Fermi-Like Behavior of Weakly Vibrated Granular Matter

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(Received 13 January 2005; published 1 September 2005)

Vertical movement of zirconia-yttria stabilized 2 mm balls is measured by a laser facility at the surface of a vibrated 3D granular matter under gravity. Realizations $z(t)$ are measured from the top of the container by tuning the fluidized gap with a 1D measurement window in the direction of the gravity. The statistics obeys a Fermi-like configurational approach which is tested by the relation between the dispersions in amplitude and velocity. We introduce a generalized equipartition law to characterize the ensemble of particles which cannot be described in terms of a Brownian motion. The relation between global granular temperature and the external excitation frequency is established.

DOI: 10.1103/PhysRevLett.95.108003 PACS numbers: 45.70.−n, 05.40.−a, 47.50.+d, 81.05.Rm

A vibrated granular medium (GM) exhibits a wealth of intriguing physical properties [1–5]. Since energy is constantly being added to the system a nonequilibrium steady state (SS) is reached [6]. In [7] we have studied the spectrum properties of vibrated GM under gravity, and shown that in the weakly excited regime the dynamics of the fluidized particles cannot be described as *simple* Brownian particles, this fact leads us to the conclusion that in order to describe the cooperative dissipative dynamics of the GM particles, it must be done in terms of generalized Langevin particles [8,9].

Recently Hayakawa and Hong [10] introduced the approach of thermodynamics of a weakly excited granular matter, in particular, vibrated GM by mapping the nonequilibrium system with a "Fermion like" theory. Our experimental conditions allow us to consider a N-particles system of n rows in a cylindrical container as a 1D degenerate Fermi system. Two experiments based on a laser were set up to investigate the occupation dynamics at the fluidized gap of the n-row GM. The first one considers a realization $z(t)$ of one particle from the top of the container by tuning the fluidized gap with a 1D window in the gravity direction; see Fig. 1(a). It is clear that these realizations mainly correspond to *macroscopic* Fermi-like particles (MFLP) from near the "Fermi level". The second one concerns the measurement after a long integration time of the mass profile which corresponds to the Fermi-like profile; see Fig. 1(b).

By focusing on the configurational properties of an excluded volume theory, the SS mass profile can be understood in terms of a configurational maximum principle assumption. Excluded volume interactions of the GM do not allow two grains to occupy the same state (gravitational energy), thus the number of configurations is $W = \Pi_i[\Omega!/N_i!(\Omega - N_i)!]$. Following Landau, to study a nonequilibrium system, the maximization of $S = \ln W$ yields that the profile is $\phi(\epsilon) = [1 + Q\exp(\beta\epsilon)]^{-1}\Omega/mgD$ where m, D are the mass and diameter of our balls; β is a Lagrange multiplier parameter, $Q^{-1} = \exp(\beta\mu)$ the fugacity, and $\epsilon = mgDs$ with $s = 0, 1, 2, 3\ldots$. Introducing the normalization: $\int_0^\infty \phi(\epsilon)d\epsilon = N$, we get $\exp(N\beta mgD/\Omega) = 1 + \exp(\beta\mu)$. Therefore the *zero-point* "chemical potential" is $\mu_0 = mgDN/\Omega$, where N/Ω is the number of balls in an elementary *column* of diameter D. From these considerations it is trivial to see that without vibration the center of mass (c.m.) is characterized by $z_{\mathrm{c.m.}} = \mu_0/2mg \equiv h/2$. For vibrated GM the Lagrange parameter β is a nontrivial function of the velocity fluctuations. Before going ahead, let us denote $\phi(\epsilon)/N$ as the cumulative probability that the energy ϵ will be occupied in an *ideal* GM layer at the nonequilibrium SS characterized by the *global temperature* β^{-1}.

From $\phi(\epsilon)$ it is possible to calculate the c.m. expansion as a function of β, the mean energy per particle, its square dispersion σ_ϵ^2, etc., Many questions concerning the GM layer system are still open, in fact *the* stochastic motion of the fluidized particles is not entirely known [11]. For

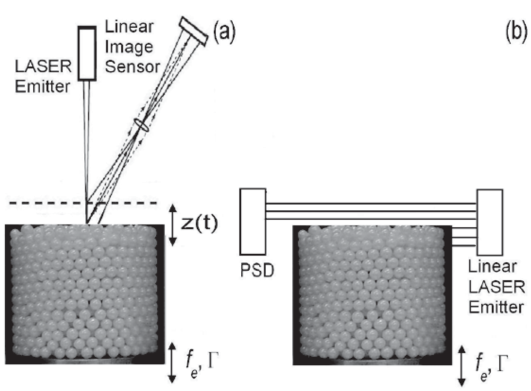

FIG. 1. Distance measurement [amplitude realizations $z(t)$] corresponding to 13 layer GM ($h = 21$ mm) with 1.99 mm $Z_rO_2 - Y_2O_3$ balls in a glass container of 30 mm diameter. Setup for (a) the single particle measurement and (b) the laser light barrier experiment.

example, it is important to test that the spectrum of the realization $S_z(f)$ does not behave as Brownian particles ($1/f^2$), but it has a more complex behavior, $1/f^\nu$, related to a cooperative dynamics [7,9]. Thus an exhaustive analysis of the realizations $z(t)$ of these *macroscopic* Fermi-like particles should be made. We have measured $z(t)$ and we calculate the Lagrange parameter β to show its complex relation to the kinetic energy.

Amplitude dispersion vs the velocity dispersion.—A sinusoidal vibration is driven by a vibration plate on the GM bed (with intensity $\Gamma = A\omega^2/g$, where A is the amplitude, g is the acceleration of gravity, and $\omega = 2\pi f_e$ the frequency of the plate). The vibration apparatus is set up by an electromagnetic shaker (TIRAVIB5212) which allows [12] for feedback through a piezoelectric accelerometer the control of f_e and Γ in the range of 10–7000 Hz, and 2–40g, respectively. The control loop is completed by an Oscillator Lab-works SC121 and a TIRA $19/z$ amplifier of 1 kw. The n-row GM bed setup was $Z_rO_2 - Y_2O_3$ balls with $D = 1.99$ mm and $m = 26.8 \pm 0.1$ mg, into a glass container of 30 mm of diameter with steel bottom; see Fig. 1(a). The experiments were carried out in a chamber at 1 atm of air with 5.8 ± 0.2 g/m^3 of water vapor. The absolute humidity was controlled by using a Peltier condenser and a control loop through a thermo-hygrometer. The humidity is of major relevance in order to control the particle-particle and particle-wall contact forces [7,12,13]. Under such humidity controlled conditions, no surface convection or convection rolls were observed in the GM, nor rotational movement of the bed with respect to the container, which is typical for a content of water vapor >10 g/m^3.

The $z(t)$ of one particle was followed in a window of 12 mm with a laser device by using a triangulation method; see Fig. 1(a). A laser emitter with a spot of 70 μm and a linear image sensor (CCD-like array) enables a high speed measurement with 100 μ sec sampling. The linear image sensing method measures the peak position values for the light spots and suppresses the perturbation of secondary peaks, which makes possible a resolution of 1 μm. The shaker and the laser displacement sensor were placed on vibration-isolated tables to isolate them from the external vibrations, and the displacement sensor from the experiment vibrations. The $z(t)$ is a measure of the variations of the distance (difference) between the particle and the sensor around the surface of the GM bed (fluidized gap). The measured $z(t)$ without excitation reveals a white noise <10 μm. Then our setup effective resolution is no higher than 10 μm. We have shown [7] that depending on the external excitation the $z(t)$ can show from quasi-nonerratic parabolas, for the movement under gravity, to realizations of larger rugosity.

The registers of $z(t)$ were taken with a 9354C Le Croy Oscilloscope of 500 MHz. The velocity $V(t) = dz/dt$ of the MFLP was calculated numerically for $\Delta t = 100$ μ sec from $z(t)$ registers. The dispersions $\sigma_z = \sqrt{\langle z(t)^2\rangle - \langle z(t)\rangle^2}$ and $\sigma_V^2 = \langle V(t)^2\rangle - \langle V(t)\rangle^2$ were obtained from a window of 2 s for each pair of registers $\{z(t), V(t)\}$. In Fig. 2(a) we report σ_z against σ_V^2 for fixed $\Gamma = 10, 20$ and several f_e from 60 to 180 Hz for GM beds of $h = 21$ and 12 mm.

For weakly excited GM the displacement of the fluidized particles, in the gap, can be studied from the profile $\phi(\epsilon)$. In fact, a nonequilibrium SS density $P(\epsilon = mgz)$, characterizing the motion of the fluidized MFLP, is sustained by the input of energy from the plate colliding periodically with the GM bed; i.e., a current of particles near μ_0—which is proportional to a gradient of $\phi(\epsilon)$—will be balanced by the random input of mass coming from the periodic movement of the plate. It is clear that $P(\epsilon)$ will be a narrow density around μ_0, so we characterize the movement of the MFLP at the Fermi-like sea by

$$P(\epsilon) \propto -\frac{d\phi(\epsilon)}{d\epsilon}, \quad \text{where } \epsilon = mgz = mgDs. \tag{1}$$

At the nonequilibrium SS the dispersion σ_z can be calculated from $P(\epsilon = mgz)$, but a rather simple and analytical expression for a characteristic length scale z^* can be obtained by solving ϵ^* from the following consideration

$$qP(\mu_0) = P(\mu_0 + \epsilon^*); \qquad 0 < q < 1, \tag{2}$$

where q is a "cumulant" parameter. If $P(\epsilon)$ were Gaussian the value $q = 1/\sqrt{e}$ would give the exact dispersion $\epsilon^* = +\sigma_\epsilon$. We have tested that our conclusions are not changed for values $q \sim 1/\sqrt{e}$. Using $\phi(\epsilon)$ in the expression for $P(\epsilon)$ we get for the characteristic scale ϵ^*

$$\exp\left(\frac{\beta\epsilon^*}{2}\right) = \frac{(2\mathcal{A} - 1) + \sqrt{(2\mathcal{A} - 1)^2 - 4\mathcal{A}q(\mathcal{A} - 1)}}{2\mathcal{A}\sqrt{q}}, \tag{3}$$

where $\mathcal{A} = e^{\beta\mu_0}$, then by putting $q \sim 1/\sqrt{e}$ in (3) it gives the amplitude dispersion $z^* = \epsilon^*/mg$ as a function of β. Now the task is to determine β as a function of the kinetic energy of the MFLP in the fluidized gap.

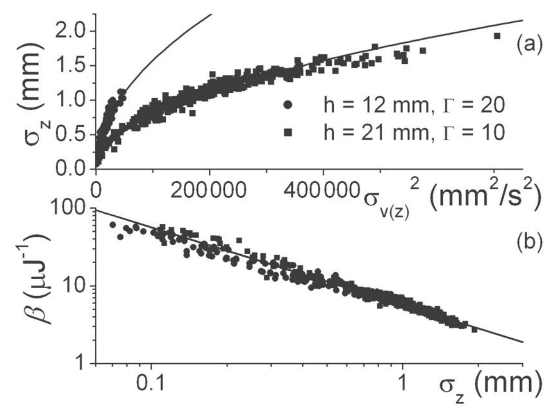

FIG. 2. (a) σ_z vs σ_V^2, and the corresponding fittings by using ϵ^* vs σ_V^2 solved from (6), or from (8) in the LT approximation. The excitation frequency was from 60–180 Hz. (b) Lagrange parameter β vs σ_z.

If collisions were elastic, in a 1D ideal gas the equipartition theorem says that total kinetic energy per particle is related to the Lagrange parameter by $m\sigma_V^2/2 = \beta^{-1}/2$. Our conjecture for a weakly excited GM is to generalize *the equipartition law* to

$$\frac{1}{2}m^*\sigma_V^2 = \frac{1}{2}\frac{\Delta N}{N}\beta^{-1}, \tag{4}$$

where $m^* = \delta m$ accounts for inelastic factors, and $\Delta N/N$ is a relative factor that counts the thermodynamically "active" MFLP in the fluidized gap. In fact, a *granular gas* since its non-Gaussian velocity distribution reveals an inelastic gas heated in a nonuniform way, with the expected high energy tail $e^{-\mathrm{constant}V^{3/2}}$.

The factor $\Delta N/N$ can be calculated from

$$\Delta N = \int_{\mu_0}^{\infty}\phi(\epsilon)d\epsilon = N\left(\frac{1}{\beta\mu_0}\ln[2e^{\beta\mu_0}-1]-1\right). \tag{5}$$

So the implicit equation to solve β is

$$\frac{1}{\beta}\left(\frac{1}{\beta\mu_0}\ln[2e^{\beta\mu_0}-1]-1\right) = \delta m\sigma_V^2. \tag{6}$$

Note that in the high temperature limit $\beta\mu_0 \ll 1$ and for the elastic case $\delta = 1$ we recover the equipartition theorem. This situation is just what we have found experimentally for one steel ball in a narrow glass cylinder [7]. In that experiment, when we compared the relation σ_z vs σ_V^2, we reduced the dissipation during the vibration and assured that there is no rotation of the ball during its movement $z(t)$, then from energetic considerations: $mg\sigma_z = \frac{1}{2}m\sigma_V^2$, predicting a line with slope $1/2g$. The opposite situation is in the limit $\beta\mu_0 \gg 1$; in this case we arrive at the *low temperature* (LT) scaling.

$$\beta^{-1} \simeq |\sigma_V|\sqrt{\delta m\mu_0/\ln 2}. \tag{7}$$

Because of the fact that dissipation and degrees of freedom are functions of the external parameters, we expect that the analysis of the complex behavior of vibrated GM will be enlightened from the study of $\sigma_z = \sigma_z(\sigma_V^2)$. Thus an important point would be to test experimentally our theoretical predictions. Noting that $z^* = \sigma_z = \epsilon^*/mg$ it is simple to see that a LT (3) gives

$$\beta \simeq 2\frac{\ln(1+\sqrt{1-q}) - \ln\sqrt{q}}{mg\sigma_z}, \tag{8}$$

then using (7) we arrive at

$$\sigma_z \simeq [\ln(1+\sqrt{1-q}) - \ln\sqrt{q}]\sqrt{\frac{4h\delta}{g\ln 2}}|\sigma_V|. \tag{9}$$

Thus we got an explicit LT formula $\sigma_z = \sigma_z(\sigma_V^2)$ as a function of the dissipative parameter δ, which in fact is a function of the external parameters Γ and f_e.

In Fig. 2(a) we report the measurement of σ_z against σ_V^2 for two experimental studies of a GM bed with $h = 21$ and 12 mm at $\Gamma = 10$ and 20, respectively, for several f_e from 60 to 180 Hz. In that figure we also show the fit with our theoretical prediction (resolved per least squares) showing a very good agreement for $\delta \sim 0.0009$ and $\delta \sim 0.0064$ ($\Gamma = 10$ and 20, respectively). In Fig. 2(b) we show the corresponding β against σ_z, where the $\{\beta\}$ data set was calculated from the $\{\sigma_V\}$ experimental data set for the two experimental studies, using (7). By considering the mass of the $Z_rO_2 - Y_2O_3$ ball and $q \sim 1/\sqrt{e}$, we represent in Fig. 2(b) the log-log plot of the Eq. (8), showing an excellent agreement between the experimental data and our theory. The two experimental data sets are on the same curve due to the fact that we use for the two experiments the same $Z_rO_2 - Y_2O_3$ balls.

Note that if all stochastic realizations could be understood in terms of a Brownian oscillating movement around μ_0, the $P(z)$ would correspond to $\exp(-z^2C\beta)$, with C a constant and β^{-1} proportional to the temperature. Then we would have obtained $\sigma_z \propto \beta^{-1/2}$ which is not the case reported experimentally in Fig. 2(b). Here we point out that in order to describe the spectrum of the fluidized particles we should use a non-Markovian description [7]. Unfortunately we still do not have a time-dependent statistical description for the MFLP.

Global temperature against the external excitation.— Equation (7) is the LT approximation of our generalized equipartition theorem for a GM experiment out of equilibrium. Now we would like to find a relation between the Lagrange parameter β and the external parameters characterizing the input of energy. The maximum kinetic energy, per particle, transferred by the oscillating plate must be proportional to the effective mass m^* and the dimensionless velocity $A\omega/\sqrt{gd}$, on the other hand the maximum potential energy related to the fluidized gap is proportional to the variation of the c.m. at the global temperature β^{-1}. Thus we get the relation

$$\frac{\delta}{2}\left(\frac{A\omega}{\sqrt{gd}}\right)^2 = \frac{mg\Delta z_{\mathrm{c.m.}}}{\mu_0}. \tag{10}$$

Where $\Delta z_{\mathrm{c.m.}} = U/N - \mu_0/2$, with $U = \int_0^\infty \epsilon\phi(\epsilon)d\epsilon$. At LT we get $\Delta z_{\mathrm{c.m.}} \simeq (\pi/\beta)^2/6\mu_0$, then

$$\beta^{-1} \simeq \frac{mh\sqrt{3g\delta/d}}{\pi}A\omega, \qquad h \equiv \mu_0/mg. \tag{11}$$

Using the relation $A\omega = \Gamma g/2\pi f_e$ we can transform (11) in terms of the variables that we have fixed in our experiment. From these considerations it is trivial to see that for a given intensity Γ and increasing frequency $f_e \to \infty$ the "temperature"$\to 0$, (i.e., $\sigma_z^2 \to 0$ and $z_{\mathrm{c.m.}} \to h/2$). In Fig. 3(a) we present the behavior of the global temperature as a function of the excitation frequency, showing an agreement with our theoretical prediction. In Fig. 3(a) a least squares fitting is also shown giving a slope of 0.06 $\mu\mathrm{J}^{-1}$ s, while (11) gives 0.09 $\mu\mathrm{J}^{-1}$ s. The dispersion in Fig. 3(a) is mainly introduced by the numerical calculation of dz/dt.

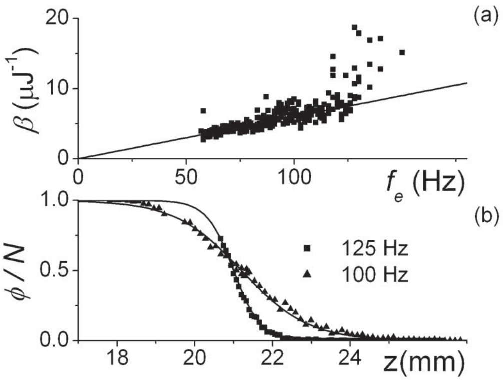

FIG. 3. Comparison of the two experiments for a GM bed of $h = 21$ mm under an acceleration $\Gamma = 10$. (a) the Lagrange parameter $\beta(f_e)$ by measuring the realizations $z(t)$ from the top of the container. (b) profile $\phi(z)/N$ integrated from the laser light barrier during 3 h corresponding to $\beta = 4.4 \pm 0.2\ \mu\mathrm{J}^{-1}$ for $f_e = 100$ Hz, and $\beta = 10.5 \pm 0.5\ \mu\mathrm{J}^{-1}$ for $f_e = 125$ Hz.

The nonequilibrium SS Fermi-like profile.—We have remarked that the profile $\phi(\epsilon)/N$ gives the probability that the energy ϵ will be occupied in an *ideal* GM layer at the nonequilibrium SS characterized by *the global temperature* β^{-1}. Note that $\phi(\epsilon = mgz)/N$ decreases monotonically with z from 1 to 0. In fact we can write $\phi(\epsilon)/N$ as a cumulative probability $\phi(\epsilon)/N = 1 - \int_0^\epsilon \psi(\epsilon')d\epsilon'$, and interpret the density $\psi(\epsilon)$ as associated to the fluidized gap. We write the SS mass profile as

$$\phi(z)/N = \frac{(\exp(\beta mgh) - 1)}{(\exp(\beta mgz) + \exp(\beta mgh) - 1)}. \qquad (12)$$

To measure the profile (12) we have implemented a second experiment on a GM bed ($h = 21$ mm) with a laser light barrier of 10 mm wide, Fig. 1(b). The voltage signal from the position sensitive detector runs from 0 to 10 V, which means a vertical window from 10 to 0 mm. We take measurements every 3 s during 3 h, where the normalized frequency count integrated from such a register is equal to the occupation number ϕ/N for $z \geq \mu_0/mg$ and to $1 - \phi/N$ for $z < \mu_0/mg$. At a fixed f_e and for $\Gamma = 10$ two registers were obtained for $f_e = 100$ Hz and $f_e = 125$ Hz, which were integrated and normalized to get the corresponding profile $\phi(z)/N$. In Fig. 3(b) we show the profile and our theoretical prediction (12). From this data we obtain the values $\beta = 4.4 \pm 0.2\ \mu\mathrm{J}^{-1}$ for $f_e = 100$ Hz, and $\beta = 10.5 \pm 0.5\ \mu\mathrm{J}^{-1}$ for $f_e = 125$ Hz, that we compare with the results of the first experiment Fig. 3(a). Not only is the agreement good, but this procedure also allows a self-consistent test.

Discussion.—In Fig. 2(a) we show the amplitude dispersion σ_z against the velocity squared dispersion σ_V^2 of the realizations $z(t)$. For weak amplitude the slope σ_V^2/σ_z shows a linear behavior and the departure from a linear behavior is a clear evidence of the complex behavior of the GM bed. This indicates that for this regime it is necessary to introduce a description in terms of our theory.

The corresponding global temperature $(k_B\beta)^{-1}$ for a fluidized gap to occur happens to be at $T = 9.2 \pm 0.5 \times 10^{15}$ K, which means $f_e = 130$ Hz for $\Gamma = 10$; see the *transition* in Fig. 3(a). Feitosa *et al.* measure for a dilute granular gas a range of temperatures of the order of $T \sim 500$ PK (petakelvin); see Fig. 5 of Ref. [5]. This range is higher than our measurements; however, it is in agreement with them since our corresponds to a weakly fluidized GM. From [7] we know that the movement of the MFLP when the fluidized gap appears can be approximated by a Brownian motion, but this description changes to a more complex stochastic behavior by decreasing f_e (for fixed Γ) when the temperature reaches $T \sim 15$ PK. This global temperature should be understood, indeed, as equivalent to an *order parameter* of the stochastic process in the energy configuration. We remark that around the fluidization transition where the stochastic dynamics start to apply dz/dt occurs with larger rugosity. Then a proper description in terms of differentiable realizations $z(t)$ is well defined in the weakly excited region ($T > 9$ PK) where the dynamics start to be non-Markovian. For lower temperatures ($T \lesssim 9$ PK) despite of the larger rugosity of realizations $z(t)$, the calculation of σ_V^2 from a time window of 2 s makes it reliable. This numerical calculation only introduces, for such range, a larger dispersion of the data in the curve of Fig. 2(b), but again in good agreement with Eq. (8).

J. E. F. thanks Professor Frank Mücklich, the von Humboldt Foundation, and ADEMAT for the support and Dr. Graciela Rodriguez for her clever suggestions. M. O. C. thanks the associated regime at the ICTP, and many interesting discussions with A. García Faure.

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