Transmission of Light through a Single Rectangular Hole

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We show that a single rectangular hole in a metallic film exhibits transmission resonances that appear near the cutoff wavelength of the hole waveguide. For light polarized with the electric field pointing along the hole's short axis, it is shown that the normalized-to-area transmittance at resonance is proportional to the ratio between the long and short sides, and to the dielectric constant inside the hole. Importantly, this resonant transmission process is accompanied by a huge enhancement of the electric field at both entrance and exit interfaces of the hole.

Since the pioneering work by Ebbesen *et al.* [1] reporting extraordinary optical transmission (EOT) through two-dimensional (2D) hole arrays perforated in optically thick silver films, the study of the transmission properties of subwavelength apertures has become a very active area of research in electromagnetism. Very recently, several experiments have focused on the influence of hole shape on the optical transmission properties of both 2D hole arrays [2–4] and single subwavelength holes [5]. These studies showed that transmission through a rectangular hole presents strong polarization dependencies and higher transmittance than square or circular holes with the same area. Interestingly, it was also found that single rectangular holes can support transmission resonances, even in the subwavelength regime.

In this Letter we present—to our knowledge—the first theoretical study about the dependence on hole shape of the transmittance through a single hole. We find that, as a difference with circular holes [6], single rectangular holes can exhibit strong transmission resonances. One of these resonances appears close to cutoff, with a peak transmittance controlled by the ratio between the long and short sides of the rectangle. Additionally, we show that the presence of a dielectric filling the hole greatly boosts the transmittance. Associated to these transmission resonances, there is a very strong enhancement of the electric field at the hole.

Figure 1 shows schematically the system under study: a rectangular hole of sides a_x and a_y perforated on a metallic film of thickness h. The system is illuminated by a p-polarized plane wave with wavelength λ, the in-plane component of the electric field pointing along the x direction. The metal is treated within the perfect conductor approximation (PCA), so our results have quantitative value in the THz or microwave frequency regimes. In the optical regime PCA is approximate, failing when the dimensions of the structure are of the order of (or smaller than) the skin depth [7]. Even in this case, the range of validity of PCA can be greatly extended by simply considering effective hole dimensions enlarged by the (metal and

wavelength dependent) skin depth [8]. What PCA does not capture are effects related to absorption and surface plasmons. However, PCA serves both as the starting point for more elaborated approximations (as the one considering surface impedance boundary conditions) and for clarifying which effects are due to geometry and which have a dielectric origin. With these caveats in mind, the results presented in this Letter apply to different frequency regimes, by simply rescaling all lengths by the same factor.

Let us briefly describe the formalism used for calculating the transmittance through the structure (a detailed account of this method which was developed in order to treat an arbitrary number of indentations, can be found in [9]). In this method, electromagnetic (EM) fields in both reflection (I) and transmission (III) vacuum regions are expressed in terms of the EM eigenmodes $|\vec{k}\sigma>$, characterized by the in-plane component of the wave vector $\vec{k}$, and the polarization σ. Inside the hole, the EM field is expanded in terms of all EM waveguide eigenmodes. After matching appropriately the EM fields at the two interfaces ($z = 0$ and $z = h$), the formalism provides the full EM
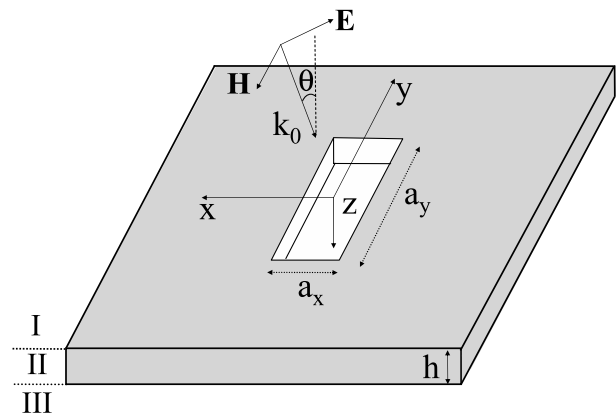

FIG. 1. Diagram of a single rectangular hole of sides a_x and a_y perforated on a metal film of thickness h. The structure is illuminated by a p-polarized plane wave with its angle of incidence with respect to the normal being θ.

field in all spatial points as a function of the projection onto waveguide eigenmodes of the electric field at both hole entrance and exit interfaces. In all calculations a_x, $a_y < \lambda$ and we have checked that considering just the first TE eigenmode ($|\text{TE}\rangle$) is enough to obtain very accurate results for the transmittance so, for simplicity, we present our formalism just for this case. In this way, the electric field bivector $\vec{E} = (E_x, E_y)^T$ (T standing for transposition) at the hole entrance and exit can be written in terms of the modal amplitudes E and E' as $|\vec{E}(z = 0)\rangle = E|\text{TE}\rangle$ and $|\vec{E}(z = h)\rangle = -E'|\text{TE}\rangle$, respectively. Here we have used Dirac's notation, with the wave field in real space of the first $|\text{TE}\rangle$ mode, $\langle\vec{r}|\text{TE}\rangle$, written as $(1, 0)^T \sin[\pi(y/a_y + 1/2)]/\sqrt{\mathcal{N}}$, $\mathcal{N} = a_x a_y/2$ being a normalization factor. The equations that E and E' must satisfy are [10]

$$(G - \Sigma)E - G_V E' = I_0, \quad -G_V E + (G - \Sigma)E' = 0, \quad (1)$$

where I_0 takes into account the external illumination. Normalizing the incident EM field ($|\vec{k}_0 p\rangle$) such that the incoming energy flux over the hole area is unity, we obtain

$$I_0 \equiv 2i Y_{\vec{k}_0 p}\langle\vec{k}_0 p|\text{TE}\rangle = \frac{4\sqrt{2}}{i\pi} \frac{\text{sinc}[k_\omega a_x \sin\theta/2]}{\sqrt{\cos\theta}}, \quad (2)$$

where θ is the angle of incidence and $k_\omega = 2\pi/\lambda$ [11]. In Eq. (1), Σ and G_V are magnitudes which only depend on the characteristics of the TE mode inside the hole: $\Sigma = Y_{\text{TE}}/\tan(q_z h)$ and $G_V = Y_{\text{TE}}/\sin(q_z h)$; $q_z = \sqrt{k_\omega^2 - (\pi/a_y)^2}$ is the propagation constant of the fundamental TE mode and $Y_{\text{TE}} = q_z/k_\omega$ its admittance.

The self-illumination of the hole, via vacuum modes, is controlled by $G = [i/(2\pi)^2]\Sigma_\sigma \int d\vec{k} Y_{\vec{k}\sigma}|\langle\text{TE}|\vec{k}\sigma\rangle|^2$. For the case of rectangular holes,

$$G = \frac{ia_x a_y}{8\pi^2 k_\omega}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} dk_x dk_y \frac{k_\omega^2 - k_y^2}{\sqrt{k_\omega^2 - k^2}}\text{sinc}^2\left(\frac{k_x a_x}{2}\right)$$
$$\times \left[\text{sinc}\left(\frac{k_y a_y + \pi}{2}\right) + \text{sinc}\left(\frac{k_y a_y - \pi}{2}\right)\right]^2, \quad (3)$$

where $k^2 = k_x^2 + k_y^2$. Notice that, in our formulation, $\text{Re}(G)$ comes from the coupling to evanescent modes in vacuum and $\text{Im}(G)$ from the radiative modes. The z component of the Poynting vector inside the hole can be expressed as a function of E and E' yielding to a normalized-to-area transmittance $T = G_V \text{Im}[E^* E']$.

Figure 2 renders $T(\lambda)$ for normal incident radiation for the case $h = a_y/3$ and several values of the ratio a_y/a_x. As clearly shown in this figure, a transmission peak develops at approximately $\lambda_c = 2a_y$, with increasing maximum transmittance and decreasing linewidth as a_y/a_x increases. In the case of square or circular holes there is also a resonance close to cutoff, but a very faint one (see inset of Fig. 2). In these last two cases, below cutoff the normalized-to-area maximum transmittance is of the order
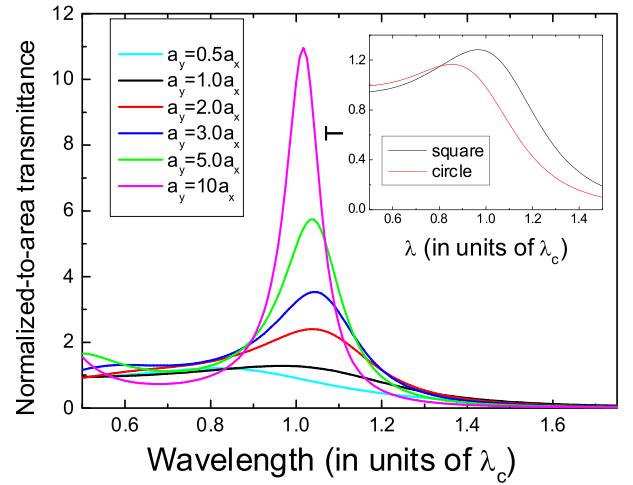

FIG. 2 (color online).   Normalized-to-area transmittance (T) versus wavelength (in units of the cutoff wavelength, $\lambda_c = 2a_y$), for a normal incident plane wave impinging on a rectangular hole, for different ratios a_y/a_x. Metal thickness is $h = a_y/3$. For comparison, the inset shows T versus wavelength for a single square (black line) and circular (red line) holes.

of 1, i.e., approximately the amount of light that is directly impinging on the hole. In all cases, above cutoff T decreases strongly with λ, due to the fact that the fields inside the hole are evanescent and that, in the extreme subwavelength regime, an incident wave couples very poorly to the hole [12].

It is interesting to comment here on the dependence of the transmission on the angle of incidence. In the set of Eqs. (1), the only term that depends on θ is I_0. Then, the location of the resonant peaks observed in Fig. 2 does not shift when θ is increased. Moreover, in the subwavelength limit, the term sinc in I_0 approaches 1 yielding to a simple $1/\cos\theta$ dependence for the peak heights in the normalized-to-area transmittance spectra. Note that this means that if instead of considering unit incoming energy flux, it is the total intensity of the impinging light that is fixed, we predict that the transmittance spectra would be almost independent of θ.

The resonant characteristics of the transmittance through rectangular holes, and their dependence on geometrical parameters can be worked out analytically from the set of Eqs. (1). For the case we are analyzing (a symmetric structure with respect to the plane $z = h/2$), maximum transmission appears when the electromagnetic energy at the entrance and exit sides of the aperture are equal, i.e., $|E| = |E'|$. From (1), this occurs when $|G - \Sigma| = |G_V|$, a condition that, after some algebra, implies

$$2\,\text{Re}(G) = \frac{|G|^2 - Y_{\text{TE}}^2}{Y_{\text{TE}}}\tan(q_z h). \quad (4)$$

There are several wavelengths at which this transcendent equation is satisfied. Ignoring the shift in the spectral dependence due to the EM coupling to vacuum modes (this is, setting $G \to 0$, which is the appropriate limit for $a_x, a_y \ll \lambda$), Eq. (4) transforms into $Y_{TE} \tan(q_z h) = 0$, which is the usual Fabry-Perot condition for the existence of a standing wave inside the hole. Note that this last equation allows the solution $q_z = 0$, so a transmission peak located at around the cutoff wavelength is expected, irrespective of the geometrical parameters a_y/a_x (see Fig. 2) and h. Equation (4) also predicts the emergence of transmission resonances appearing at $\lambda < \lambda_c$ as the depth of the hole is increased. These are canonical Fabry-Perot resonances ($q_z \neq 0$), similar to those found in subwavelength 1D slits [13–15].

Using the resonance condition [Eq. (4)], we obtain that the normal-incidence T at resonance, T_{res} is given by

$$T_{res} = \frac{|I_0|^2}{4 \, \mathrm{Im}(G)}. \qquad (5)$$

A very accurate analytical approximation to T_{res} can be obtained realizing that, in the extreme subwavelength limit ($a_x, a_y \ll \lambda$), Eq. (3) gives $\mathrm{Im}(G) \approx 32 a_x a_y / (3\pi\lambda^2)$. We have checked that this expression holds even for $a_y \approx \lambda/2$ therefore, for $\lambda > 2a_y$ we find

$$T_{res} \approx \frac{3}{4\pi} \frac{\lambda_{res}^2}{a_x a_y}. \qquad (6)$$

Recalling that T is the normalized-to-area transmission, this expression implies that the *total* amount of light emerging from a rectangular hole is, at least for the resonance appearing close to cutoff, independent of the length of the short side. Although derived for rectangular holes, Eq. (6) seems to be more general as the same expression was found for circular holes [16], with the term $a_x a_y$ replaced by the area of the circular hole. The important point in rectangular holes is that, for the polarization chosen, the transmittance peak appearing at cutoff only depends on the long side ($\lambda_{res} = 2a_y$), resulting in a transmittance $T_{res} \approx (3/\pi) a_y/a_x$ close to cutoff. This is the main result of this Letter, as it predicts a huge transmission enhancement in a *single* rectangular hole with large aspect ratio.

Additionally, even for a fixed aspect ratio a_y/a_x, Eq. (6) gives us a clue for further enhancing the transmission: namely, filling the hole with a material with dielectric constant $\epsilon > 1$, as this increases the cutoff wavelength. In fact, in the definition of quantities appearing in Eq. (1), the only place in which ϵ enters is in the propagation constant associated to mode $|TE\rangle$ which now reads $q_z = \sqrt{\epsilon k_\omega^2 - (\pi/a_y)^2}$. Therefore, the spectral position of resonances depend on q_z (and therefore on ϵ) but the transmittance at resonance is still given by Eq. (5). As a result, filling the hole with a dielectric would redshift the

transmission peak appearing close to cutoff to $\approx 2\sqrt{\epsilon} a_y$ and, more importantly, increase its transmittance. This is illustrated in Fig. 3, which renders the transmission spectra for rectangular holes of aspect ratio $a_y/a_x = 10$, in a metallic film of thickness $h = a_y/3$, for several values of ϵ. Note that this way of increasing the transmission through the hole by filling it with material with $\epsilon > 1$ can be also operative for the case of circular [17] or square holes. Remarkably, in rectangular holes this mechanism acts almost independently of the enhancement due to the aspect ratio, so T_{res} is proportional to both a_y/a_x and ϵ.

Associated to this resonant phenomenon, there is an enhancement of the EM fields. Naively, one would expect that the intensity of the E field at the entrance and exit sides of the hole ($|E|^2$ and $|E'|^2$) should be proportional to the transmittance. However, the direct evaluation of $|E|^2 = |E'|^2$ at the resonant condition given by Eq. (4) yields

$$|E|_{res}^2 = |E'|_{res}^2 = \frac{|I_0|^2}{4[\mathrm{Im}(G)]^2}, \qquad (7)$$

leading to an enhancement of the intensity of the E field (with respect to the incident one) that scales with λ_{res} as $\lambda_{res}^4/(a_y a_x)^2$, much larger than the enhancement in the transmittance (see inset of Fig. 3). This implies that in the process of resonant transmission, light is highly concentrated on the entrance and exit sides of the hole but only a small fraction of this light is finally transmitted. This finding opens the possibility of using rectangular holes for spectroscopic purposes and for exploring nonlinear effects.

In order to illustrate this E-field enhancement, we plot in Fig. 4 the amplitude of the E field at two different planes ($y = 0$ top panel and $z = 0^-$ bottom panel) for the case
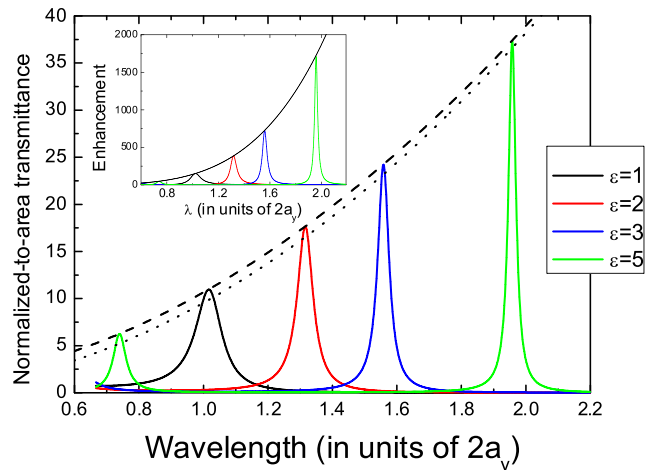

FIG. 3 (color online). T for a normal incident plane wave versus wavelength for a rectangular hole with $a_y/a_x = 10$ and different values of ϵ inside the hole. Metal thickness is $h = a_y/3$. Dashed and dotted lines show the behavior of Eqs. (5) and (6), respectively. Inset: enhancement of the E-field intensity obtained for the previous cases; black curve renders Eq. (7).
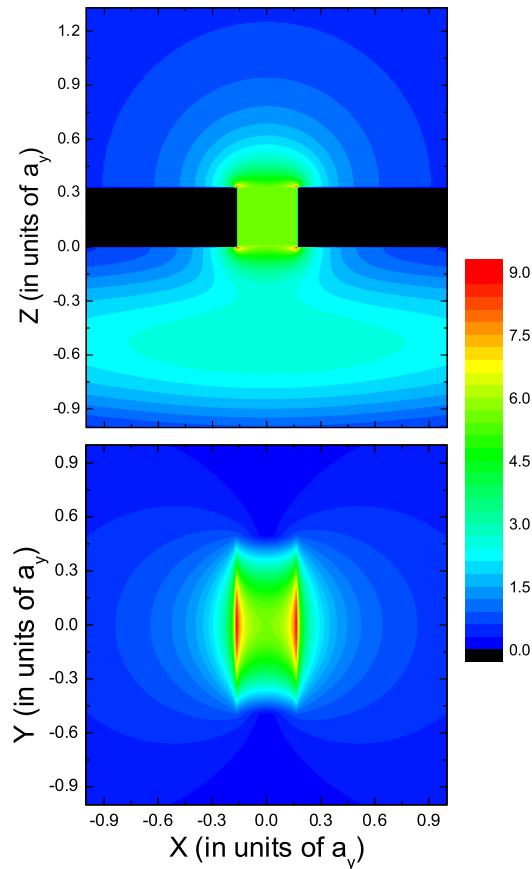
FIG. 4 (color online). Enhancement of the E-field amplitude with respect to the incident one, for a rectangular hole with $a_y/a_x = 3$ and $h = a_y/3$ at two different planes: cut through the center of the rectangle ($y = 0$, top panel; wave impinging from the bottom) and entrance surface ($z = 0^-$, bottom panel). In both cases, the E-field amplitude is evaluated at the resonant wavelength.

$a_y = 3a_x$ and $h = a_y/3$ (blue curve in Fig. 2) evaluated at the resonant wavelength. The cut through $y = 0$ shows that the E field amplitude is practically constant inside the hole, as corresponds to the excitation of a standing wave with zero propagation constant. On the other hand, the pattern of the E-field amplitude at the entrance surface clearly reveals the localized character of the resonance and its dipolar nature. This figure also shows that the E-field intensity maxima are at the ridges of the hole. An analysis of the vector components shows that inside the hole only the x component of the E field is present, whereas at the ridges there is an additional enhancement coming from the z component.

Some comments on the comparison of our results with published experimental data in the optical regime [5] are pertinent. Our theoretical results agree with the phenomenology found in these experiments (increase of transmit-

tance at resonance with increased aspect ratio and field enhancement localized at the ridges of the hole), which indicates that the localized modes analyzed in this Letter capture the basic ingredients of the transmission resonances observed in the optical regime for single holes. However, the authors of Ref. [5] attributed their experimental results to the excitation of localized surface plasmons. As explained before, surface plasmons are not included within our approach. Therefore, the role played by these modes in the optical regime is a point that deserves further theoretical investigation.

Additionally to its interest in the optical regime, these effects (huge enhancements of both transmission and E-field intensity) should be readily observable in the microwave or THz frequency ranges, which present the advantage that holes with very large aspect ratio can be manufactured and, furthermore, dielectric materials presenting large positive dielectric functions are available.

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[10] Note that although we work with the E-field amplitudes at both entrance and exit sides of the hole (E and E'), retardation is automatically included in our formalism.
[11] $Y_{\tilde{k}s} = k_z/k_\omega$ and $Y_{\tilde{k}p} = k_\omega/k_z$ (for s and p polarization, respectively) where $|k|^2 + k_z^2 = k_\omega^2$.
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