Sensitive Measurement of Forces at the Micron Scale Using Bloch Oscillations of Ultracold Atoms

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We show that Bloch oscillations of ultracold fermionic atoms in the periodic potential of an optical lattice can be used for a sensitive measurement of forces at the micrometer length scale, e.g., in the vicinity of a dielectric surface. In particular, the proposed approach allows us to perform a local and direct measurement of the Casimir-Polder force which is, for realistic experimental parameters, as large as 10^{-4} gravity.

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The study of forces at small length scales constitutes one of the challenges of current frontier research in physics. The interest spans from fundamental issues, like the search of possible deviations from Newtonian gravity, to the phenomenology of forces close to surfaces. Accurate investigations have been performed using a variety of experimental techniques, e.g., micro-cantilevers [1,2], torsion pendula [3], micromechanical torsional oscillators [4], and atomic force microscopes [5]. The laser production and manipulation of micron-size ultracold atomic samples has provided a new tool for the investigation of forces at small length scales [6,7]. More recently, also trapped atomic Bose-Einstein condensates have been applied to the study of forces close to surfaces [8–10].

The possibilities offered by quantum degenerate gases have been enriched by the combination with periodic trapping potentials, such as those produced by optical lattices. As demonstrated previously [11–13], this combination allows us to study the general phenomenon of Bloch oscillations [14]. A key feature of this phenomenon is that the measurement of an oscillation frequency, which can be made very precise, can be turned into a direct measurement of a force. An important step in this direction has been the observation of temporally resolved Bloch oscillations of a degenerate Fermi gas of atoms trapped in an optical lattice and subjected to gravity [15]. This has shown the feasibility of a high sensitivity force sensor.

In this Letter we explore the effect of a weak inhomogeneous force close to a surface, such as the Casimir-Polder (CP) force [16], on atomic Bloch oscillations (BO). We make a complete theoretical analysis for a Fermi gas trapped in an optical lattice in proximity of a dielectric surface, using realistic parameters from the existing experiment [15]. We discover that this atomic Bloch oscillator realizes a powerful sensor for the detection of forces with a spatial resolution of the order of few microns.

The physical system that we consider here consists of a sample of ultracold atoms trapped in a 1D lattice aligned along gravity, as shown in Fig. 1. The atoms are initially cooled in a harmonic potential, and then transferred to the optical lattice. A harmonic horizontal confinement is assumed to be present, e.g., given by the same red-detuned lattice beams. As soon as the vertical harmonic confinement is switched off, the atoms start to perform BO in the lattice potential under the action of gravity. If now a surface is brought close to the atomic sample, additional forces between the atoms and the surface will affect the dynamics of BO. We will now investigate how such forces can be measured from the shift of the Bloch oscillation frequency as the distance between atoms and surface is varied.

In particular, we consider here the CP force, which is exerted on an atom by the modified fluctuations of the electromagnetic field in the vicinity of a dielectric surface [16]. As a first step, we have worked out the simplified model in which the CP force $F_{\rm CP}$ is assumed to be spatially homogeneous. This is equivalent to assuming that the size of the sample along the vertical direction is small compared to the actual spatial variation of the force. In this case the quasimomentum q of the atoms simply evolves as

$$\hbar \dot{q} = F = mg + F_{\rm CP}, \qquad (1)$$

F being the total force acting on the atom. As q is defined only modulo the Bragg momentum $q_B = 2\pi/\lambda$ (λ is the wavelength of the optical lattice), each time q gets to the

FIG. 1. Sketch of the physical situation considered in this Letter: an ultracold sample of fermionic atoms is trapped in a vertical optical lattice in proximity of a dielectric surface and performs Bloch oscillations under the combination of gravity and of the CP force.

edge of the first Brillouin zone at q_B, it reappears on the other side at $-q_B$. The period of BO is thus $T_B = 4\pi\hbar/F\lambda$. Following the evolution of BO for many periods one can infer their frequency, and therefore the CP force, with high accuracy. The fact that the frequency shift is here proportional to the force is an important difference of the present setup with respect to previous work [9]. In these experiments, as well as in the ones actually in progress in the group of Cornell at JILA [17], the shift in the collective oscillation frequencies of a Bose-Einstein condensate confined in a harmonic trap is in fact proportional to the gradient of the CP force. The possibility of directly addressing the force rather than its gradient opens new possibilities for the measurement, e.g., of the thermal effects in the surface-atom force [10,18,19].

To observe BO one can study the momentum distribution of the atomic sample after a ballistic expansion. The typical evolution of the momentum is the one depicted in Fig. 2. At the initial time $t = 0$ the momentum distribution is centered at $k = 0$, with symmetric lateral peaks at multiples of $2q_B$ [20]. As time goes on, the comb of peaks moves rigidly with constant velocity, while their relative intensity follows the Fourier transform of a Wannier state [21] as an envelope. In particular, when $q(t)$ reaches the edge of the first Brillouin zone at $t = T_B/2$, the distribution contains a pair of symmetric peaks at $\pm q_B$. At $t = T_B$, the distribution has regained the initial shape. Note that the dynamics of BO in momentum space, being governed by the evolution Eq. (1) for the quasimomentum, is independent of the lattice height, while the corresponding oscillations in real space have an amplitude of order δ/F, where 2δ is the width of the first energy band of the lattice. The presence of a strong, known in advance, force, such as gravity, is therefore necessary to reduce the amplitude of the oscillation and consequently to achieve a high spatial resolution in the measurement of a weak force. The use of a vertical geometry is helpful also to prevent the atoms from hitting the surface during the ballistic expansion.

This discussion, originally developed for a single particle, is immediately extended to the case of a many-atom gas simply by convolving the single-particle prediction by the initial quasimomentum distribution of the gas: a fundamental requirement to clearly follow BO is that the quasimomentum spread of the sample must not be much larger than q_B. This implies that the atomic temperature must be at most of the order of 2δ. As the system size is limited by the spatial resolution we intend to obtain, a high density sample has to be used to increase the number of atoms and thus improve the signal. This motivates the use of a quantum degenerate gas.

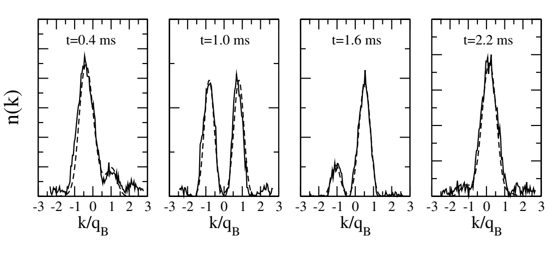

FIG. 2. Momentum distribution of a Fermi gas performing Bloch oscillations at different times within an oscillations period: solid lines are experimental data, dashed lines are theoretical calculations including adiabatic switch-off of the lattice. $E_F/2\delta = 0.9$.

Although a Bose-Einstein condensate would certainly offer a much narrower initial quasimomentum spread than a Fermi gas, it has been shown in recent experiments [22] to suffer from instabilities due to interactions, which quickly broaden the quasimomentum distribution and make the observation of BO practically impossible in condensates at high density. As a matter of fact, for the rest of the Letter we will mainly consider an ultracold and spin-polarized Fermi gas: thanks to the absence of s-wave interactions between fermionic atoms with parallel spins, this appears to be an excellent candidate for this kind of experiment [15]. For temperatures well below the Fermi degeneracy temperature T_F, the quasimomentum distribution can be safely approximated by the zero-temperature one:

$$N_q = \frac{1}{2}\left(\frac{E_F - E_q}{\hbar\omega_r}\right)^2 \Theta(E_F - E_q) \qquad (2)$$

where E_q is the energy of the Bloch state of quasimomentum q, $E_F = k_B T_F$ is the Fermi energy, and ω_r is the frequency of the radial harmonic trapping. As long as E_F is smaller than 2δ, the quasimomentum distribution is localized in a region around $q = 0$ and is zero outside, while the contrast slowly deteriorates for larger values of E_F. Notice that this behavior is different from the case of a purely one-dimensional system, where the contrast would be strictly zero as soon as $E_F \geq 2\delta$ [23].

The contrast can be improved by adiabatically switching off the lattice potential at the end of the BO. In this case, the population of the state of quasimomentum q is projected onto the first Brillouin zone without being weighted by the Fourier transform of the Wannier state and at most two peaks are observed. In Fig. 2 we report the result of a theoretical analysis taking into account the adiabatic switch-off of the lattice, which reproduces with excellent accuracy the experimental data taken as described in [15].

Let us now study the effect of the CP force between the atoms and the dielectric surface. A general discussion of its properties has been recently given by [10], and we refer to that paper for the details of the calculations. In the present simulations, the potential plotted in Fig. 3 of [10] has been used, which takes into account all the relevant regimes (van der Waals-London, Casimir-Polder, and thermal), in particular, the thermal one

$$V_{\rm CP}^{(\rm th)}(z) = -\frac{k_B T \alpha_0}{4z^3}\frac{\epsilon_0 - 1}{\epsilon_0 + 1}, \qquad (3)$$

which dominates at distances larger than the thermal wavelength $\hbar c/k_B T$ of the photon. Here α_0 is the static atomic polarizability, ϵ_0 is the static dielectric constant of the material composing the surface, which is placed at $z = 0$, the z axis being oriented downwards. At the lowest level, one can approximate the spatially varying force with a homogeneous one equal in magnitude to the value of thermal force (3) evaluated at the center of the cloud. In this case, the BO period for an atomic cloud at a distance D from the surface is easily evaluated from Eq. (1) and has a relative shift

$$\frac{\Delta T_B}{T_B} = -\frac{F_{\mathrm{CP}}(D)}{mg} = \frac{0.1728}{D^4}(\mu\mathrm{m})^4. \tag{4}$$

The numerical value refers to the specific case of ^{40}K atoms and a sapphire surface with $\epsilon_0 = 9.4$ at $T = 300$ K. In Fig. 3 we show the relative BO period shift as a function of the distance D, computed using the approximated formula (4). Note how the effect of the Casimir potential ranges from 10^{-4} to 10^{-5} at distances D from 5 to 10 microns, which are realistic for a possible experiment. This situation is encouraging because such a level of sensitivity has been already demonstrated in a proof-of-principle experiment [15], and can be further improved by a few orders of magnitude, as we will discuss later.

It is then worth performing a deeper analysis taking fully into account the spatial inhomogeneity of the Casimir force over the atomic cloud, its full z dependence beyond the large distance approximation (3) as well as the effect of a real-space motion of the cloud during BO. We have indeed performed an exact numerical calculation for the noninteracting many fermion problem, in the combined potential of the trap, the gravity, the lattice, and the complete CP force as predicted in [10]. This has been done by first obtaining the single-particle orbitals in the initial potential of the trap, the lattice, and the CP force, by then populating them according to a Fermi distribution at a temperature well below the Fermi temperature, and by finally propagating them via the Schrödinger equation in the combined potential of the lattice, the gravity, and the CP force. This procedure provides the density profile of the gas, as well as its momentum and quasimomentum distributions at all times. From the center-of-mass of the quasimomentum distribution, it is then possible to extract a theoretical prediction for the BO frequency shift.

The results of such a calculation are shown in Fig. 3 for realistic parameters: a situation has been considered in which 2×10^4 ^{40}K atoms are prepared in a lattice with $\lambda = 873$ nm, with a vertical extension of about 4 μm, i.e., 8 lattice sites. To achieve a favorable ratio $E_F/2\delta = 0.9$ close to 1 we have chosen a lattice height $s = 5$, and a radial frequency of 10 Hz. The vertical extension in the lattice is determined by the confinement of the initial harmonic potential of a trap, which for this specific case has a frequency of 400 Hz. Justification for the slight difference with respect to the analytical formula (4) can be found in several effects that have been neglected in the previous discussion, in particular, the spatial displacement (on the order of 1 μm in the direction of going further from the surface) of the atomic cloud while performing BO, the difference between the approximate thermal potential (3) and the exact one of [10] and finally the spatial inhomogeneity of the CP force across the atomic cloud. The inhomogeneity of the force can have a further consequence: the frequency shift of BO is in fact given by an average of the force over the cloud profile, and a sort of inhomogeneous broadening is to be expected. As this latter effect may affect the accuracy of the measurement, in particular, after the large number of oscillations which are required for a high-precision measurement, we have investigated the broadening of the momentum distribution after a large number of BO periods. The results, reported in Fig. 4(a), show that the broadening is not yet significant after 1000 periods.

For the sake of completeness, a complete analysis has also been performed for the bosonic case by numerically solving the 1D Gross-Pitaevskii equation with trap and atom number parameters analogous to the Fermi case

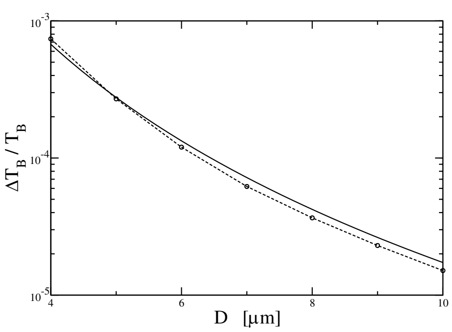

FIG. 3. Relative shift $\Delta T_B/T_B$ of the Bloch oscillation period due to the CP force. The solid line is the prediction (4) using the thermal potential (3), the circles are the result of a numerical simulation as described in the text.

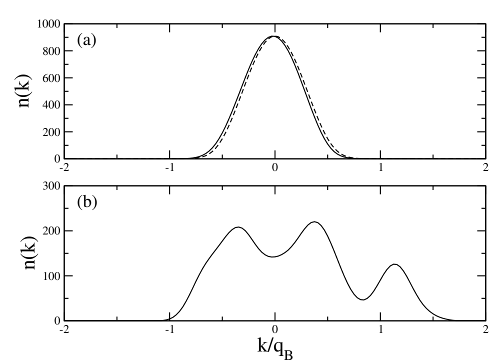

FIG. 4. Upper panel (a) momentum distribution of a Fermi gas at a distance $D = 10$ μm from the surface at beginning (dashed line) and after 1000 periods of Bloch oscillations (solid line). Lower panel (b) momentum distribution of a BEC after 2 periods of Bloch oscillations.

(that is with a peak density of $n = 4 \times 10^{13}$ cm^{-3}). The results agree with the experimental observations of [15]: the quasimomentum distribution dramatically broadens as soon as the quasimomentum of the Bose condensate enters the dynamically instable region [24] and after only a few BO periods, it is not possible to follow the oscillations any longer [Fig. 4(b)]. To overcome this problem, one could think of setting the scattering length a to zero by means of a Feshbach resonance [25]. As the characteristic time for the onset of the instability is of the order of $\tau_{\text{inst}} \approx \hbar/gn_{\text{max}}$ [24], $gn_{\text{max}} = 4\pi\hbar^2 an_{\text{max}}/m$ being the mean-field interaction energy, an upper bound on the absolute value of a is set by the fact that τ_{inst} has to be much longer than the actual duration of the measurement. For instance, a measurement lasting 100 BO periods, performed with the atomic density here considered, would require a reduction of the scattering length of a factor of the order of 10^3.

Let us now discuss the sensitivity that can be realistically achieved in an experiment. In [15] we demonstrated an overall sensitivity $\Delta T_B/T_B = 10^{-4}$, by following about 100 BO periods of ^{40}K atoms in a lattice with $\lambda = 873$ nm. This sensitivity was limited by various sources of decoherence due to the imperfect nature of the optical lattice, e.g., the phase noise due to mechanical vibrations of the retroreflecting mirror, or the longitudinal inhomogeneous forces due to the focusing of the lattice beams. Most of these decoherence sources can be reduced in an optimized experiment, thus extending the number of oscillations observable with high contrast. The typical lifetime of the atomic sample of a few seconds will, however, limit the realistic number of observable BO periods to the order of 10^3. Since the sensitivity on the measurement of a single BO period can be pushed to 10^{-3}–10^{-4}, as typically achieved in this kind of experiments [26], one can expect to obtain an overall sensitivity $\Delta T_B/T_B = 10^{-6}$–10^{-7}.

Concerning the spatial resolution, the limiting factors are the actual size of the sample and its spatial displacement during the BO. The numerical data shown in the figures correspond to a case where several lattice sites are occupied and the displacement is on the order of 1 μm. Analogous simulations starting from a tighter trapping potential (1600 Hz) along the z direction show that a cloud can be obtained which is almost completely concentrated in two sites, while the spatial displacement during the BO can be suppressed by increasing the lattice height; $s = 10$ is enough to make the displacement smaller than 1 lattice site. A spatial resolution on the order of the lattice spacing can therefore be achieved with realistic experimental parameters.

In conclusion, we have discussed how Bloch oscillations of ultracold fermionic atoms can provide a sensitive and direct measurement of forces at the micron scale. The expected sensitivity and spatial resolution appear very promising for the detection of weak and inhomogeneous forces like the CP force between atoms and a dielectric surface. In particular, we expect this technique to give access to the CP force at distances larger than 5 μm, where novel thermal effects have been predicted [10,18,19] and not yet observed. Experiments of this kind might also give stricter bounds on possible short-distance deviations from Newton's gravitational law [27].

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