Four-Body Problem and BEC-BCS Crossover in a Quasi-One-Dimensional Cold Fermion Gas

C. Mora,[1] A. Komnik,[2] R. Egger,[1] and A. O. Gogolin[3]

[1]*Institut für Theoretische Physik, Heinrich-Heine-Universität, D-40225 Düsseldorf, Germany*
[2]*Physikalisches Institut, Albert-Ludwigs-Universität, D-79104 Freiburg, Germany*
[3]*Department of Mathematics, Imperial College London, 180 Queen's Gate, London SW7 2BZ, United Kingdom*

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The four-body problem for an interacting two-species Fermi gas is solved analytically in a confined quasi-one-dimensional geometry, where the two-body atom-atom scattering length $a_{\rm aa}$ displays a confinement-induced resonance. We compute the dimer-dimer scattering length $a_{\rm dd}$ and show that this quantity completely determines the many-body solution of the associated BEC-BCS crossover phenomenon in terms of bosonic dimers.

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Cold atomic quantum gases continue to attract a lot of attention due to their high degree of control, tunability, and versatility. A main topic of interest has been the exploration of the Bose-Einstein condensate (BEC)-BCS crossover in fermionic systems [1–6]. In two or three dimensions, this is still a controversial and not completely settled issue on the theory side [7–10], despite the qualitative agreement between mean-field theories and experimental data. Notably, a similar (but different) crossover phenomenon has been predicted to occur in quasi-one-dimensional (1D) systems [11,12], where a cylindrical trap leads to a confinement-induced resonance (CIR) [13,14] in the atom-atom interaction strength, analogous to the magnetically tuned Feshbach resonance [8]. In contrast to what happens in 3D, one *always* has a two-body bound state ("dimer") in 1D, regardless of the sign of the 3D atom-atom scattering length a. We solve the fermionic four-body problem in the confined geometry, and compute the dimer-dimer scattering length $a_{\rm dd}$ throughout the full BCS-BEC crossover, on each side of the CIR. On the "BEC" side, we establish contact with results for the unconfined case [15], while on the "BCS" side, a simple Bethe ansatz calculation provides exact results. The three-body problem has no trimer solution [16], and thus the *full many-body crossover solution can be expressed in terms of* $a_{\rm dd}$ alone and is thereby solved completely in this Letter. Since 1D atomic gases can be prepared and probed thanks to recent advances [17–19], our predictions could be observed in state-of-the-art experiments.

We assume two fermion hyperfine components (denoted by $\uparrow, \downarrow$) with identical particle numbers $N_\uparrow = N_\downarrow = N/2$, interacting only via s-wave interactions. At low energies, the pseudopotential approximation [20] for the 3D interaction among unlike fermions applies, $V(\mathbf{r}) = (4\pi\hbar^2 a/m_0)\delta(\mathbf{r})\partial_r(r\cdot)$ (m_0 is the mass). We consider the transverse confinement potential $U_c(\mathbf{r}) = m_0\omega_\perp^2(x^2 + y^2)/2$, with length scale $a_\perp = (2\hbar/m_0\omega_\perp)^{1/2}$. The solution of the two-body problem [13,14] reveals that a single dimer (composite boson) state exists, where the dimensionless binding energy Ω_B and (longitudinal) size a_B,

$$\Omega_B = -\frac{E_B}{2\hbar\omega_\perp} = (a_\perp/2a_B)^2 > 0, \tag{1}$$

follow from $\zeta(1/2, \Omega_B) = -a_\perp/a$ with the Hurvitz zeta function. For an experimental verification, see Ref. [18]. For $a_\perp/a \to -\infty$, the BCS limit with $\Omega_B \simeq (a/a_\perp)^2 \ll 1$ and $a_B \simeq a_\perp^2/2|a|$ is reached, while for $a_\perp/a \to +\infty$, the dimer (or BEC) limit emerges, with $\Omega_B \simeq (a_\perp/2a)^2 \gg 1$ and $a_B \simeq a$. The atom-atom scattering length is

$$a_{\rm aa} = a_\perp(C - a_\perp/a)/2, \qquad C = -\zeta(1/2) \simeq 1.4603. \tag{2}$$

For low energies, this result is reproduced by the 1D atom-atom interaction $V_{\rm aa}(z, z') = g_{\rm aa}\delta(z - z')$ with $g_{\rm aa} = -2\hbar^2/m_0 a_{\rm aa}$ [13]. The CIR (where $g_{\rm aa} \to \pm\infty$) takes place for $a_\perp/a = C$, which is equivalent to $\Omega_B = 1$. In this Letter, we solve the 1D fermionic four-body ($\uparrow\uparrow\downarrow\downarrow$) problem and show that this also solves the N-body problem for arbitrary Ω_B in the low-energy regime.

Let us first discuss general symmetries of the four-body problem. We denote the positions of the $\uparrow$ ($\downarrow$) fermions by $\mathbf{x}_{1,4}$ ($\mathbf{x}_{2,3}$), respectively, and then form distance vectors between unlike fermions, $\mathbf{r}_1 = \mathbf{x}_1 - \mathbf{x}_2$, $\mathbf{r}_2 = \mathbf{x}_4 - \mathbf{x}_3$, and $\mathbf{r}_+ = \mathbf{x}_1 - \mathbf{x}_3$, $\mathbf{r}_- = \mathbf{x}_4 - \mathbf{x}_2$. The distance vector between dimers {12} and {34} is $\mathbf{R}/\sqrt{2} = (\mathbf{x}_1 + \mathbf{x}_2 - \mathbf{x}_3 - \mathbf{x}_4)/2$. After an orthogonal transformation, the center-of-mass coordinate decouples and the four-body wave function Ψ depends only on $\mathbf{r}_{1,2}$ and $\mathbf{R}$. With respect to dimer interchange, Ψ is symmetric,

$$\Psi(\mathbf{r}_1, \mathbf{r}_2, \mathbf{R}) = \Psi(\mathbf{r}_2, \mathbf{r}_1, -\mathbf{R}), \tag{3}$$

while under the exchange of identical fermions,

$$\Psi(\mathbf{r}_1, \mathbf{r}_2, \mathbf{R}) = -\Psi(\mathbf{r}_\pm, \mathbf{r}_\mp, \pm(\mathbf{r}_1 - \mathbf{r}_2)/\sqrt{2}). \tag{4}$$

The four-body Schrödinger equation then reads

$$\Big[-\frac{\hbar^2}{m_0}(\Delta_{\mathbf{r}_1} + \Delta_{\mathbf{r}_2} + \Delta_{\mathbf{R}}) + U_c(\mathbf{r}_1) + U_c(\mathbf{r}_2) + U_c(\mathbf{R}) + V(\mathbf{r}_2) - E\Big]\Psi = -\sum_{i=1,\pm} V(\mathbf{r}_i)\Psi. \tag{5}$$

The pseudopotentials on the right-hand side are incorporated via Bethe-Peierls boundary conditions imposed when a dimer is contracted, e.g.,

$$\Psi(\mathbf{r}_1, \mathbf{r}_2, \mathbf{R})|_{\mathbf{r}_1\to 0} \simeq \frac{f(\mathbf{r}_2, \mathbf{R})}{4\pi r_1}(1 - r_1/a). \tag{6}$$

All other boundary conditions can also be expressed in terms of $f(\mathbf{r}, \mathbf{R})$ using Eqs. (3) and (4), where

$$f(\mathbf{r}, \mathbf{R}) = f(-\mathbf{r}, -\mathbf{R}) \tag{7}$$

expresses (parity) invariance of Eq. (5) under $\mathbf{r}_{1,2} \to -\mathbf{r}_{1,2}$ and $\mathbf{R} \to -\mathbf{R}$ in combination with Eq. (3). In order to appreciate the importance of Eq. (7), it is instructive to expand $f(\mathbf{r}, \mathbf{R})$ in terms of the single-particle eigenfunctions $\psi_\lambda(\mathbf{r})$ and the two-body scattering states $\Phi_\lambda(\mathbf{r})$ in the presence of the confinement,

$$f(\mathbf{r}, \mathbf{R}) = \sum_{\mu\nu} f_{\mu\nu}\Phi_\mu(\mathbf{r})\psi_\nu(\mathbf{R}). \tag{8}$$

The quantum numbers λ include the 1D momentum k [21], the (integer) angular momentum m, and the radial quantum number $n = 0, 1, 2, \ldots$. Explicit expressions for ψ_λ and Φ_λ can be found in Refs. [13,16]. While both have the same energy E_λ, the Φ_λ now include the dimer bound state (denoted by $\lambda = 0$) $\Phi_0(\mathbf{r})$. For relative longitudinal momentum k of the two dimers, the total energy is (excluding zero-point and center-of-mass motion)

$$E = -2\hbar\omega_\perp\Omega_B + \frac{\hbar^2 k^2}{2m_0}. \tag{9}$$

We consider the low-energy regime $ka_\perp < 1$, where the relative dimer motion is in the lowest transverse state ($n = m = 0$) when dimers are far apart. We then have to deal with a 1D dimer-dimer scattering problem in this "open" channel, where the asymptotic 1D *scattering state* $f_0(Z)$ for $|Z| \gg \max(a_\perp, |a_{\rm aa}|)$ follows from Eq. (8) as

$$f(\mathbf{r}, \mathbf{R}) = \Phi_0(\mathbf{r})\psi_{\perp,00}(\sqrt{X^2 + Y^2})f_0(Z), \tag{10}$$

where $\psi_{\perp,00}$ is the transverse part of $\psi_{n=0,m=0}$. The symmetry relation (7) now enforces $f_0(Z) = f_0(-Z)$, reflecting the fact that two (composite) *bosons* collide, i.e.,

$$f_0(Z) = e^{-ik|Z|} + [1 + 2\tilde{f}(k)]e^{ik|Z|}. \tag{11}$$

As long as only s-wave scattering is important, symmetry considerations thus rule out odd-parity solutions normally present in 1D scattering problems [13,16]. This crucial observation implies that, assuming analyticity, the 1D scattering amplitude can be expanded in terms of a 1D *dimer-dimer scattering length* $a_{\rm dd}$ [22],

$$\tilde{f}(k) = -1 + ika_{\rm dd} + \mathcal{O}(k^2). \tag{12}$$

For $|ka_{\rm dd}| \ll 1$, this also follows from the zero-range 1D dimer-dimer potential

$$V_{\rm dd}(Z, Z') = g_{\rm dd}\delta(Z - Z'), \qquad g_{\rm dd} = -\frac{2\hbar^2}{(2m_0)a_{\rm dd}}. \tag{13}$$

We stress that Eq. (13) holds for arbitrary $a_\perp/a$, and therefore 1D dimer-dimer scattering at low energies is always characterized by a simple δ interaction.

Let us then analyze the *BCS limit,* $\Omega_B \ll 1$, where the scattering problem is kinematically 1D on length scales exceeding $a_\perp$. Projecting Eq. (5) onto the transverse ground state, the 1D Schrödinger equation for four attractively interacting fermions reads with $a_{\rm aa} = a_\perp^2/2|a| \gg a_\perp$ [see Eq. (2)],

$$\left(\frac{2m_0E}{\hbar^2} + \sum_{i=1}^{4}\partial_{z_i}^2 + \frac{4}{a_{\rm aa}}\sum_{i<j}\delta(z_i - z_j)\right)\Psi = 0, \tag{14}$$

where the second sum excludes identical fermion pairs, (i, j) corresponding to $\{14\}$ and $\{23\}$. The Bethe ansatz expresses the wave function as a sum of products of plane waves [23]. Let us choose the momenta $a_{\rm aa}k_{1,4} = \mp i - u/2$ and $a_{\rm aa}k_{3,2} = \mp i + u/2$ to describe dimer-dimer scattering, and measure lengths in units of $a_{\rm aa}$. The energy of this state is $E = \hbar^2(-2 + u^2/2)/(m_0a_{\rm aa}^2)$ and u the relative momentum of the two dimers. Up to an overall normalization constant, the wave function in the domain $\mathcal{D}_1 = \{(z_1, z_4) < (z_3, z_2)\}$ must then be given by

$$\Psi_1 = e^{-(z_2+z_3-z_4-z_1)}\big(e^{iu(z_2+z_4-z_3-z_1)/2} - e^{iu(z_2+z_1-z_3-z_4)/2} + e^{iu(z_3+z_1-z_2-z_4)/2} - e^{iu(z_3+z_4-z_2-z_1)/2}\big)$$

to ensure a normalizable and antisymmetric solution under the exchange of identical fermions. Consider next a second domain, $\mathcal{D}_2 = \{z_1 < z_3 < z_4 < z_2\}$, where z_3 and z_4 are exchanged compared to $\mathcal{D}_1$. At the boundary between $\mathcal{D}_1$ and $\mathcal{D}_2$, $z_3 = z_4$, Eq. (14) implies $\Psi_1 = \Psi_2$ and $(\partial_{z_3} - \partial_{z_4})(\Psi_1 - \Psi_2) = -4\Psi_1$ [24], leading to

$$\Psi_2 = 2\mathrm{Re}\Bigg[e^{-(z_2+z_3-z_4-z_1)}\frac{iu}{2+iu}e^{iu(z_2+z_4-z_3-z_1)/2} + e^{-(z_2+z_4-z_3-z_1)} \times\left(\frac{2}{2+iu}e^{iu(z_2+z_3-z_4-z_1)/2} - 2e^{iu(z_2+z_1-z_4-z_3)/2)}\right)\Bigg].$$

The wave function in other domains can be found in a similar manner. As a result, for a large dimer-dimer distance Z, $\Psi \propto e^{-|z_+|}e^{-|z_-|}f_0(Z)$, where $e^{-|z_\pm|}$ is the 1D wave function of the dimers $\{13\}$ and $\{24\}$, respectively. This result shows explicitly that even in the BCS limit, the *two dimers are not broken* in the collision even for large k. There is no coupling to additional fermionic states, and the *composite* nature of the dimer is not apparent in Ψ. The 1D scattering state $f_0(Z)$ [see Eq. (11)] has the *exact* scattering amplitude

$$\tilde{f}(k) = -\frac{1}{1 + ika_{\rm dd}}, \qquad a_{\rm dd} = \frac{a_{\rm aa}}{2} = \frac{a_\perp^2}{4|a|}, \tag{15}$$

which reproduces the full scattering amplitude derived from Eq. (13) and not just the first order as in Eq. (12). The bound state at imaginary k predicted by Eq. (15) is, however, unphysical, since the corresponding Bethe ansatz solutions are then not normalizable. It would correspond to a nonexistent bound four-fermion (tetramer) state, and hence Eq. (15) is restricted to the real axis.

Let us now turn to the many-body problem, starting with the BCS limit. Since dimers are not broken in the collision, the ground state can be described in terms of $N/2$ bosons ("bosonization") with the interaction (13) and $a_{\rm dd} = a_{\rm aa}/2$. The attractively interacting Bose gas is stabilized by the real-k restriction, implying the omission of many-body bosonic bound states. Bosonization is possible for $\rho a_\perp < 1$, since typical momenta are $k \approx \rho$ for total 1D fermionic density ρ. This reasoning immediately leads to the famous Lieb-Liniger (LL) equations [25],

$$\frac{E_0}{N} = -\hbar\omega_\perp\Omega_B + \frac{1}{\rho}\int_{-K_0}^{K_0} dk \frac{\hbar^2k^2}{4m_0} f(k), \tag{16a}$$

$$2\pi f(k) = 1 - \frac{4}{a_{\rm dd}}\int_{-K_0}^{K_0} dp \frac{f(p)}{4/a_{\rm dd}^2 + (p-k)^2}, \tag{16b}$$

where E_0 is the ground state energy and K_0 is fixed by $\rho/2 = \int_{-K_0}^{K_0} dk f(k)$. Notably, since $a_{\rm dd} = a_{\rm aa}/2$, the LL equations coincide with Yang-Gaudin equations for N attractively interacting 1D fermions, thereby explaining a deep connection noticed previously [11,12,26]. Moving towards the dimer limit, Eq. (13) still applies, but now only for sufficiently small k such that Eq. (12) holds, and $a_{\rm dd} \neq a_{\rm aa}/2$. For $a_{\rm dd} \lesssim a_\perp$, one leaves the BCS regime and enters the "crossover regime," while (once $a_{\rm dd} < 0$) the dimer regime is realized for $|a_{\rm dd}| \gtrsim a_\perp$. Within the crossover regime, $|a_{\rm dd}| \lesssim a_\perp$, we have hard-core bosons that can effectively be fermionized [11,12], again implying typical momenta $k \approx \rho$. For $\rho a_\perp < 1$, the condition $|ka_{\rm dd}| \ll 1$ imposed by Eq. (12) is therefore safely fulfilled throughout the crossover regime. Finally, in the *dimer limit*, $a < a_\perp$, fermions form very tightly bound dimers. The confinement can then not influence the four-body collision, which is therefore described by a 3D zero-range interaction with $a_{\rm dd}^{\rm 3D} \approx 0.6a$ [15]. However, for dimer-dimer distance larger than $a_\perp$, dimers eventually must occupy the transverse ground state; see Eq. (10). In effect, for $\rho a_\perp < 1$, we recover a 1D (bosonic) two-body problem, where Eq. (2) gives the answer (exact for $\Omega_B \gg 1$),

$$a_{\rm dd} = -\frac{a_{{\rm red},\perp}^2}{2(0.6a)}, \qquad a_{{\rm red},\perp} = (\hbar^2/m_0\omega_\perp)^{1/2}, \tag{17}$$

where $a_{{\rm red},\perp}$ is the transverse oscillator length for dimers. To summarize this discussion, we have shown that (a) as long as the single condition $ka_\perp < 1$ holds, dimer-dimer scattering is described by Eq. (13) for *arbitrary* $a_\perp/a$, and (b) knowledge of $a_{\rm dd}$ and hence the solution of the 1D four-body problem is sufficient to completely solve the 1D BEC-BCS many-body problem for dilute systems, $\rho a_\perp < 1$, in terms of the LL Eq. (16).

Next we discuss the 1D *four-body problem*. Enforcing the boundary condition (6) or the other equivalent ones, Eq. (5) leads to an integral equation for $f(\mathbf{r},\mathbf{R})$ [15,16]. Using Eq. (8), some algebra [27] yields

$$\left[\zeta\left(1/2, \frac{E_\mu + E_\nu - E}{2\hbar\omega_\perp}\right) - \zeta(1/2,\Omega_B)\right] f_{\mu\nu} = \frac{4\pi\hbar^2 a_\perp}{\sqrt{2}m_0}\sum_{\mu'\nu'} \mathcal{G}_{\mu\nu}^{\mu'\nu'} f_{\mu'\nu'},$$
$$\mathcal{G}_{\mu\nu}^{\mu'\nu'} = \sum_\pm \int d\mathbf{r}d\mathbf{R}\, G_{E-E_\mu-E_\nu}((\mathbf{r}\pm\sqrt{2}\mathbf{R})/2, 0)\Phi_\mu^*\left(\frac{\mathbf{r}\mp\sqrt{2}\mathbf{R}}{2}\right)\psi_\nu^*\left(\mp\frac{\mathbf{r}}{\sqrt{2}}\right)\Phi_{\mu'}(\mathbf{r})\psi_{\nu'}(\mathbf{R}). \tag{18}$$

The two-body Green's function $G_E(\mathbf{r},0)$ can be found in Ref. [16]. The 2 degrees of freedom in $f(\mathbf{r},\mathbf{R})$ imply two different types of "closed" channels that may be excited in a dimer-dimer collision: (i) *scattering states* above the bound state for each dimer [corresponding to $\mathbf{r}$ or μ in Eq. (8)], and (ii) *excited states in the transverse direction* for the relative motion of two dimers [corresponding to $\mathbf{R}$ or ν in Eq. (8)]. Neglecting both types of closed-channel excitations, Eq. (18) can be solved numerically for arbitrary $a_\perp/a$ as in Ref. [16]. The result is shown in the inset of Fig. 1. In addition, this approximation allows one to extract $a_{\rm dd}$ in both limits analytically: in the dimer limit, we find $a_{\rm dd} = -\kappa_0 a_\perp^2/(2a) + 2\kappa_1 a$, where $\kappa_0 = 1/4$ and $\kappa_1 \simeq 0.319$, while in the BCS limit, $a_{\rm dd} = \eta_0 a_\perp^2/|a|$ with $\eta_0 \simeq 0.402$. The exact (numerical) result for arbitrary $a_\perp/a$ agrees to within ± 0.05 in $a_{\rm dd}/a_\perp$ with a simple interpolation formula obtained by simply adding these two limiting results. For practical purposes, the interpolation is therefore virtually exact. Let us then turn to the effects of closed-channel excitations. In the BCS limit, excitations of type (ii) are irrelevant [16], but type-(i) excitations are important. Their inclusion results in the exact value $\eta_0 = 1/4$ [see Eq. (15)], which also follows from the solution of Eq. (18) including type-(i) excitations [27]. In the dimer limit, inclusion of the closed channels leads to the correct value $\kappa_0 \approx 0.83$; see Eq. (17). Incidentally, the two excitation types can be disentangled [27], and we find $a_{\rm dd}^{\rm 3D} \approx 0.66a$ by just neglecting type-(i) excitations, which is already close to the exact value $a_{\rm dd}^{\rm 3D} \approx 0.6a$ [15]. Type-(ii) excitations are obviously important in the dimer limit, which may be valuable input for diagrammatics [11,28]. The exact limiting results for $a_{\rm dd}$ are shown in the main part of Fig. 1 as dashed curves. For the full crossover, the additive interpolation formula is again expected to be highly accurate. Notably, this predicts $a_{\rm dd} = 0$ for $\Omega_B \approx 0.3$. At this point, a *CIR for dimer-dimer scattering* occurs [see Eq. (13)] where the interaction strength $g_{\rm dd}$ diverges and changes sign. Interestingly, the dimer-

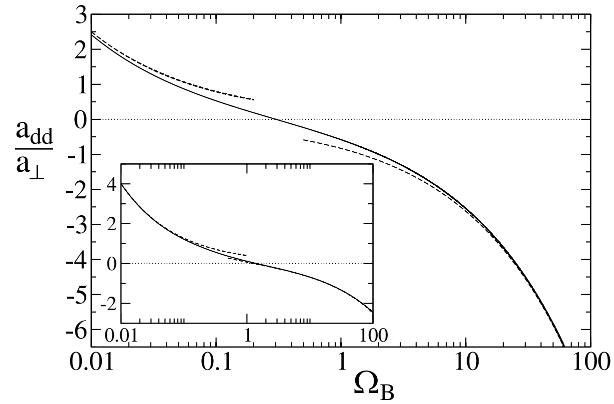

FIG. 1. Scattering length $a_{\rm dd}$ as a function of Ω_B. Dashed curves give exact limiting results, and the solid curve interpolates by adding these. Inset: Same but neglecting all closed-channel excitations. Here the solid curve gives the exact result.

dimer CIR takes place at a different value for Ω_B (and hence $a_\perp/a$) than the atom-atom CIR.

In experiments, quasi-1D regimes can be obtained in arrays of very elongated traps with a shallow confinement in the longitudinal direction. Typical trap frequencies are $\omega_\perp/2\pi \approx 70$ kHz and $\omega_z/2\pi \approx 250$ Hz, with $N \approx 100$ atoms per tube to ensure the 1D condition $N < \omega_\perp/\omega_z$ [18]. The BCS-BEC crossover can be investigated using a Feshbach resonance, which leads to changes in the density profile [11], excitation gaps [12], and ground state energy that can be probed via release energy [5] and rf spectroscopy measurements [6,18]. A probably more precise approach is to measure collective axial modes. The dipole mode frequency is always ω_z, irrespective of interactions. Using a sum rule approach [29], we calculated the frequency of the lowest compressional (breathing) mode from $\omega^2 = -2(d\ln\langle z^2\rangle/d\omega_z^2)^{-1}$ (see Fig. 2) by solving Eq. (16) using our results for $a_{\rm dd}$. Limiting values are $\omega = \sqrt{3}\omega_z$ in the dimer limit, and $\omega = 2\omega_z$ both in the BCS limit and close to $a_{\rm dd} = 0$. We hope that this prediction will soon be tested.

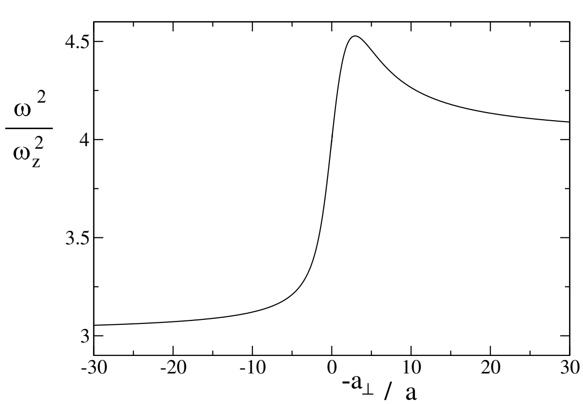

FIG. 2. Squared ratio of breathing and dipole mode frequency as a function of $-a_\perp/a$. Here we have chosen $N\omega_z/\omega_\perp = 1/3$.

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