Collisions of Halo Nuclei within a Dynamical Eikonal Approximation

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The dynamical eikonal approximation unifies the semiclassical time-dependent and eikonal methods. It allows calculating differential cross sections for elastic scattering and breakup in a quantal way by taking into account interference effects. Good agreement is obtained with experiment for ^{11}Be breakup on ^{208}Pb. Dynamical effects are weak for elastic scattering.

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The experimental study of the breakup of weakly bound exotic nuclei is one of the main tools for determining their properties. The analysis of those experiments requires an accurate description of collision processes leading to the continuum. Starting with some model of the loosely bound nucleus, different cross sections corresponding to feasible measurements need to be calculated with the best possible reaction mechanism.

An efficient approach to breakup involves a semiclassical approximation [1]. At high enough velocities, the relative motion between the target and projectile can be treated in a classical way. In the target frame of reference, the projectile center of mass follows a classical trajectory that can be well approximated by a straight line or a Rutherford trajectory. When following the trajectory, the projectile experiences a Coulomb and nuclear field from the target that varies in time. This time variation induces excitation and breakup. This approach is a natural extension of the Coulomb excitation theory [1]. The generalization involves taking into account not only the nuclear interaction but also, and mostly, excitations into the continuum.

The semiclassical approximation leads to the resolution of a time-dependent Schrödinger equation [2–12]. It is a fully dynamical theory where all couplings, not only between the bound states and the continuum but also couplings inside the continuum, are properly taken into account. A purely numerical resolution of the time-dependent Schrödinger equation presents the advantage that no simplifying assumptions need to be done about the description of the continuum. Nevertheless, the validity of the physical results rely on the convergence and the accuracy of the solution. In recent years, much progress has been achieved in this numerical resolution and several groups have developed accurate three-dimensional codes for the study of breakup [3–12]. Several numerical techniques lead to consistent results for the total breakup cross sections and for momentum distributions.

The semiclassical approach, however, has the drawback not to be a fully quantal approximation. It does not fulfill energy conservation. Differential cross sections are usually not accurate because they lack some interference effects. This is especially true for elastic scattering for which only crude estimations are available.

Different quantal theories have been proposed or extended to allow a treatment of excitations in the continuum. Let us mention the continuum discretized coupled-channels method [13,14], the adiabatic approximation [15], the distorted-wave Born approximation [16,17], and the eikonal approximation [18,19].

The eikonal approximation presents a number of interesting features and some drawbacks. To some extent, it takes into account the few-body degrees of freedom at all orders. It has a rather simple physical interpretation and allows a decomposition of the amplitudes into different components. However, the eikonal approximation consists in an approximate calculation of the phase of the scattering wave function. By lack of dynamical effects, the projectile internal probability density is not modified when all potentials are real. This approximation is mostly valid for peripheral reactions. Some technical difficulties appear related to the long range of the Coulomb interaction.

The aim of this Letter is to develop a common generalization of the semiclassical and eikonal methods. The dynamical eikonal method is a purely quantal method that combines the advantages of both approaches. It improves the eikonal method in the sense that missing dynamical effects are now taken into account. It is also an improvement of the semiclassical approach as it allows in a simple and natural way to calculate differential cross sections, including for elastic scattering. The principle of this generalization has been proposed [20] and applied [21] for inelastic scattering in the context of the close-coupling method in atomic physics. Other eikonal-inspired generalizations are encountered in nuclear physics [22–24]. Here, we present a simple general derivation of the dynamical eikonal method and apply it to the elastic scattering and breakup of a halo nucleus. We calculate differential cross sections that were not reachable in the semiclassical approach. We also compare the traditional eikonal approximation with its dynamical extension in order to assess its validity and the role of dynamical effects.

A two-body projectile made up of a structureless core with mass m_c and a structureless fragment with mass m_f ($m_P = m_c + m_f$) collides a target with mass m_T. Let $\boldsymbol{R}$ be the coordinate of the center of mass (c.m.) of the projectile with respect to the target and $\boldsymbol{r}$ the coordinate of the fragment with respect to the core. The corresponding momenta are denoted as $\boldsymbol{P}$ and $\boldsymbol{p}$, respectively. The core-fragment interaction V_{cf} provides the projectile properties through the internal Hamiltonian

$$H_0 = \frac{p^2}{2\mu_{cf}} + V_{cf}(\boldsymbol{r}), \tag{1}$$

where μ_{cf} is the core-fragment reduced mass. The ground-state wave function and energy of the projectile are ϕ_0 and E_0, respectively. After removing its c.m. motion, the Hamiltonian of the three-body system reads [19]

$$H = \frac{P^2}{2\mu} + H_0 + V_{cT}\left(\boldsymbol{R} - \frac{m_f}{m_P}\boldsymbol{r}\right) + V_{fT}\left(\boldsymbol{R} + \frac{m_c}{m_P}\boldsymbol{r}\right), \tag{2}$$

where μ is the projectile-target reduced mass. The core-target and fragment-target interactions are described with the optical potentials V_{cT} and V_{fT}, respectively.

In the Schrödinger equation $H\Psi(\boldsymbol{R}, \boldsymbol{r}) = E_T\Psi(\boldsymbol{R}, \boldsymbol{r})$, let us introduce the usual eikonal ansatz [18,19]

$$\Psi(\boldsymbol{R}, \boldsymbol{r}) = e^{iKZ}\hat{\Psi}(\boldsymbol{R}, \boldsymbol{r}), \tag{3}$$

where the wave number K is related to the total energy E_T of the system by

$$E_T = \frac{\hbar^2 K^2}{2\mu} + E_0. \tag{4}$$

Hence, the Schrödinger equation becomes

$$\frac{\hbar^2}{2\mu}\Delta_{\boldsymbol{R}}\hat{\Psi} + i\hbar v\frac{\partial\hat{\Psi}}{\partial Z} = (H_0 + V_{cT} + V_{fT} - E_0)\hat{\Psi}, \tag{5}$$

where $v = \hbar K/\mu$ is the asymptotic relative velocity. The eikonal approximation consists in neglecting the first term in Eq. (5) [18]. In the standard eikonal theory, it is complemented by the adiabatic approximation where H_0 is replaced by E_0 [19]. At this step, the dynamical effects due to H_0 are thus lost. Here we do not perform this second approximation. Rather we introduce the auxiliary timelike variable

$$t = Z/v. \tag{6}$$

The eikonal solution is then obtained by solving

$$i\hbar\frac{\partial\hat{\Psi}}{\partial t} = (H_0 + V_{cT} + V_{fT} - E_0)\hat{\Psi} \tag{7}$$

with the initial condition $\hat{\Psi} \to_{t\to-\infty} \phi_0(\boldsymbol{r})$. Equation (7) is formally equivalent to the semiclassical time-dependent Schrödinger equation with straight-line trajectories [11]. The transverse part $\boldsymbol{b}$ of the quantal coordinate $\boldsymbol{R}$ plays the role of the impact parameter in Eq. (7) although we are here in a purely quantal context. The dynamical eikonal wave function is given by Eq. (3) as a function of the time-dependent solution $\hat{\Psi}(\boldsymbol{b}, Z/v, \boldsymbol{r})$ of Eq. (7).

With this approximate wave function, one can calculate the different transition matrix elements for elastic or inelastic scattering and dissociation. We sketch here the derivation for elastic scattering. The transition matrix element reads [25]

$$T_{fi} = \langle e^{i\boldsymbol{K}\cdot\boldsymbol{R}}\phi_0(\boldsymbol{r})|V_{cT} + V_{fT}|\Psi(\boldsymbol{R}, \boldsymbol{r})\rangle. \tag{8}$$

By using Eqs. (3) and (7), and $H_0\phi_0 = E_0\phi_0$, one obtains

$$\begin{aligned} T_{fi} &= i\hbar v\langle e^{i\boldsymbol{K}\cdot\boldsymbol{R}}\phi_0(\boldsymbol{r})|e^{iKZ}\frac{\partial}{\partial Z}\hat{\Psi}(\boldsymbol{R}, \boldsymbol{r})\rangle \\ &\approx i\hbar v\int d\boldsymbol{R}e^{-i\boldsymbol{q}\cdot\boldsymbol{b}}\frac{\partial}{\partial Z}\langle\phi_0(\boldsymbol{r})|\hat{\Psi}(\boldsymbol{R}, \boldsymbol{r})\rangle. \end{aligned} \tag{9}$$

The transferred momentum $\boldsymbol{q} = \boldsymbol{K} - K\hat{\boldsymbol{Z}}$ is assumed to be purely transverse. With the notation

$$S_0(\boldsymbol{b}) = \langle\phi_0(\boldsymbol{r})|\hat{\Psi}(\boldsymbol{R}, \boldsymbol{r})\rangle_{Z=+\infty} - 1, \tag{10}$$

the transition matrix element reads

$$T_{fi} = i\hbar v\int d\boldsymbol{b}e^{-i\boldsymbol{q}\cdot\boldsymbol{b}}S_0(\boldsymbol{b}). \tag{11}$$

As S_0 does not depend on the orientation of $\boldsymbol{b}$, it becomes

$$T_{fi} = i2\pi\hbar v\int_0^\infty bdbJ_0(qb)S_0(b). \tag{12}$$

The elastic differential cross section is then easily deduced.

In a similar way, one can derive the dissociation transition matrix element

$$T_{fi} = i2\pi\hbar v\int_0^\infty bdbJ_0(qb)S(\boldsymbol{k}, b) \tag{13}$$

with

$$S(\boldsymbol{k}, \boldsymbol{b}) = \langle\chi_{\boldsymbol{k}}^{(-)}(\boldsymbol{r})|\hat{\Psi}(\boldsymbol{R}, \boldsymbol{r})\rangle_{Z=+\infty}, \tag{14}$$

where $\chi_{\boldsymbol{k}}^{(-)}(\boldsymbol{r})$ is an ingoing-wave solution of $H_0\chi_{\boldsymbol{k}}^{(-)} = E\chi_{\boldsymbol{k}}^{(-)}$ at energy $E > 0$ and $\boldsymbol{k}$ is the corresponding wave vector. From these expressions, one deduces the dissociation cross sections.

The usual eikonal approximation is obtained by neglecting $H_0 - E_0$ in Eq. (7), i.e., by the replacement

$$\hat{\Psi}(\boldsymbol{R}, \boldsymbol{r}) \to \exp\left[-\frac{i}{\hbar}\int_{-\infty}^{Z}(V_{cT} + V_{fT})dZ'\right]\phi_0(\boldsymbol{r}) \tag{15}$$

in Eqs. (10) and (14). Coulomb effects are separated analytically with the help of the Coulomb phase-shift function [18,19].

As an application, we study collisions of ^{11}Be on ^{12}C and ^{208}Pb. The angular-momentum dependent core-fragment potential V_{cf} reproduces the bound states and $5/2^+$ resonance of ^{11}Be [12]. The V_{cT} and V_{fT} optical potentials are taken from the literature [26–28] as described in Refs. [11,12]. Hence the calculations presented

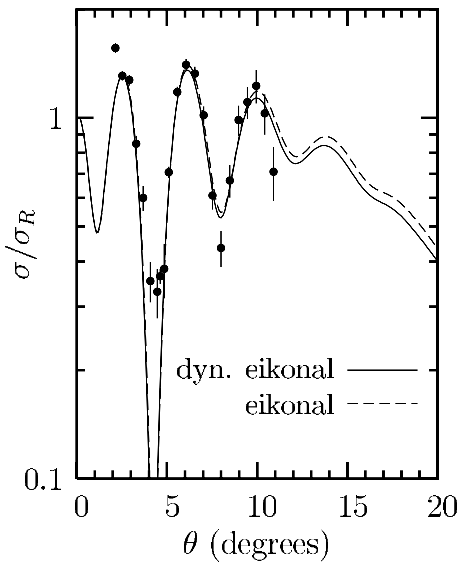

FIG. 1. Ratio of elastic cross section to Rutherford for ^{11}Be scattering on ^{12}C at 49.3 MeV/nucleon: dynamical eikonal (full line) and eikonal (dashed line) approximations. Experimental data are from Ref. [29].

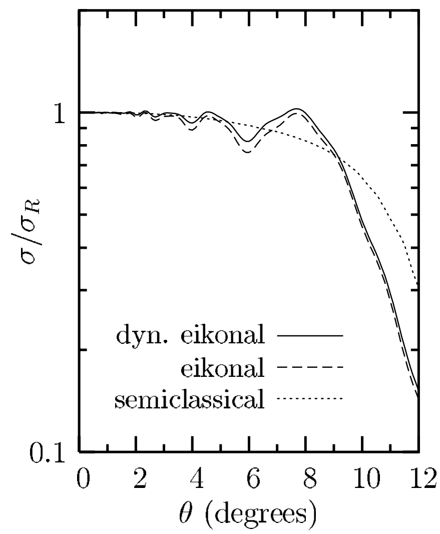

FIG. 2. Ratio of elastic cross section to Rutherford for ^{11}Be scattering on ^{208}Pb at 20 MeV/nucleon: dynamical eikonal (full line), eikonal (dashed line), and semiclassical (dotted line) approximations.

below do not contain any adjustable parameter. Dynamical eikonal cross sections are calculated with Eqs. (12) and (13) after solving numerically Eq. (7) with the technique explained in Ref. [11].

Figure 1 displays elastic cross sections of ^{11}Be on ^{12}C at 49.3 MeV/nucleon as a function of the projectile c.m. scattering angle θ. The full line corresponds to the dynamical eikonal calculation. The agreement with experiment [29] is quite good. Its quality is identical to that obtained with the adiabatic approximation [27]. The results from the usual eikonal approximation (dashed line) are close to those of the dynamical method. This means that dynamical effects are weak for this elastic collision. However, the shapes of the functions S_0 defined in Eq. (10) are not identical for both approximations. Their behavior at large impact parameters b are quite different. While the eikonal S_0 decreases as a power of $1/b$, the dynamical S_0 decreases exponentially. But this does not affect much the elastic cross section at these high velocities.

Significant dynamical effects at large impact parameters imply an important role of the Coulomb interaction. As an attempt to amplify this influence, we present in Fig. 2 a calculation of the elastic scattering of ^{11}Be on the heavier target ^{208}Pb. We also choose the lower energy of 20 MeV/nucleon. Here the difference between both calculations is a little larger but the traditional eikonal approximation remains surprisingly accurate. It underestimates the elastic cross section by less than 10% over the considered angular range. The classical relation between impact parameter and angle on a Coulomb trajectory provides a semiclassical elastic cross section (dotted line) where interference effects are missing. This approximation would be meaningless in Fig. 1 because nuclear effects dominate.

Now we turn to breakup calculations, i.e., ^{11}Be dissociation on ^{208}Pb. This breakup is dominated by Coulomb effects. Differential cross sections have been measured by Fukuda *et al.* [30] at 69 MeV/nucleon. The experimental data correspond to an integration over a domain of relative energies E of the core and fragment up to 1 MeV. The same quantity calculated with the three approximations is displayed in Fig. 3. Above about 1°, both eikonal approximations essentially coincide, but they are quite different below that angle. The purely semiclassical approximation (dotted line) appears to be fairly valid below 3°. The agreement of the dynamical eikonal approximation (full line) with experiment is quite good in view of the fact that no parameter is adjusted. In particular, the magnitude of the experimental cross section is very well reproduced. The theory, however, underestimates the data beyond 4°. Notice that no correction due to experimental conditions is included in the theory. At small angles, the traditional

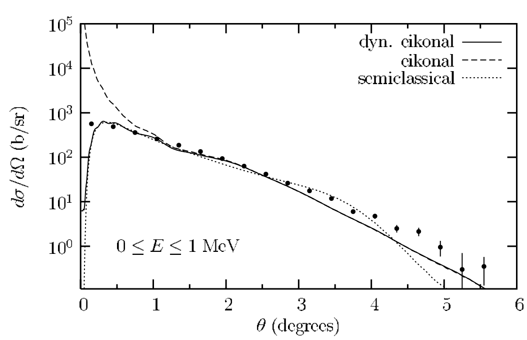

FIG. 3. Differential breakup cross section of ^{11}Be on ^{208}Pb at 69 MeV/nucleon: dynamical eikonal (full line), eikonal (dashed line), and semiclassical (dotted line) approximations. Experimental data are from Ref. [30].

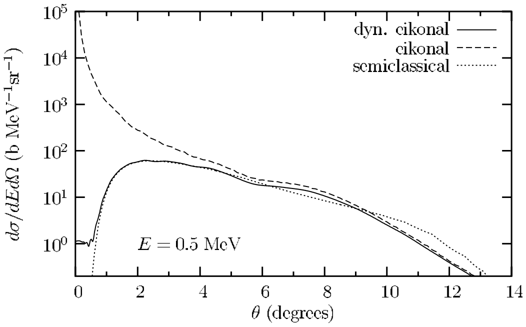

FIG. 4. Differential breakup cross section of ^{11}Be on ^{208}Pb at 20 MeV/nucleon: dynamical eikonal (full line), eikonal (dashed line), and semiclassical (dotted line) approximations.

eikonal method (dashed line) strongly overestimates the data while the semiclassical approximation tends to zero. The overestimation is related to the slow decrease of $S(\boldsymbol{k}, b)$ at large impact parameters. The large b exponential decrease of the dynamic eikonal approximation is thus much more realistic.

A similar calculation at 20 MeV/nucleon is displayed in Fig. 4. Here the results correspond to the single relative energy $E = 0.5$ MeV near the maximum of the cross section integrated over angles. The difference between both eikonal approximations is larger and remains significant up to about 8°. Dynamical effects play a more important role here. The semiclassical approximation is good between 1° and 10°.

In summary, the dynamical eikonal method unifies the semiclassical and eikonal approximations. It improves time-dependent calculations by taking interference effects into account and by providing realistic calculations of differential cross sections. It improves the traditional eikonal method by introducing dynamical effects. This method opens the way toward more extended applications of time-dependent codes and should provide a better level of accuracy by its quantum nature. Applications to elastic scattering show that the usual eikonal method is good at high projectile energies. For breakup, however, it is not valid at small angles and less good than a purely semiclassical approach. The good agreement of the dynamical method with the data of Ref. [30] is encouraging and should lead to a deeper understanding of this type of experiment. Differences and similarities between the extended and the usual eikonal approximations deserve a thorough analysis.

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