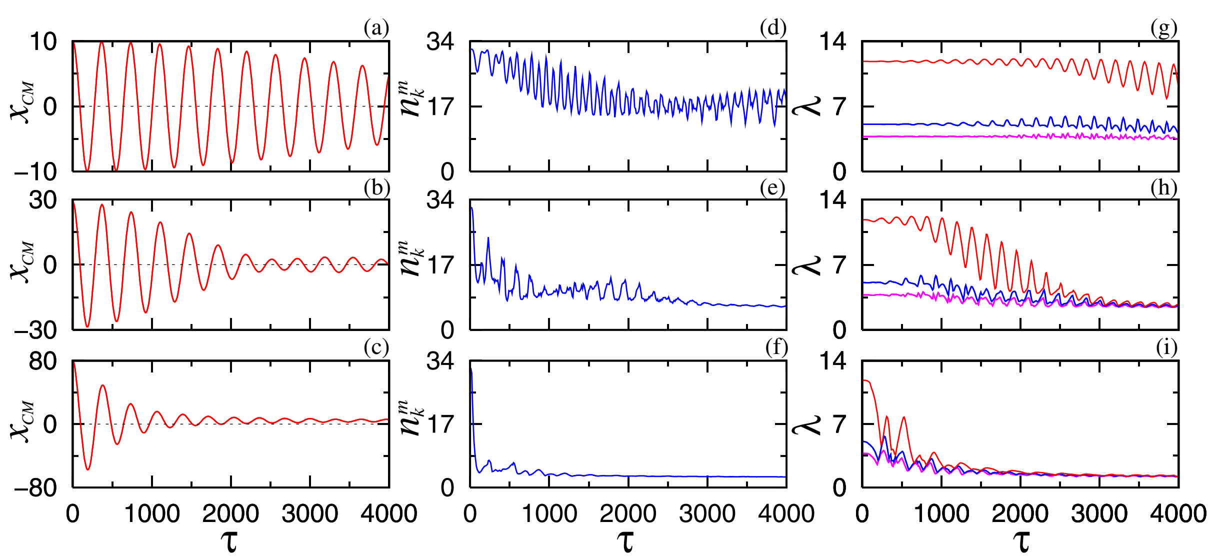

FIG. 1 (color online). Evolution of the c.m. position (a)–(c), maximum value of n_k (d)–(f), and the occupations of the three lowest NO (g)–(i), vs time (τ, in units of $\hbar/t$). The initial trap displacements are $x_0 = 10a$ ($x_0/R_0 \sim 0.07$) (a), (d), (g), $x_0 = 29a$ ($x_0/R_0 \sim 0.2$) (b), (e), (h), and $x_0 = 80a$ ($x_0/R_0 \sim 0.55$) (c), (f), (i). These systems have $N_b = 101$, and $V_2a^2 = 10^{-4}t$.

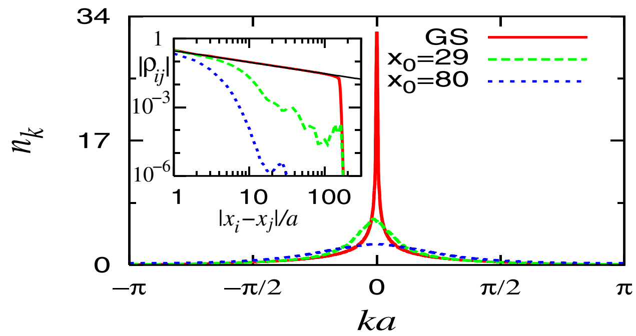

FIG. 2 (color online). n_k of the ground state (GS) compared with the ones at $\tau = 4000\hbar/t$ for two different initial displacements of the trap ($N_b = 101$, and $V_2 a^2 = 10^{-4} t$). As shown in Figs. 1(b) and 1(c), at $\tau = 4000\hbar/t$ the motion of the c.m. is totally damped. The inset shows the OPDM for the same systems, the straight line is $\rho_{ij} \sim 1/\sqrt{|x_i - x_j|/a}$.

during the oscillations of the system, even though the lowest NO itself does oscillate in a coherent manner, as would be the case for an ideal BEC when there is no lattice. Only when the c.m. energy is transfered into excitation energy, the lowest NO occupations (and the shape of their wave functions) start to change. This reflects the onset of a cutoff to the off-diagonal quasi-long-range order initially present in the OPDM, and it is similar to the appearance of a finite temperature in the system. Such effect can be seen in the inset in Fig. 2 where we show the OPDM of the ground state when the c.m. oscillations are damped. In the latter cases, the fast decays of the OPDM reflected in the plots are very close to exponentials. As expected, the largest damping produces the fastest decay. The consequent reduction of the lowest NO occupations can be seen in Fig. 1(h) and 1(i).

So far we have analyzed trapped systems with no MI phase. In what follows we study the consequences of approaching the MI regime on the damping of the c.m. oscillations. Figure 3(a) shows that after damping, when the characteristic density of the system is increased, the final c.m. position is displaced from the center of the trap. We kept the ratio between the initial trap displacement x_0 and the cloud radius R_0 constant, and equal to 0.2 as in Figs. 1(b), 1(e), and 1(h). The insets show the results for $x_{\text{c.m.}}/x_0$ vs τ in traps with $\tilde{\rho} = 2.5$, just before the MI phase appears, and for $\tilde{\rho} = 4$, when a MI domain is present in the center of the trap [Fig. 4(a)]. These results show that (i) the damping rate increases with increasing $\tilde{\rho}$, and Bragg scattering keeps the c.m. displaced from the middle of the trap, and (ii) when there is a MI domain ($\tilde{\rho} = 4$), the c.m. barely displaces from its original position. These results are in agreement with the experiments reported by Stöferle *et al.* [4], and differ from the ones in the weakly interacting regime where for small initial trap displacements no shift of the c.m. position is observed after damping [8,9].

One interesting feature that appears with the formation of the MI can be seen in the inset in Fig. 3(b). Although n_k^m at $\tau = 0$ is small compared with n_k^m for smaller $\tilde{\rho}$, its value remains almost the same during the evolution of the system. The same occurs with w, the width of n_k, as seen in Fig. 4(b). The above result can be intuitively understood by the fact that almost no c.m. energy has been converted into excitation energy, which keeps the system in a state with n_k, and NO occupations, similar to the ones in the ground state. Something similar may occur in experiments in the overdamped regime, when the c.m. almost does not change from its initial position [7]. However, we should remark that we do obtain big changes of n_k^m, and w, [Figs. 2 and 3(b)] as a consequence of the damping in systems with no MI. This is in contrast with the behavior of w one infers from the constant cloud widths after time of flight (TOF) reported experimentally in Ref. [7]. The reason for this difference may be that TOF measurements, due to inter-

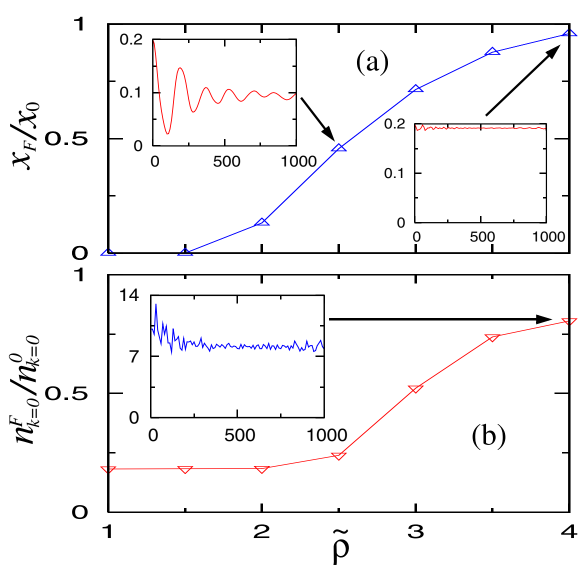

FIG. 3 (color online). (a) Ratio between the c.m. position after damping (x_F) and the initial trap displacement (x_0) vs $\tilde{\rho}$. The insets show $x_{\text{c.m.}}/R_0$ vs τ for the signaled $\tilde{\rho}$. (b) Ratio between $n_{k=0}$ after damping ($n_{k=0}^F$) and $n_{k=0}$ at $\tau = 0$ ($n_{k=0}^0$) vs $\tilde{\rho}$. The inset shows n_k^m vs $\tau = 0$ for the signaled $\tilde{\rho}$.

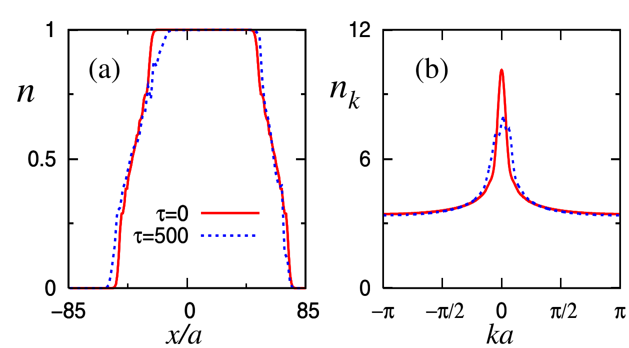

FIG. 4 (color online). Density and momentum profiles for a system with $N_b = 101$, and $\tilde{\rho} = 4$. See also results for $\tilde{\rho} = 4$ in the insets of Fig. 3.

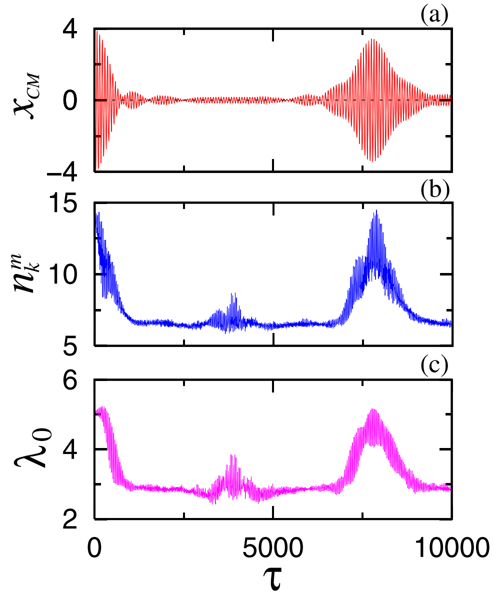

FIG. 5 (color online). Long time evolution of the c.m. position (a), maximum value of n_k (b), and the occupation of the lowest NO (c), vs time (τ, in units of $\hbar/t$). The initial trap displacement is $x_0 = 4a$ ($x_0/R_0 \sim 0.1$), and the system has $N_b = 20$, and $\tilde{\rho} = 1.0$.

particle interactions during the expansion, may not give the original n_k in the trap. (An explicit example was presented in Ref. [22].) If this is the case, other experimental techniques will be needed to determine the effects of the damping on n_k.

One final remark is in order concerning the damping discussed here. Since the experimental systems with ultracold quantum gases are almost closed systems, damping has a different meaning when compared with systems in contact with a reservoir, where the energy is truly dissipated. In the systems we have analyzed, the energy is *conserved*. In Fig. 5 we show a long time evolution of the c.m. oscillations, n_k^m, and the lowest NO occupation, after an initial displacement of the center of the trap. As seen in these figures, a revival of the c.m. oscillations occurs, with an increase of n_k^m and λ_0 to values similar to the ones at $\tau = 0$. This suggests that ultracold gases experiments could be used to study quantum Poincare recurrences [23]. The relation between long time dynamics in integrable systems, such as HCB analyzed here, and nonintegrable (finite on-site repulsion) models, is an important open question (of broad relevance to condensed matter physics) that needs further analysis.

In summary, we have presented a detailed, and exact, study of the damping of the c.m. oscillation in a system of strongly correlated bosons. We have shown that far from the Mott-insulating regime large damping occurs only for large initial displacements of the trap, and it reduces dramatically the maximum value of n_k and increases its full width. The damping also destroys the quasi-long-range order of the OPDM, reducing the occupation of the lowest NO, analogous to an increase in the temperature of the system. When the Mott insulator is present in the trap, overdamping occurs even for small initial displacements. In this case the c.m. remains very close to its initial position, and no big changes occur in n_k. Finally, we have shown that since the system is closed, after some time a revival of the c.m. oscillations occurs, and n_k^m and λ_0 also regain their initial values.

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Note added.—Complementary to this work, a related study has been recently presented in Ref. [24].

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