Casimir Zero-Point Radiation Pressure

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(Received 3 January 2005; revised manuscript received 3 August 2005; published 19 August 2005)

We analyze some consequences of the Casimir-type zero-point radiation pressure. These include macroscopic "vacuum" forces on a metallic layer in between a dielectric medium and an inert [$\epsilon(\omega) = 1$] one. Ways to control the sign of these forces, based on dielectric properties of the media, are thus suggested. Finally, the large positive Casimir pressure, due to surface plasmons on thin metallic layers, is evaluated and discussed.

DOI: 10.1103/PhysRevLett.95.080404 PACS numbers: 03.70.+k, 12.20.Fv, 42.50.Ct, 42.50.Lc

Imagine polarizable bodies that are placed in vacuum. Their interaction with the electromagnetic field (which can often be described by boundary conditions on the latter at the surfaces of the bodies) may produce a change in the zero-point energy of the field. Should that energy depend on, for example, the distance between two of these bodies, forces between these two bodies will follow. This can be regarded as the origin of the van der Waals molecular forces [1], which change at large separations due to retardation effects [2]. For the simpler case of two large parallel conducting plates, the Casimir force [3] [cf. Eq. (4) below] results at large separations (where retardation is important) between the plates and becomes the Lifshitz force [4,5] at small separations (where quasistationarity applies). The crossover between the short- and long-distance behaviors occurs for distances on the order of the velocity of light divided by the characteristic excitation frequency of the bodies (i.e., about 200 A for $\hbar\omega = 10$ eV). Even for a single body, volume- and shape-dependent [6] forces will arise when the field energy depends on these parameters. The Casimir force has by now been amply confirmed by experiment [7]. Corrections due to finite temperatures, realistic surfaces, etc., are becoming relevant [8]. The Casimir effect may be crucial to nanomechanical devices [9]. Its relevance is not limited to the electromagnetic field only. It should exist with any physical field that interacts with matter.

Besides its general interest *vis-à-vis* the observability of the (changes of the) vacuum energy [10] and genuine relevance to molecular and colloidal forces, the Casimir effect touches upon several fundamental questions of physics. These range from "vacuum friction" to the value of the cosmological constant and the modifications of classical Newtonian gravitation on small scales. The reader is referred to several books and review articles, which discuss the many aspects of the Casimir effect [11–16].

A problem of principle, which arises in the calculation of Casimir-type forces, is the well-known UV divergence of the electromagnetic vacuum energy. This divergence is clearly physically irrelevant here, since what matters are only the *differences* of energies. For a good discussion of the cutoff procedure, see [17]. Ordinary metals are basically transparent at high frequencies, above the characteristic plasma frequency ω_p, which is therefore a natural cutoff. It is clear that waves with $\omega \gg \omega_p$ do not "see" the bodies and therefore are irrelevant. In his original calculation, Casimir in fact first employed a soft cutoff as above and then made a judicious subtraction of a large energy to obtain a finite, universal, and cutoff-independent result. This subtraction procedure is rather tricky. While various physical interpretations for it have been suggested in the literature, none of them is truly satisfactory. We shall start by physically analyzing Casimir's subtraction procedure. Before that, we remark that cutoff dependence can be allowed when the cutoff is based on physical considerations. For example, the Lifshitz forces in the static limit do depend on the cutoff ω_p, where ω_p is the plasma frequency of the metals. Another example of cutoff dependence will be discussed in this Letter.

In 1948, Casimir [3] considered the force between two large metallic plates placed parallel to the x-y plane, with a distance d along the z axis between their internal faces, and $d \gg c/\omega_p$. The zero-point energy of the field between the plates is

$$E_0(d) = \hbar c \frac{L^2}{\pi^2} \int^{(c)} d^2 k_\perp \sum_{(0)}^{\infty} \left(n^2 \frac{\pi^2}{d^2} + k_\perp^2 \right)^{1/2}, \quad (1)$$

where $\int^{(c)}$ means that the integrand is multiplied by a soft cutoff function which vanishes smoothly around and above $|k_p| = \omega_p/c$, and $\sum_{(0)}^{\infty}$ means that the $n = 0$ term is multiplied by $1/2$. The corresponding subtracted quantity is

$$E_0'(d) = E_0(d) - \text{subtraction}. \quad (2)$$

The force between the plates is given by

$$F = -\frac{\partial E_0'(d)}{\partial d}, \quad (3)$$

where positive F means repulsion between the plates. Casimir chose to subtract in Eq. (2) the same expression but with the sum over n converted to an integral, as appropriate for very large d. Thus, the subtraction is that of the energy for the plates "at infinity" (questions such as whether the plates have a finite thickness and, if so, what happens beyond them are left open). Evaluating the difference between the sum and the integral over n with the Euler-Maclaurin formula, he arrived at the following celebrated result:

$$P_c = F/L^2 = -\hbar c \frac{\pi^2}{240} \frac{1}{d^4}. \tag{4}$$

For the unretarded, quasistationary limit, $d \ll c/\omega_p$, a length $\sim c/\omega_p$ replaces one power of d in the denominator of Eq. (4), as found by Lifshitz [4].

A clear physical justification for the subtraction procedure is clearly called for. It is immediately suggested [18–20] (and in fact hinted in Casimir's original paper; see also Ref. [3] of [19]) that the physical significance of the above subtraction is in obtaining the difference between the radiation pressures of the zero-point fields between the plates and outside of the plates. This idea was advocated and followed up in Refs. [18,19]. The purpose of the present Letter is to analyze some new consequences of this interpretation of the subtraction. Neither it nor the other regularization procedures are truly satisfactory. Therefore, it is of interest to compare the new results following from this interpretation of the subtraction procedure with experiments to come.

We follow Refs. [18,19] in calculating the pressure of the zero-point electromagnetic field, but here we present a somewhat different derivation. We take a large vessel [21] with conducting walls. The vessel is taken to be a box with dimensions L_x, L_y, L_z, $V \equiv L_x L_y L_z$. The pressure in the z direction is given by the momentum imparted to the wall per unit area per unit time [19]:

$$P_0 = \hbar \sum_{k_x,k_y,k_z}^{(c)} c(k) \frac{k_z^2}{k}/V, \tag{5}$$

where $k_x = n_x \pi / L_x$, with $n_x > 0$, etc., and $c(k)$ is the light (group) velocity $(d\omega_k/dk)$ as a function of $k \equiv \sqrt{k_x^2 + k_x^2 + k_z^2}$, slightly generalizing the result of Ref. [19]. The symbol (c) above the summation sign signifies an upper cutoff around the plasma frequency of the walls, necessary to control the divergence, as discussed above. To derive this result, one may calculate [22] $-(\hbar/L_y L_x)(\partial\omega_k/\partial L_z) = [\hbar c(k) k_z^2]/kV$ and sum over the levels, canceling the factor of $1/2$ in the zero-point energy and the degeneracy of each k mode.

For a large system, the sum can be replaced by an integral. We perform the angular integrations and change variables from k to frequency, ω, obtaining

$$P_0 = \frac{\hbar}{6\pi^2 c^3} \int^{(c)} d\omega\, \omega^3 \epsilon(\omega)^{3/2}, \tag{6}$$

where we used the frequency-dependent $\epsilon(\omega)$ via $\omega = [c/\epsilon(\omega)^{1/2}]k$. The superscript (c) signifies an upper cutoff around the plasma frequency of the walls, as above. By defining $\bar{c}$ as a suitable average of the light velocity (i.e., $c^3/\bar{c}^3 = [\int^{(c)} d\omega\, \omega^3 \epsilon(\omega)^{3/2}]/[\int^{(c)} d\omega\, \omega^3]$), one obtains

$$P_0 \cong \frac{\hbar\omega_p^4}{24\pi^2 \bar{c}^3}. \tag{7}$$

We used an approximate equality, since the result was given for a sharp cutoff. The basic physical assumption here is that modes below ω_p give much of their momentum to the wall, while those above do not. This assumption can be very easily justified for the 1D waveguide case.

One might try to use here [as in Eq. (3), based on [3]] the $T = 0$ thermodynamic relationship (see also [23]):

$$P_0 = -\frac{\partial E_0}{\partial V} = -\frac{\partial}{\partial V} \int^{(c)} V d\omega \frac{\omega^2}{\pi^2 c^3} \frac{\hbar\omega}{2}. \tag{8}$$

This would produce a *negative* pressure. However, Eq. (8) is valid only for a closed system, while the present system exchanges energy with the continuum levels above the cutoff at ω_p. When the volume varies, this happens via zero-point photon levels moving through the cutoff. Interestingly, Casimir used the same relationship to calculate the net pressure on each plate. We believe that this may be justified for pressure differences, but *only* when the media on the two sides of each metallic plate are equivalent. This point will be more fully discussed elsewhere.

The pressure of Eq. (7) is not so large but quite significant. It is convenient to express it in terms of a Bohr (or Fermi) pressure $P_B \equiv 10\ \text{eV/A}^3 \simeq 1.6 \times 10^8\ \text{N/cm}^2$. For $\hbar\omega_P = 10$ eV and $\epsilon = 1$, we find $P_0 \sim 0.5 \times 10^{-9} P_B \sim 0.1\ \text{N/cm}^2$. For comparison, the ordinary Casimir force/unit area at a distance of 100 nm is on the order of $10^{-3}\ \text{N/cm}^2$.

Since the ordinary Casimir force is the result of the near cancellation of much larger quantities, its sign is notoriously difficult to predict, except via detailed calculations [20,24]. We suggest that some control of the sign can be achieved [25] by employing polarizable materials as the electromagnetic vacuum in some part of the system. For a material with a dielectric constant $\epsilon(\omega)$, Eq. (6) suggests that if the suitably averaged value $\bar{\epsilon} > 1$ (where $\bar{c} \equiv c/\sqrt{\bar{\epsilon}}$), which should often happen, the pressure of the dielectric will exceed that of the vacuum by ΔP:

$$\Delta P \equiv P_0(\epsilon) - P_0(1) \cong \frac{\hbar\omega_p^4}{24\pi^2}\left(\frac{1}{\bar{c}^3} - \frac{1}{c^3}\right). \tag{9}$$

Thus, for example, a metallic wall having a dielectric medium with such an $\epsilon(\omega)$ on one side and a medium with $\epsilon = 1$ on the other, both having the same mechanical pressure, will be attracted into the vacuum [26]. As a weak example, we take $\epsilon(\omega) = 2$ up to 0.05 of the metallic ω_p and going to 1 above that frequency; this will give a net force per unit area of $\sim 10^{-6}\ \text{N/cm}^2$. Thus, for example, the volume and the pressure of a fluid *depend* on both the optical properties of another fluid separated from the first by a solid slab and the optical properties of that slab. It may be possible to observe the displacement of the slab's position and interfaces by changing the dielectrics. One may also contemplate changing the optical properties of a semiconducting slab in the relevant range by using, e.g., induced photoconductivity or gating. The resulting small changes of the macroscopic dimensions or pressures may then be observed, in principle, with an interferometric method or using piezoelectric detectors. This would constitute a *macroscopic* version of the Casimir effect.

Things become rather interesting also for the ordinary, mesoscopic-scale Casimir effect. A good check of the pressure interpretation of the Casimir subtraction is the following: Consider the case where the medium outside the plates is "inert" ($\epsilon = 1$) and the medium between them has an $\epsilon(\omega)$, with $\bar{\epsilon} > 1$. The conventional calculations treat the case in which these two media are identical [with the same $\epsilon(\omega)$] [5]. Let us then start with both the inside and outside media identical and having an $\epsilon(\omega)$. The Casimir pressure in this case was calculated in Ref. [5]. We denote it by $P_c(\epsilon)$. In the case of interest to us, the medium outside is inert, so we have to subtract the pressure of the vacuum rather than the pressure of the dielectric medium. We then find that the net Casimir pressure is, in our case,

$$P_c(\epsilon \text{ inside}, 1 \text{ outside}) = P_c(\epsilon) - P_0(\epsilon = 1) + P_0(\epsilon). \tag{10}$$

Therefore, for sufficiently large $\epsilon(\omega)$ the sign of the force will change and it will push the plates away. For the aforementioned example, considered below Eq. (9), this repulsion will win against the Casimir attraction around a distance of about 0.8μ. For larger distances, the full Casimir force should ideally be repulsive; see, however, [26]. (For smaller distances the Casimir force will remain attractive.) This change of sign is due to the larger "volume force" due to the dielectric inside.

At distances below c/ω_p, where quasistationarity holds, the outside pressure P_0 may again be smaller than the inside Lifshitz pressure. Interesting effects due to dielectric media placed between or outside of the plates are possible and will be discussed elsewhere.

We conclude this Letter by examining the Casimir vacuum forces on a single flat metallic plate of thickness d. For large thicknesses, we simply have the two pressures, P_0, from the two sides of the metallic layer [18]. These will slightly decrease the thickness of the layer, a very interesting effect which can be increased with dielectric materials as discussed above and might be observable some day. In addition to the ordinary electromagnetic modes considered so far, there will be surface plasmons [4,5,28–30] running on the two interfaces of the layer. For a thick layer, the energy of these modes will be independent of d, but once d becomes comparable to the decay lengths of the modes, their energies will depend on d and lead to a significant further positive pressure on the metallic plate.

To calculate that pressure, we consider a metallic slab with a dielectric constant $\epsilon(\omega) = 1 - \frac{\omega_p^2}{\omega^2}$ and of thickness $d = 2a$, larger than atomic dimensions, between the planes $z = \pm a$. Following Ref. [28], we approximate in the quasistationary limit, $d \lesssim c/\omega_p$, the full wave equation by the Laplace one for the electrostatic potential ϕ. We take without loss of generality a wave propagating in the x direction, $\phi(x, z) = \exp(ikx)u(z)$, and find $u'' = ku$. Thus $u = \sum_\pm A_\pm \exp(\pm kz)$ inside the film, and u is exponentially decaying in the two vacua (with $\epsilon = 1$) on the two sides of the film. On the surfaces of the film, ϕ and $\epsilon \frac{\partial\phi}{\partial z}$ are continuous. By symmetry, we choose even and odd solutions with respect to $z = 0$, and find the surface plasmons' dispersion relations:

$$\omega_\pm(k) = \frac{\omega_p}{\sqrt{2}}\sqrt{1 \mp e^{-kd}}, \tag{11}$$

where the upper (lower) sign is for the even (odd) modes. In the extreme quasistationary limit, $d \ll c/\omega_p$, we may neglect the polariton effect—the coupling of the above modes with the "light modes" $\omega = ck$. The dispersion of the latter is extremely steep and intersects the $\omega_\pm(k)$ dispersion only at very small values of k.

To obtain the force, one needs the derivative with respect to d of the d-dependent total zero-point energy of these plasmons. One may either directly take the derivative with respect to d or first integrate the energies subtracting from each branch an infinite d-independent constant, which is the $k \to \infty$ limit of both dispersion curves:

$$E_0(d) = \frac{1}{2}\sum_\pm \hbar\left(\frac{L}{2\pi}\right)^2 \int d^2k\left(\omega_\pm(k) - \frac{\omega_p}{\sqrt{2}}\right), \tag{12}$$

$$F(d) = -\frac{\partial}{\partial d}E_0(d). \tag{13}$$

In both ways, we find for the (positive) pressure exerted by the vacua on the metallic film a result resembling the Lifshitz pressure in the nonretarded regime [4,5]:

$$P(d) = \frac{F(d)}{L^2} = 0.0078\frac{\hbar\omega_p}{d^3}. \tag{14}$$

This pressure is quite substantial and increases markedly with decreasing d. It would be on the order of 2×10^6 N/cm^2 for a 1 A thin film—almost approaching the Fermi pressure scale for atomic thicknesses. The Fermi (including the Coulomb) pressure will eventually stabilize the very thin layer against squeezing by the vacuum pressure. The contraction proportional to d^{-3} of the film in the thin direction may well be observable on top of other thin-film effects. These considerations are clearly relevant for the physics of very thin films.

To summarize, we considered the radiation pressure of bulk zero-point electromagnetic modes. The dependence of the force on the dielectric constant of the electromagnetic vacuum leads to a novel type of force in asymmetric situations where the conducting slab has different dielectrics on its two sides. Options for controlling the sign in the Casimir-type geometry are suggested. Finally, the substantial positive pressure, associated with the surface plasmons, exerted by the electromagnetic vacuum on a thin metallic film was evaluated and discussed.

The author thanks M. Aizenman, A. Aharony, O. Entin-Wohlman, U. Gavish, Y. Levinson, S. Nussinov, S. Rubin, A. Schwimmer, A. Stern, and Z. Vager, and especially T. Emig and W. Kohn, for discussions and comments on the manuscript. In particular, I thank M. Milgrom for pointing out the error in using Eq. (8), which was made in a previous, unpublished version of this Letter. This work was supported by a Center of Excellence of the Israel Science Foundation and by the German Federal Ministry of Education and Research (BMBF), within the framework of the German-Israeli Project Cooperation (DIP).

[1] F. London, Z. Phys. **63**, 245 (1930).
[2] H. B. G. Casimir and D. Polder, Phys. Rev. **73**, 360 (1948).
[3] H. B. G. Casimir, Proc. K. Ned. Akad. Wet. **51**, 793 (1948).
[4] E. M. Lifshitz, Sov. Phys. JETP **2**, 73 (1956).
[5] I. E. Dzyaloshinskii and L. P. Pitaevskii, Sov. Phys. JETP **9**, 1282 (1959); I. E. Dzyaloshinskii, E. M. Lifshitz, and L. P. Pitaevskii, Sov. Phys. JETP **10**, 161 (1960); Adv. Phys. **10**, 165 (1961).
[6] R. Balian and B. Duplantier, quant-ph/0408124.
[7] B. V. Deriaguin and I. I. Abrikosova, Sov. Phys. JETP **3**, 819 (1957); M. J. Sparnaay, Physica (Utrecht) **24**, 751 (1958); A. Anderson, S. Haroche, E. A. Hinds, W. Jhe, and D. Meschede, Phys. Rev. A **37**, 3594 (1988); S. K. Lamoreaux, Phys. Rev. Lett. **78**, 5 (1997); **81**, 5475(E) (1998); U. Mohideen and A. Roy, Phys. Rev. Lett. **81**, 4549 (1998); B. W. Harris, F. Chen, and U. Mohideen, Phys. Rev. A **62**, 052109 (2000); G. Bressi, G. Carugno, R. Onofrio, and G. Ruoso, Phys. Rev. Lett. **88**, 041804 (2002).
[8] S. Reynaud, A. Lambrecht, and C. Genet, quant-ph/0312224.
[9] H. B. Chan, V. A. Aksyuk, R. N. Kleiman, D. J. Bishop, and F. Capasso, Science **291**, 1941 (2001); E. Buks and M. L. Roukes, Europhys. Lett. **54**, 220 (2001).
[10] The very existence of the Casimir effect still seems provocative to some. It might be useful to point out that it is not really that different from including the dependence of the ground state energy of the electrons in a molecule on the nuclear coordinates, in the potential energy of the latter. This follows from the basic principles of quantum mechanics, as long as the Born-Oppenheimer conditions hold, including the fermionic case. For an example of a fermionic Casimir effect, see A. Bulgac and A. Wirzba, Phys. Rev. Lett. **87**, 120404 (2001).
[11] P. W. Milonni, *The Quantum Vacuum* (Academic, New York, 1994).
[12] V. M. Mostepanenko and N. N. Trunov, *The Casimir Effect and Its Applications* (Clarendon, Oxford, 1997).
[13] M. Bordag, U. Mohideen, and V. M. Mostepanenko, Phys. Rep. **353**, 1 (2001).
[14] Y. S. Barash and V. Ginzburg, Sov. Phys. Usp. **27**, 467 (1984).
[15] G. Plunien, B. Muller, and W. Greiner, Phys. Rep. **134**, 87 (1986).
[16] K. A. Milton, *The Casimir Effect: Physical Manifestations of Zero-Point Energy* (World Scientific, Singapore, 2001).
[17] H. W. J. Blöte, J. L. Cardy, and M. P. Nightingale, Phys. Rev. Lett. **56**, 742 (1986).
[18] A. Gonzalez, Physica (Amsterdam) **131A**, 228 (1985).
[19] P. W. Milonni, R. J. Cook, and M. E. Goggin, Phys. Rev. A **38**, 1621 (1988).
[20] V. Hushwater, Am. J. Phys. **65**, 381 (1997); quant-ph/9909084.
[21] On the right-hand side of the Casimir geometry, this vessel is usually taken to consist of the right-hand side infinite plate and another parallel infinite plate very far away on the right.
[22] For the nth photon eigenstate of an infinite square well of size L, a momentum $\frac{2\hbar n\pi}{L}$ is given to each wall in each period $2L/c$. This is identical, as it should, to (minus) the derivative with respect to L of the energy $\frac{\hbar c n\pi}{L}$.
[23] G. Barton, J. Phys. A **34**, 4083 (2001); R. L. Jaffe, AIP Conf. Proc. **687**, 3 (2003)..
[24] T. H. Boyer, Phys. Rev. **174**, 1764 (1968).
[25] A change of sign, due to different signs of dielectric function differences with the middle layer, has been found in Ref. [5] in the asymmetric three-layer situation. It appears to be due to a different physical origin from the change of sign discussed here in the symmetric five-layer situation.
[26] This force will lead to a finite displacement of the slab, balanced by the elasticity (or the surface tension) of the medium; see also [4,5,27].
[27] L. P. Pitaevskii, cond-mat/0505754.
[28] N. G. van Kampen, B. R. A. Nijboer, and K. Schram, Phys. Lett. **26A**, 307 (1968).
[29] E. A. Stern and R. A. Ferrel, Phys. Rev. **120**, 130 (1960).
[30] H. Raether, *Surface Plasmons* (Springer-Verlag, Berlin, 1988), Vol. 111.