Nongeometric Conditional Phase Shift via Adiabatic Evolution of Dark Eigenstates: A New Approach to Quantum Computation

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We propose a new approach to quantum phase gates via the adiabatic evolution. The conditional phase shift is neither of dynamical nor geometric origin. It arises from the adiabatic evolution of the dark state itself. Taking advantage of the adiabatic passage, this kind of quantum logic gates is robust against moderate fluctuations of experimental parameters. In comparison with the geometric phase gates, it is unnecessary to drive the system to undergo a desired cyclic evolution to obtain a desired solid angle. Thus, the procedure is simplified, and the fidelity may be further improved since the errors in obtaining the required solid angle are avoided. We illustrate such a kind of quantum logic gates in the ion trap system. The idea can also be realized in other systems, opening a new perspective for quantum information processing.

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Recently, much attention has been paid to quantum computers, which are based on the fundamental quantum mechanical principle and can provide more powerful computational ability than the classical computers [1]. Shor [2] has shown that the problem of factorizing a large integer, which is the basis of the security of many cryptographic systems and takes exponentially increasing time on a classical computer, can be solved in polynomial time using a quantum computer. Moreover, Grover [3] has discovered a quantum mechanical algorithm for searching for an item from a disordered system that is polynomially faster than any classical algorithm. Because of these advantages many efforts have been devoted to the implementation of practical quantum information processors.

It has been shown [4] that the building blocks of quantum computers are two-quantum-bit (qubit) gates. So far, two kinds of two-qubit phase gates have been proposed. One is based on the conditional dynamical phase shift [4–6]. The other is based on the geometric operation: driving the qubits to undergo appropriate cyclic evolutions conditional on the state of the qubits to acquire the geometric phase. In comparison with the dynamical gates, geometric phase gates have practical advantages since they are resilient to certain small errors. Schemes have been proposed to construct adiabatic geometric gates using NMR [7], superconducting nanocircuits [8], and trapped ions [9].

In this Letter, we propose a new class of quantum phase gates via the adiabatic evolution. Unlike normal dynamical and geometric phase gates, neither does the qubit system undergo any dynamical phase shift since it works in the dark space nor does the Hamiltonian need to change along the suitable loop involving a required solid angle. Thus, the conditional phase is of neither dynamical nor geometric origin. It arises from the adiabatic evolution of the dark eigenstate itself under certain conditions. To our best knowledge, this is the first scheme for quantum logic gates by using the adiabatic evolution of the dark eigenstate itself without forcing the system to evolve along a suitable cyclic loop in the parameter space to obtain the required solid angle. This kind of quantum logic gates are robust against moderate fluctuations of experimental parameters by using the adiabatic passage [10]. In comparison with the geometric phase gate, it does not require the parameters to sweep a required solid angle, and thus the procedure is simplified and the errors in obtaining the required solid angle are avoided. The idea may open a new perspective for quantum computation.

Consider a two-particle system. The particles have four states $|e\rangle$, $|g\rangle$, $|e'\rangle$, and $|g'\rangle$. The quantum information of the first qubit is encoded on the states $|e_1\rangle$ and $|g_1\rangle$, while the quantum information of the second qubit is encoded on $|g_2\rangle$ and $|g_2'\rangle$. The two qubits are coupled to a third subsystem, whose states are denoted by $|0\rangle$ and $|1\rangle$. The operation procedure is divided into two parts. During the first stage, the coupling between the two qubits is governed by the Hamiltonian

$$H_1 = \lambda_1|e_1 0\rangle\langle g_1 1| - \lambda_2|e_2 0\rangle\langle g_2 1| - \lambda_3|e_2' 0\rangle\langle g_2' 1| + \text{H.c.}, \tag{1}$$

where $|e_2\rangle$ and $|e_2'\rangle$ are two auxiliary states of the second particle. In the subspace $\{|e_1\rangle|g_2\rangle|0\rangle, |g_1\rangle|e_2\rangle|0\rangle, |g_1\rangle|g_2\rangle|1\rangle\}$ the dark state is

$$|D_1\rangle = \cos\theta|e_1\rangle|g_2\rangle|0\rangle + \sin\theta|g_1\rangle|e_2\rangle|0\rangle, \tag{2}$$

where

$$\cos\theta = \frac{\lambda_2}{\sqrt{\lambda_1 + \lambda_2^2}}, \tag{3}$$

$$\sin\theta = \frac{\lambda_1}{\sqrt{\lambda_1^2 + \lambda_2^2}}. \tag{4}$$

On the other hand, in the subspace $\{|e_1\rangle|g'_2\rangle|0\rangle, |g_1\rangle|e'_2\rangle|0\rangle, |g_1\rangle|g'_2\rangle|1\rangle\}$ the dark state is

$$|D'_1\rangle = \cos\theta'|e_1\rangle|g'_2\rangle|0\rangle + \sin\theta'|g_1\rangle|e'_2\rangle|0\rangle, \quad (5)$$

where

$$\cos\theta' = \frac{\lambda_3}{\sqrt{\lambda_1^2 + \lambda_3^2}}, \quad (6)$$

$$\sin\theta' = \frac{\lambda_1}{\sqrt{\lambda_1^2 + \lambda_3^2}}. \quad (7)$$

We adiabatically vary θ and θ' from 0 to $\pi/2$, and thus the state $|e_1\rangle|g_2\rangle|0\rangle$ adiabatically evolves to $|g_1\rangle|e_2\rangle|0\rangle$ according to Eq. (2), while $|e_1\rangle|g'_2\rangle|0\rangle$ adiabatically evolves to $|g_1\rangle|e'_2\rangle|0\rangle$ according to Eq. (5).

During the second stage, the evolution of the system is governed by the Hamiltonian

$$H_2 = \lambda_1|e_1 0\rangle\langle g_1 1| - \lambda_2|e_2 0\rangle\langle g_2 1| + \lambda_3|e'_2 0\rangle\langle g'_2 1| + \text{H.c.} \quad (8)$$

In this case the dark state in the subspace $\{|e_1\rangle|g_2\rangle|0\rangle, |g_1\rangle|e_2\rangle|0\rangle, |g_1\rangle|g_2\rangle|1\rangle\}$ is again given by Eq. (2), while the dark state in the subspace $\{|e_1\rangle|g'_2\rangle|0\rangle, |g_1\rangle|e'_2\rangle|0\rangle, |g_1\rangle|g'_2\rangle|1\rangle\}$ is

$$|D'_2\rangle = -\cos\theta'|e_1\rangle|g'_2\rangle|0\rangle + \sin\theta'|g_1\rangle|e'_2\rangle|0\rangle. \quad (9)$$

We now adiabatically vary θ and θ' from $\pi/2$ to 0. Then $|g_1\rangle|e_2\rangle|0\rangle$ adiabatically evolves to $|e_1\rangle|g_2\rangle|0\rangle$ according to Eq. (2), while $|g_1\rangle|e'_2\rangle|0\rangle$ to $-|e_1\rangle|g'_2\rangle|0\rangle$ according to Eq. (9). During the two stages the dark state in the first subspace evolves along the same path and finally returns to the initial state without undergoing any phase change, while the dark state in the second subspace evolves along the difference path due to a different setting of coupling between $|e'_2 0\rangle$ and $|g'_2 1\rangle$, which produces a conditional phase shift. Since no solid angle is swept in the parametric space, no Berry geometric phase [11] is involved. On the other hand, the states $|g_1\rangle|g_2\rangle|0\rangle, |g_1\rangle|g'_2\rangle|0\rangle$ are not affected by H_1 and H_2. This leads to

$$\begin{aligned} &|g_1\rangle|g_2\rangle \longrightarrow |g_1\rangle|g_2\rangle, \qquad |g_1\rangle|g'_2\rangle \longrightarrow |g_1\rangle|g'_2\rangle, \\ &|e_1\rangle|g_2\rangle \longrightarrow |e_1\rangle|g_2\rangle, \qquad |e_1\rangle|g'_2\rangle \longrightarrow -|e_1\rangle|g'_2\rangle. \end{aligned} \quad (10)$$

This transformation corresponds to the phase gate between the two qubits. Neither does the conditional phase shift result from the eigenenergy dependent dynamical phase nor does it result from the Berry phase during the evolution. It comes from the evolution of the dark state itself.

We now show how we can adiabatically vary θ and θ' and how the adiabatic passage plays its role in order to improve the gate fidelity. Initially we switch on the couplings $|e_2 0\rangle\longleftrightarrow|g_2 1\rangle$ and $|e'_2 0\rangle\longleftrightarrow|g'_2 1\rangle$. At this moment, the coupling $|e_1 0\rangle\longleftrightarrow|g_1 1\rangle$ is off and thus $\lambda_1 = 0$, which means that $\theta = \theta' = 0$. We then adiabatically increase the coupling $|e_1 0\rangle\longleftrightarrow|g_1 1\rangle$ and decrease the couplings $|e_2 0\rangle\longleftrightarrow|g_2 1\rangle$ and $|e'_2 0\rangle\longleftrightarrow|g'_2 1\rangle$ until they are switched off, and thus $\lambda_2 = \lambda_3 = 0$, changing θ and θ' to $\pi/2$. Finally, we adiabatically increase the couplings $|e_2 0\rangle\longleftrightarrow|g_2 1\rangle$ and $|e'_2 0\rangle\longleftrightarrow|g'_2 1\rangle$ and decrease the coupling $|e_1 0\rangle\longleftrightarrow|g_1 1\rangle$ until it is switched off, and thus $\lambda_1 = 0$, varying θ and θ' back to 0. We note that the operation is insensitive to fluctuations of the experimental parameters. For example, suppose that we expect λ_1 to be λ_e at the end of the first stage. Because of the fluctuation it is actually $\lambda_e + \Delta\lambda_e$. At this moment the couplings $|e_2 0\rangle\longleftrightarrow|g_2 1\rangle$ and $|e'_2 0\rangle\longleftrightarrow|g'_2 1\rangle$ are switched off, and thus the condition $\theta = \theta' = \pi/2$ is still satisfied according to Eqs. (3), (4), (6), and (7). Therefore, the state evolution is not affected. In the following we illustrate the idea in the ion trap system, but it should be applicable to other systems since what is required is that the parameters of the Hamiltonian are adiabatically varied. For example, the conditional adiabatic evolution has been experimentally demonstrated in NMR [7].

We consider N ions confined in a linear trap. We show how we can perform the new kind of conditional phase shift between two ions. Assume the ions have two excited states $|e\rangle$ and $|e'\rangle$ and two electronic ground states $|g\rangle$ and $|g'\rangle$. The quantum information of the first ion is stored in $|e\rangle$ and $|g\rangle$, while the quantum information of the second ion is stored in $|g\rangle$ and $|g'\rangle$. We drive the transition $|g\rangle \rightarrow |e\rangle$ for each ion with a traveling-wave laser beam with the frequency equal to $\omega_0 - \nu$, where ω_0 is the transition frequency between the states $|g\rangle$ and $|e\rangle$ and ν is the frequency of the center-of-mass vibrational mode. Meanwhile, we drive the transition $|g'\rangle \rightarrow |e'\rangle$ for the second ion with a traveling-wave laser beam with the frequency equal to $\omega'_0 - \nu$, where ω'_0 is the transition frequency between the states $|g'\rangle$ and $|e'\rangle$. In the resolved sideband limit, where the vibrational frequency ν is much larger than other characteristic frequencies of the problem, the interaction of the ion with the laser can be treated by using a nonlinear Jaynes-Cummings model [12]. In this case the Hamiltonian for such a system, in the interaction picture, is given by

$$\begin{aligned} H = {} & e^{-\eta^2/2}\Omega_1 e^{-i\phi_1}\sum_{j=0}^{\infty}\frac{(i\eta)^{2j+1}}{(j!)^2}a^{\dagger j}a^{j+1}|e_1\rangle\langle g_1| \\ & + e^{-\eta^2/2}\Omega_2 e^{-i\phi_2}\sum_{j=0}^{\infty}\frac{(i\eta)^{2j+1}}{j!(j+1)!}a^{\dagger j}a^{j+1}|e_2\rangle\langle g_2| \\ & + e^{-\eta^2/2}\Omega_3 e^{-i\phi_3}\sum_{j=0}^{\infty}\frac{(i\eta)^{2j+1}}{j!(j+1)!}a^{\dagger j}a^{j+1}|e'_2\rangle\langle g'_2| + \text{H.c.}, \end{aligned} \quad (11)$$

where $a^\dagger$ and a are the creation and annihilation operators for the collective vibrational mode, Ω_l and ϕ_l $(l = 1, 2, 3)$ is the Rabi frequency and phase of the lth laser, and η is the Lamb-Dicke parameter. Define the excitation number operator

$$N_e = \sum_{j=1}^{2} |e_j\rangle\langle e_j| + \sum_{j=1}^{2} |e'_j\rangle\langle e'_j| + a^\dagger a. \tag{12}$$

The interaction Hamiltonian commutes with N_e, and thus the excitation number conserves during the evolution. If the vibrational mode is initially in the vacuum state $|0\rangle$, it remains in the subspace $\{|0\rangle, |1\rangle\}$ during the interaction. This is due to the fact that the quantum information of the second ion is encoded on the two ground states, and thus the total excitation number of the whole system does not exceed 1. In this case the Hamiltonian reduces to

$$\begin{aligned} H = {} & i\eta e^{-\eta^2/2}\Omega_1 e^{-i\phi_1}|e_1 0\rangle\langle g_1 1| \\ & + i\eta e^{-\eta^2/2}\Omega_2 e^{-i\phi_2}|e_2 0\rangle\langle g_2 1| \\ & + i\eta e^{-\eta^2/2}\Omega_3 e^{-i\phi_3}|e'_2 0\rangle\langle g'_2 1| + \text{H.c.} \end{aligned} \tag{13}$$

We divide the evolution time into two parts. During the first stage, we choose $\phi_1 = \pi/2$ and $\phi_2 = \phi_3 = 3\pi/2$. Then the Hamiltonian has the same form of Eq. (1) with $\lambda_l = \eta e^{-\eta^2/2}\Omega_l$. The collective vibrational mode acts as the third subsystem, and the laser fields couple the two qubits to the vibrational mode. The corresponding dark states are given by Eqs. (2) and (5). We adiabatically increase θ and θ' from 0 to $\pi/2$ by adjusting the Rabi frequencies of the laser fields, leading to the evolution $|e_1\rangle|g_2\rangle|0\rangle \to |g_1\rangle|e_2\rangle|0\rangle$ and $|e_1\rangle|g'_2\rangle|0\rangle \to |g_1\rangle|e'_2\rangle|0\rangle$. During the second stage, we choose $\phi_1 = \phi_3 = \pi/2$ and $\phi_2 = 3\pi/2$. In this case the Hamiltonian has the same form of Eq. (8), with the corresponding dark states given by Eqs. (2) and (9). We now adiabatically vary θ and θ' from $\pi/2$ to 0, leading to $|g_1\rangle|e_2\rangle|0\rangle \to |e_1\rangle|g_2\rangle|0\rangle$ and $|g_1\rangle|e'_2\rangle|0\rangle \to -|e_1\rangle|g'_2\rangle|0\rangle$. In this way, we obtain the phase gate of Eq. (10) between the two ions.

The dynamical phase gates between two trapped ions via resonant sideband excitations have been proposed [13] and experimentally demonstrated [14]. In comparison with the dynamical proposal, the operation here is insensitive to small changes of experimental parameters by using the adiabatic passage [10]. Therefore, the gate fidelity can be greatly improved. Another advantage of the scheme is that it works beyond the Lamb-Dicke regime. Furthermore, the present scheme does not use the vibrational mode as the memory. It is unnecessary to transfer the quantum state of one ion to the vibrational mode, and then transfer back to the ion after the conditional phase shift. The vibrational mode is unexcited throughout the procedure.

According to the experiments of the Innsbruck group [14,15], two Zeeman levels of the $S_{1/2}$ ground state of $^{40}Ca^+$ ions can act as two ground states, while two Zeeman levels of the metastable $D_{5/2}$ state can act as the excited states. The lifetime of the metastable state is very long, and thus the spontaneous emission is negligible. For the setup of the NIST group, one can use the Raman transitions between two pairs of the hyperfine $S_{1/2}$ ground states of $^9Be^+$ ions through virtual excited states to suppress the spontaneous emission [16].

We now give a quantitative analysis of the experimental implementation. First, the second and third laser fields are switched on and the first laser is off, and thus $\theta = \theta' = 0$. We then adiabatically increase Ω_1 and decrease Ω_2 and Ω_3 until the second and third laser fields are switched off, resulting in $\theta = \theta' = \pi/2$. Finally, we adiabatically increase Ω_2 and Ω_3 and decrease Ω_1 until the first laser field is switched off, leading to $\theta = \theta' = 0$. By this way we adiabatically vary θ from 0 to $\pi/2$ during the first stage and $\pi/2$ to 0 during the second stage. Suppose that we expect Ω_1 to be 0.1ν at the end of the first stage. Because of the error it may be 0.11ν or 0.09ν instead of 0.1ν. However, at this moment the second and third laser fields are switched off and thus θ and θ' are still changed to $\pi/2$. Thus the operation is insensitive to fluctuations of the experimental parameters, which are the main source of gate error in the recent experiment [14]. Since the errors arising from the fluctuations of experimental parameters are suppressed by taking advantage of the adiabatic passage, the main error sources are the residual thermal excitation, addressing errors, and off-resonant excitations, which are about 2%, 3%, and 4% in the experiment of Ref. [14]. Technical improvements such as the improved control of the addressing beam will increase the fidelity. It should be noted that here we just illustrate the new kind of phase gates in the ion trap system. This kind of phase gates are also realizable in other physical systems.

In conclusion, we have presented a new class of two-qubit quantum phase gates. This new kind of phase gates does not depend on the dynamical phase shift since they work in the dark space. Neither is the phase of geometric origin. The conditional phase shift is achieved by the adiabatic evolution of the dark state itself. This kind of phase gates has the advantage of being insensitive to small fluctuations of experimental parameters. In comparison with the adiabatic geometric gates, the nontrivial cyclic loop is unnecessary, and thus the errors in obtaining the required solid angle are avoided, which makes this new kind of phase gates superior to the geometric gates. We illustrate the idea in the ion trap system. However, it can also be realized in other systems. The idea opens a new prospect for realizing high-fidelity phase gates in various physical systems.

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