Superconducting π Qubit with a Ferromagnetic Josephson Junction

T. Yamashita,[1] K. Tanikawa,[1] S. Takahashi,[1] and S. Maekawa[1,2]

[1]*Institute for Materials Research, Tohoku University, Sendai, Miyagi, 980-8577, Japan*
[2]*CREST, Japan Science and Technology Agency (JST), Kawaguchi, Saitama, 332-0012, Japan*

Solid-state qubits have the potential for the large-scale integration and for the flexibility of layout for quantum computing. However, their short decoherence time due to the coupling to the environment remains an important problem to be overcome. We propose a new superconducting qubit which incorporates a spin-electronic device: the qubit consists of a superconducting ring with a ferromagnetic π junction which has a metallic contact and a normal Josephson junction with an insulating barrier. Thus, a quantum coherent two-level state is formed without an external magnetic field. This feature and the simple structure of the qubit make it possible to reduce its size leading to a long decoherence time.

The quantum computer is an innovative device in the sense that it would make it possible to solve problems which require unrealistically long computation times on a classical computer [1]. In the quantum computer, the information is stored in a basic element called the qubit, which is a quantum coherent two-level system. The superposition of the two-level state is utilized in the process of quantum computing. For the physical realization of the qubit, various systems have been proposed, e.g., ion traps, nuclear spins, and photons. Among the proposals, solid-state devices have the advantage of large-scale integration and flexibility of layout. On the other hand, a challenging problem for the solid-state qubits is the reduction of the decoherence effect, since the solid states qubits in general have a short decoherence time due to their coupling to the environment. In recent years, several qubits based on the Josephson effect have been proposed [2–15]. One of the proposals involves a Cooper-pair box type of qubit [2,3]. In this case, quantum oscillations between the quantum two-level states (Rabi oscillations) have been detected [2], and the operation of coupled two qubits has been demonstrated [3]. Another example is a flux qubit which uses the superconducting phase. For this proposal, a circuit with a single and relatively large Josephson junction has been demonstrated [4]. Mooij *et al.* have also proposed a flux qubit which consists of a superconducting loop with three Josephson junctions [5–12]. In this qubit, degenerate double minima form in the superconducting phase space when an external magnetic field, which corresponds to the half of the unit magnetic flux, is applied. The bonding and antibonding states which form due to the tunneling between these two states can be used as the two quantum coherent states. Experimentally, microwave-induced transitions between the two quantum states, a coupling between two qubits, and entangled states, have been observed for this qubit [5–11]. Another proposal is a qubit which does not require an external magnetic field and uses an *s*-wave/*d*-wave/*s*-wave (*sds*) superconducting junction [13]. In addition, a five-junction device with one ferromag-

netic π junction and four normal Josephson junctions has been discussed in analogy with the *sds* qubit [14].

Recent advances in microprocessing techniques have yielded a variety of spin-electronic devices [16]. For instance, in ferromagnet/superconductor (FM/SC) junctions, novel phenomena due to the competition between ferromagnetism and superconductivity are expected [17,18]. There have also been extensive theoretical studies of Josephson π junctions consisting of a ferromagnet sandwiched between two superconductors (SC/FM/SC) [19–22]. In this respect, several experimental observations of the π state have been reported [23–26]. At the interface between an SC and an FM, Cooper pairs penetrating into the FM have a finite momentum $Q \propto h_{\mathrm{ex}}/v_F$, where v_F is the Fermi velocity, because of the exchange splitting h_{ex} between the up and the down spin bands [22]. Consequently, the superconducting order parameter ψ oscillates as $\psi \propto \cos(2Qx)$ along the direction x which is perpendicular to the interface. In SC/FM/SC junctions, the order parameters in the two SC's take different signs, when the thickness of the FM is about half of the period of the oscillation. This causes the current-phase relation to be shifted by π from that of a normal Josephson junction. This is the so-called π junction [19–26].

In this Letter, we study a qubit which consists of a superconducting ring with an insulator and a ferromagnet as shown in Fig. 1. In the ring, the superconductor/insulator/superconductor (SC/I/SC) junction is a normal Josephson junction with Josephson energy $U_0 = -E_0 \cos\theta$. Here, E_0 is the coupling constant in the SC/I/SC junction and θ is the phase difference between the SC's. On the other hand, the SC/FM/SC junction is a metallic π junction. In order to realize this qubit, it is required to fabricate a clean π junction with a nonsinusoidal current-phase relation. Before starting the discussion on the qubit, it is useful to first derive the Josephson energy U_π of the metallic π junction by using the Bogoliubov–de Gennes (BdG) equation [27]. Solving the BdG equation, we obtain the Andreev bound state energy E_σ for spin σ [28]. The
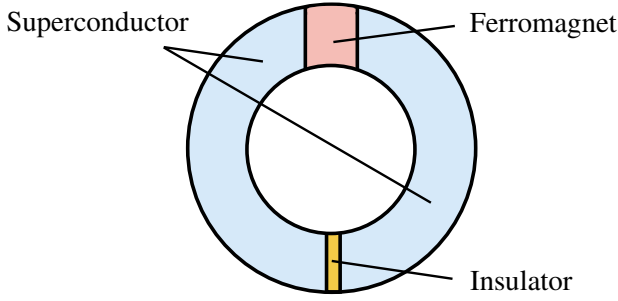
FIG. 1 (color online).   Schematic diagram of a superconducting
ring with an insulator and a ferromagnet. The superconductor/
insulator/superconductor (SC/I/SC) junction is a 0 junction (a nor-
mal Josephson junction), and the superconductor/ferromagnet/
superconductor (SC/FM/SC) junction is a metallic π junction.

phase, θ, dependent part of the free energy in the SC/FM/
SC junction is then expressed in terms of E_σ as

$$F = -k_B T \sum_\sigma \sum_{E_\sigma > 0} \ln[2\cosh(E_\sigma/2k_B T)]. \qquad (1)$$

As usual, T is the temperature and k_B is the Boltzmann
constant. At low temperatures, the free energy [Eq. (1)]
reduces to $F = -(1/2)\Sigma_\sigma \Sigma_{E_\sigma>0} E_\sigma$.

Figures 2(a) and 2(b) show the θ dependence of the free
energy F and of the Josephson current $I = (2e/\hbar)(\partial F/\partial\theta)$
in the SC/FM/SC junction, respectively. Here, results are
presented for two different values of the interfacial barrier
$Z = mV/\hbar^2 k_F$, where V is the strength of the δ-function
type of potential at the interfaces, m is the electron mass,
and k_F is the Fermi wave number [28]. The solid and the
dashed curves in Figs. 2(a) and 2(b) correspond to the cases
of $Z = 0$ and 3, respectively. In obtaining these results, we
have assumed that the exchange field is $h_{ex} = 0.31\mu_F$,
where μ_F is the Fermi energy, the thickness of the FM is
$L = 10/k_F$, and the coherence length at zero temperature
is $\xi_0 = 1000/k_F$. As shown in Fig. 2(a), F has a minimum
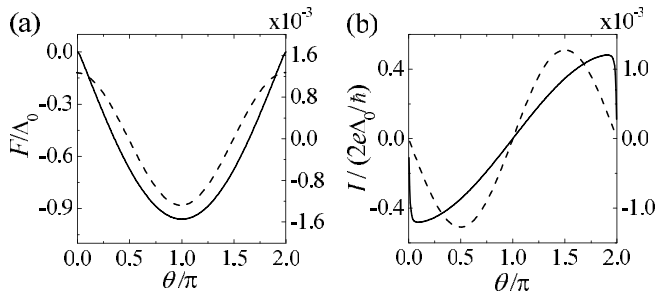at $\theta = \pi$ (π junction), and the variation of F with θ is

FIG. 2.   (a) Free energy F as a function of θ, and (b) current-
phase relation in a SC/FM/SC junction for $h_{ex} = 0.31\mu_F$. The
solid and the dashed lines are for the metallic contact ($Z = 0$)
and the tunnel junction ($Z = 3$), respectively, and Δ_0 is the
superconducting gap at zero temperature. In these figures, the
left vertical axes are for $Z = 0$ and the right vertical axes are for
$Z = 3$. In addition, in (a) the origin of the vertical axes is
arbitrary.

strongly dependent on Z. In the tunnel junction for $Z = 3$,
we have $F \approx -E_\pi \cos(\theta + \pi)$, where E_π is the Josephson
coupling constant. This gives the current with the π-shifted
sinusoidal form which is shown in Fig. 2(b) and described
by $I \approx I_\pi \sin(\theta + \pi)$ with $I_\pi = 2eE_\pi/\hbar$ the critical cur-
rent. On the other hand, in the metallic contact ($Z = 0$),
we have approximately $F \approx -E_\pi|\cos[(\theta + \pi)/2]|$. This
leads to a current with the nonsinusoidal form $I \approx
-(I_\pi/2)\sin[(\theta + \pi)/2]$ for $0 \le \theta \le 2\pi$. The solid curve
in Fig. 2(b) shows that there are deviations from this form
when θ is near 0 or 2π. However, these deviations do not
affect the following discussion, and the approximate rela-
tions given here are valid for the cases of $0.28\mu_F \lesssim h_{ex} \lesssim
0.34\mu_F$ and $0.79\mu_F \lesssim h_{ex} \lesssim 0.87\mu_F$. In addition, it can
be shown that these relations are valid for more realistic
cases with small but finite Z [29]. Therefore, if we choose
the appropriate values for h_{ex}/μ_F and $k_F L$, this form for
the Josephson energy of the metallic π junction is a good
approximation, and in the following it will be used for the
Josephson energy of the metallic π junction in the super-
conducting ring.

Now, we discuss the superconducting qubit which is
shown in Fig. 1. In the ring, the SC/I/SC (0 junction) and
the SC/FM/SC (π junction) junctions have the Josephson
energies $U_0 = -E_0 \cos\theta_0$ and $U_\pi = -E_\pi|\cos[(\theta_\pi +
\pi)/2]|$, respectively. Here, $E_{0(\pi)}$ is the coupling constant
in the 0 (π) junction. The superconducting phase differ-
ence is θ_0 for the 0 junction, and θ_π for the π junction. In
this case, the total flux in the ring Φ satisfies the relation
$\theta_\pi - \theta_0 = 2\pi\Phi/\Phi_0 - 2\pi l$, where $\Phi_0 = h/2e$ is the unit
flux and l is an integer. The total Hamiltonian of the ring
is expressed as $H = K + U_0 + U_\pi + U_L$, where $K =
-4E_c(\partial^2/\partial\theta_0^2)$ is the electrostatic energy, $E_c = e^2/2C$ is
the Coulomb energy for a single charge, C is the capaci-
tance of the 0 junction, and U_L is the magnetic energy
stored in the ring. Here, the electrostatic energy in the π
junction is neglected. The magnetic energy U_L is given by
$U_L = (\Phi - \Phi_{ext})^2/2L_s$, where L_s is the self-inductance of
the ring and Φ_{ext} is the external magnetic flux. The total
Hamiltonian H is analogous to that describing the motion
of a particle with kinetic energy K and in a potential $U_{tot} =
U_0 + U_\pi + U_L$. Using the relation between the phase and
the total flux, the potential U_{tot} becomes a function of θ_π
and Φ: $U_{tot} = U_{tot}(\theta_\pi, \Phi)$. In order to obtain the state of
the ring, we seek the solution at which U_{tot} is minimum.
First, we minimize U_{tot} with respect to Φ, i.e., $\partial U_{tot}/\partial\Phi =
0$, which yields $\Phi(\theta_0) = \beta(\Phi_0/2\pi)\sin\theta_0 + \Phi_{ext}$, where
$\beta = 2\pi I_0 L_s/\Phi_0$ and $I_0 = 2eE_0/\hbar$ is the critical current in
the 0 junction. Substituting this equation in the expression
for U_{tot}, we obtain

$$U_{tot}/E_0 = -\cos\theta_0 + \frac{\beta}{2}\sin^2\theta_0$$
$$- \alpha\left|\cos\left(\frac{\theta_0 + \pi}{2} + \frac{\beta}{2}\sin\theta_0 + \pi\frac{\Phi_{ext}}{\Phi_0}\right)\right|, \quad (2)$$

where $\alpha = E_\pi/E_0$.

Figure 3(a) shows the θ_0 dependence of the normalized U_{tot} for various values of α when there is no external magnetic field ($\Phi_{\text{ext}} = 0$). Here, we have taken $L_s = 2.0$ pH for the ring with a diameter of 2 μm and $I_0 = 500$ nA, which leads to $\beta \approx 3.1 \times 10^{-3}$ [5,8]. As shown in Fig. 3(a), U_{tot} has double minima located at $\theta_0 \approx \pi/2$ and $3\pi/2$, and the barrier height between the two degenerate states, $|\uparrow\rangle$ ($\theta_0 \approx \pi/2$) and $|\downarrow\rangle$ ($\theta_0 \approx 3\pi/2$), is controlled by α. The value of α can be adjusted by changing the thickness and the area of the insulating barrier or of the ferromagnet. For $|\uparrow\rangle$ and $|\downarrow\rangle$ states, currents I of magnitude $\lesssim I_0$ flow in the clockwise and counterclockwise directions, respectively, inducing flux $\Phi = L_s I \approx \pm 4.8 \times 10^{-4}\Phi_0$. Because of the quantum tunneling between $|\uparrow\rangle$ and $|\downarrow\rangle$, the bonding $|0\rangle \propto |\uparrow\rangle + |\downarrow\rangle$ and the antibonding $|1\rangle \propto |\uparrow\rangle - |\downarrow\rangle$ states are formed, hence yielding a two-level quantum system. For an Al_2O_3 insulator with junction area 0.1 μm^2 and thickness 1 nm, it is formed that $E_c \approx 1.7 \times 10^{-24}$ J, and $E_0/E_c \approx 96$. In this case, from numerical calculations for $\alpha = 3$, we estimate that the energy gap ΔE between the ground state $|0\rangle$ and the first exited state $|1\rangle$ is $\Delta E \approx 3.3 \times 10^{-24}$ J, which corresponds to a frequency of $\Delta E/h \approx 5.0$ GHz. Microwave absorption measurements can be used to confirm this two-level quantum state. Figure 3(b) shows the θ_0 dependence of the normalized U_{tot} for $\alpha = 3$ within external magnetic fields. This figure shows that the degeneracy of the states $|\uparrow\rangle$ and $|\downarrow\rangle$ is lifted by applying a small external magnetic field to the ring. For $\Phi_{\text{ext}} = +(-)0.01\Phi_0$, the component of $|\uparrow\rangle$ ($|\downarrow\rangle$) is larger than that of $|\downarrow\rangle$ ($|\uparrow\rangle$) in the ground state $|0\rangle$, and vice versa in the first excited state $|1\rangle$. Within the presence of the larger external magnetic field ($\Phi_{\text{ext}} = \pm 0.05\Phi_0$), the double-well potential disappears and the ground state is either $|\uparrow\rangle$ or $|\downarrow\rangle$. Therefore, finite currents flow in opposite directions for $|0\rangle$ and $|1\rangle$ states, when there is a finite external magnetic field. As a result of this, it is possible to distinguish the $|0\rangle$ and $|1\rangle$ states by measuring the current flowing in the ring with a superconducting quantum interference device (SQUID) placed around the ring. Our qubit has coherent states which require no exter-

nal bias magnetic field; thus only a small external magnetic field is needed for the manipulation and the detection of the states, as compared to the half unit flux $\Phi_0/2$ required in the proposal of Ref. [5]. For instance, even if the dimension of the qubit is several 100 nm's, only magnetic fields of the order of a millitesla are needed for the manipulation of our qubit. This feature enables us to make qubits with smaller size which is advantageous in large-scale integration. This type of qubit is also resistant to external noise and has longer decoherence time [5]. Here we discuss the dissipation in the metallic π junction. In the small metallic π junction, the discrete Andreev bound states are formed. For $L \ll \xi_0$, only one Andreev bound state exists. The Andreev bound state energy is $E_\sigma \approx 0$ (gapless) for $\theta_\pi = 0$ and 2π, and therefore all quantum transitions and qubit operations should avoid these phases at which noise due to thermally excited quasiparticles will increase [30]. As discussed above, in the superconducting ring, the quantum tunneling occurs between $\theta_0 \approx \theta_\pi \approx \pi/2$ and $\approx 3\pi/2$ (Fig. 3). In this phase region, the Andreev bound state energy is in the region of $0.75\Delta \lesssim E_\sigma \lesssim \Delta$, where Δ is the superconducting gap. This indicates that the metallic π junction is well gapped in the phase region where the quantum tunneling occurs. Therefore, the quasiparticle tunneling which can cause the dissipation is strongly suppressed at low temperatures and low voltages. In order to realize the universal quantum logic gates, a controlled-NOT gate is needed in addition to the single qubit rotations discussed above. Using our qubit, the controlled-NOT gate is realized in the following way: when two qubits A and B have an inductive coupling, the energy gap in qubit A depends on the state of qubit B. In other words, the energy gap for qubit A is ΔE_{A0} or ΔE_{A1} when qubit B is in state $|0\rangle$ or $|1\rangle$, respectively. Now, if a microwave pulse with the frequency $\Delta E_{A1}/h$ is applied to qubit A, the state of qubit A can be changed only if qubit B is in state $|1\rangle$. This is our proposal for the realization of the controlled-NOT gate.

We also propose a qubit which consists of a superconducting ring with a metallic superconductor/normal metal/superconductor (SC/NM/SC) junction and a ferromagnetic π junction. In the metallic SC/NM/SC junction, the phase dependence of the free energy is π shifted from that of a metallic π junction. The double minima are formed in the ring regardless of the interfacial barrier height in the π junction, and hence the ring has potential as a qubit.

The qubits which we propose have the following advantages: (i) the geometry is simple; i.e., only two Josephson junctions are used, (ii) the qubit is constructed without an external magnetic field, and only a small external magnetic field is needed in the measurement of the state. Furthermore, because of these advantages, (iii) the size of the qubit can be reduced. This makes large-scale integration possible, and leads to a reduction of the decoherence due to the coupling to the environment. Spin-electronic devices have been extensively studied. The qubits offer a new route for spin-electronics to quantum computing.
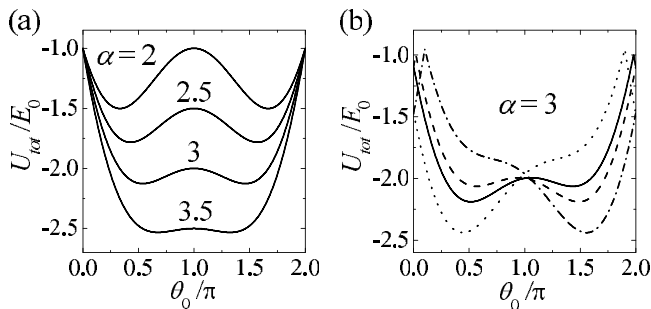
FIG. 3.   Normalized U_{tot} as a function of θ_0 for $\beta = 3.1 \times 10^{-3}$ shown in (a) when there is no external magnetic field ($\Phi_{\text{ext}} = 0$), and shown in (b) when the field is finite: $\Phi_{\text{ext}}/\Phi_0 = 0.01$ (solid line), -0.01 (dashed line), 0.05 (dotted line), and -0.05 (dot-dashed line).

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