Dephasing of Mesoscopic Interferences from Electron Fractionalization

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(Received 1 April 2005; published 8 August 2005)

We investigate the dephasing of mesoscopic interferences by electron-electron interactions in a strictly one-dimensional geometry composed of two weakly coupled (clean and very long) Luttinger liquids. We demonstrate that interactions can produce a visible attenuation of Aharonov-Bohm oscillations. Through a Nyquist-noise–type description of the interactions and a direct (exact) calculation based on the Luttinger theory, we firmly stress that the dephasing time results from the electron fractionalization time.

DOI: 10.1103/PhysRevLett.95.076801 PACS numbers: 73.23.−b, 71.10.Pm, 73.21.−b

Interesting phenomena in mesoscopic systems are known to result from quantum interferences: weak localization corrections to the conductivity, universal fluctuations of the conductance, and Aharonov-Bohm oscillations, for example [1]. The understanding of "dephasing" processes, i.e., the physical causes which suppress those interference effects, constitutes a topic of perpetual interest in mesoscopic systems. From the perspective of possible mesoscopic phase-coherent mesoscopic devices, knowledge of phase-breaking length or time is of great importance. On the other hand, the loss of the electron phase coherence is interesting in its own right because this reveals information about the fundamental physics of the electron scattering mechanisms or electron decoherence in a correlated medium. An interfering particle coupled to some environment fatally loses its phase. Notice that by environment we mean either some external dissipative bath [2–4] or still the electromagnetic field driven by the random thermal motion of other electrons in the system. It indeed has been well established that the effect of interactions in a disordered Fermi liquid can be embodied by a fluctuating electromagnetic field or Nyquist noise [5]. At low temperatures, the predominant process generating dephasing in metals is irrefutably electron-electron interactions. In two dimensions, experiments consistent with the electron scattering time [6] $\tau_\phi \propto (T^2 \ln T)^{-1}$ have been carried out in clean samples [7], T being the temperature. Phase-breaking mechanisms in ballistic mesoscopic systems of dimensionality less than two are presently not completely understood and therefore would deserve some intensive theoretical and experimental endeavors. Recent Aharonov-Bohm oscillations measured on very clean (ballistic) quasi-1D rings support a dephasing time which varies as [8] $\tau_\phi \propto T^{-1}$. For quasi-1D disordered wires, one would rather expect [1,5] $\tau_\phi^{-1} \propto T^{2/3}$, as observed in Ref. [9]. Of interest to us is to study dephasing in a non-Fermi liquid such as the Luttinger liquid.

More precisely, the Luttinger liquid (LL) is well known to exhibit fractional quasiparticles [10] which correspond to genuine excitations of 1D ballistic systems with nonzero charge or/and current with respect to the ground state (with no plasmon excited) [11]. In Ref. [12], exploiting the appropriate fractionalization mechanism, we have precisely derived the temperature and interaction dependence of the electron lifetime in one dimension. For spinless electrons, we report that the electron lifetime obeys $\tau_{Fc}^{-1} \propto \pi T[(g + g^{-1})/2 - 1]$, $g < 1$ being the well-known Luttinger exponent. For sufficiently weak interactions, in agreement with the Refs. [13,14], we have found $\tau_{Fc}^{-1} \propto \pi T(Ua/\hbar v_F)^2$, where U is the on-site interaction, a the short-distance cutoff, and v_F the Fermi velocity. On the other hand, after Ref. [12] it was still unclear which controlled experimental setup could eventually detect this fractionalization time [15]. In this Letter, we propose to *unambiguously* reveal the latter via the dephasing of Aharonov-Bohm (AB) interferences built out from two weakly coupled very long and spin-polarized Luttinger liquids (which can be realized with quantum wires possessing a single conducting channel). First, through two complementary approaches, we show that in the geometry of Fig. 1 interactions inside the quantum wires can effectively suppress the AB oscillations, and second, we firmly demonstrate that the resulting dephasing time τ_ϕ can be identified as the electron fractionalization time τ_{Fc}. A right-moving electron tunneling from one LL to the other, e.g., at $x = 0$, decomposes itself into a right-moving charge [10–12] $Q_+ = (1 + g)/2$ and a left-moving charge $Q_- = (1 - g)/2$, inevitably provoking the loss of quantum interferences (in Refs. [11,12], $Q_\pm$ has been normalized to $-e$ for convenience). For spinful electrons and weak interactions, the most important source of electron decoherence is spin-charge separation, and the electron lifetime gets modified as $\tau_{Fs}^{-1} = \tau_\phi^{-1} \propto TUa/(\hbar v_F)$.

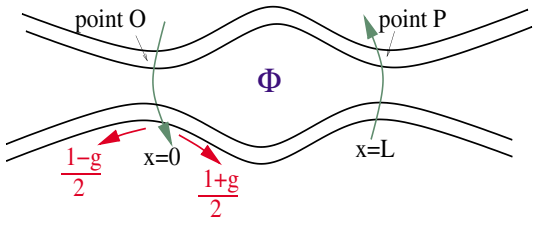

FIG. 1 (color online). Electron wave interferences with weakly coupled Luttinger wires. The electrons can tunnel from one lead to the other at two point contacts located at $x = 0$ and $x = L$.

Interactions as Nyquist noise.—It has been emphasized for more than a decade that an Ohmic environment could fake the electronic interactions in a one-channel mesoscopic conductor [16]. In this picture, a one-channel conductor in series with a resistance is equivalent to a one-dimensional interacting system described by the LL. More precisely, the random thermal motion of electrons produces a fluctuating potential [17] $\delta V_i(t)$, which from the fluctuation-dissipation theorem is equivalent to an effective resistance R_j in each lead, $j = 1$ being the upper lead and $j = 2$ the lower lead; an analogous point of view has been explored in the context of disordered Fermi liquids [5]. Such a correspondence in one dimension has been already established, e.g., in the presence of a single impurity [18] or a quantum dot [19], and we propose to extend it for this setup of weakly coupled LLs. In a given lead, a right-moving electron which is located at $x = 0$ at the time $t = 0$ will propagate ballistically with a velocity v_F. Note that throughout the Ohmic environment concept, at a frequency ω and for $0 < x \leq L$, the related retarded electron Green's function takes a relatively simple form:

$$\mathcal{G}_j(x, \omega) = \exp\left[i\left(\frac{L\omega}{v_F} + m_j\pi\varphi\right)\frac{x}{L} + i\mathcal{K}_j\left(\frac{x}{v_F}\right)\right]. \quad (1)$$

The variable x measures the position inside each wire. The first term is the dynamical phase, whereas the second term is the chirality-dependent AB phase, and the third term is induced by the fluctuating potential $\delta V_j(t)$ which results in an extra phase $\pm\mathcal{K}_j(t) = \pm\frac{e}{\hbar}\int_0^t \delta V_j(t')dt'$ in the electron annihilation-creation operator. Here, $\varphi = \Phi/\Phi_0$ is related to the enclosed magnetic flux, with Φ_0 being the flux quantum, and the second term must exhibit a sign change (e.g., $m_j = \pm$) for $j = 1$ and $j = 2$, respectively.

Let us consider an electron wave packet at the point O and examine the interference phenomenon at the point P. The electron wave packet can take either the lead 1 or the lead 2, and the related transmission amplitudes are defined as $\mathcal{A}_1 = \sqrt{(1 - T_0)(1 - T_L)}\mathcal{G}_1(L, \omega)$ and $\mathcal{A}_2 = \sqrt{T_0T_L}\mathcal{G}_2(L, \omega)$; $T_i \ll 1$ with $i = 0$ or L denotes the tunneling probability at each point contact. Since $T_i \ll 1$, the transmission probability $\mathcal{T} = |\mathcal{A}_1 + \mathcal{A}_2|^2$ from the point O to the point P hence can be approximated as

$$\mathcal{T} \approx 1 - (T_0 + T_L) + \sqrt{T_0T_L}(e^{i2\pi\varphi + iK_1 - iK_2} + \text{H.c.}), \quad (2)$$

$T_0 \approx T_L$ and $K_j = \mathcal{K}_j(\frac{L}{v_F})$, the crucial point being how the environments affect the partial waves. Similar to Refs. [4,16–18], we can resort to a set of harmonic oscillators to mimic the fluctuations of δV_i; this is especially well funded in one dimension since interactions are known to produce bosonic-type excitations (plasmons). Now, we can average $\mathcal{T}$ with respect to the unperturbed set of oscillators exploiting the identity $\langle e^{iK_1 - iK_2}\rangle = e^{-(\langle K_1^2 + K_2^2\rangle)/2}$. Furthermore, the variance $\langle K_j^2\rangle = \langle\mathcal{K}_j(\frac{L}{v_F})^2\rangle = (e^2/\hbar^2)\int_0^{L/v_F} dt' \int_0^{L/v_F} dt''\langle\delta V_j(t')\delta V_j(t'')\rangle$ can be easily evaluated in the case of an Ohmic environment (Nyquist noise). In the high-temperature regime, we must identify [4] $\langle\delta V_j(t')\delta V_j(t'')\rangle = R_j\beta^{-1}\delta(t' - t'')$, which gives $\langle K_j^2\rangle = 2\pi\beta^{-1}\frac{L}{\hbar v_F}r_j$, where $r_j = e^2R_j/h$ represents the dimensionless resistance and $\beta = 1/(k_BT)$.

The final step attempts to relate the resistance R_j with the corresponding Luttinger parameter g_j which typically measures the strength of the interactions in each wire. This can be performed by investigating the form of the local electron tunneling density of states (TDOS) ρ_j of each wire at one of the two contacts. From the environmental theory [16,17], in which the interactions between electrons are embodied by a fluctuating potential, we extract [20] $\rho_j(T) \propto T^{r_j}$ when $\beta^{-1} \ll \hbar v_F/a$. On the other hand, in the bulk of a LL, we have $\rho_j(T) \propto T^{-1+(g_j+g_j^{-1})/2}$; this results in $r_j = \frac{e^2R_j}{h} = -1 + (g_j + g_j^{-1})/2$ and then in

$$\langle K_j^2\rangle = 2\pi\beta^{-1}\frac{L}{\hbar v_F}\left(\frac{g_j + g_j^{-1}}{2} - 1\right). \quad (3)$$

Keep in mind that this formula is appropriate only in the weak-tunneling regime after identification between the exact TDOS of a LL and that obtained from the environmental-type theory. For $T_i \ll 1$, the average transmission probability $\langle\mathcal{T}\rangle$ exhibits the following form:

$$\langle\mathcal{T}\rangle \approx \eta + \sqrt{4T_0T_L}\cos(2\pi\varphi)e^{(-\pi L/\hbar v_F\beta)\sum_j[-1+(g_j+g_j^{-1})/2]}, \quad (4)$$

where $\eta = 1 - (T_0 + T_L)$. Applying the Landauer formalism, the total conductance of the wire 1 (similarly the conductance of the wire 2) then obeys $\mathcal{G} = (e^2/h)\langle\mathcal{T}\rangle$. The first term stems from $(e^2/h)|\mathcal{A}_1|^2$. Moreover, as a blatant signature of dephasing due to electron-electron interactions the interference (flux-dependent) part of the conductance irrefutably exhibits an exponential suppression versus the temperature. The latter takes the specific form $\exp(-2L/l_\phi)$, where the dephasing length l_ϕ satisfies

$$l_\phi^{-1} = \frac{\pi}{2\hbar v_F\beta}\sum_{j=1}^{j=2}\left(\frac{g_j + g_j^{-1}}{2} - 1\right). \quad (5)$$

Remember that in one dimension the dephasing length varies linearly with the thermal length $L_T \approx \hbar v_F\beta$. The visibility of the interference pattern will be suppressed when $L \geq l_\phi$ due to the fluctuations in the phases K_1 and K_2. We like to stress that the Nyquist noise description provides a relatively simple explanation of the result that in a 1D wire the dephasing length grows *linearly* with L_T; this stems from the fact that for an Ohmic environment in the high-temperature limit the fluctuations of the potential δV_j are proportional to the temperature [3]. This might be relevant to explain the experiment of Ref. [8] (however, their geometry is distinct from ours). For free electrons, implying $g_j = 1$, we recover $l_\phi^{-1} = 0$ and then perfect AB oscillations. It could be anticipated that, like the suppression of the TDOS in one dimension, dephasing may be

attributed to electron fractionalization. Below, we like to enrich this Nyquist noise approach by a more direct (exact) calculation based on the Luttinger theory.

Luttinger-type calculation.—More precisely, the flux-dependent part of the current I_φ in the wire 1 (or in the wire 2) passing between the points O and P as a function of the applied potential difference V may be calculated applying the Luttinger formalism for small tunneling amplitudes at the point contacts. The Hamiltonian $H = H_0 + H_{\rm tun}$ is the sum of the well-known Luttinger Hamiltonian H_0 as well as the tunneling part, $H_{\rm tun} = \Sigma_i \Gamma_{i\pm} \Psi^\dagger_{2\pm}(x = i)\Psi_{1\pm}(x = i) + \text{H.c.}$, acting only at the point $i = 0$ or L; the index $\pm$ refers to right and left movers, respectively, and for convenience we have denoted $\Gamma_{0\pm} = \sqrt{T_0}\exp(i\mu_{1\pm}t/\hbar)$ and $\Gamma_{L\pm} = \sqrt{T_L}\exp(i\mu_{1\pm}t/\hbar)\times \exp(2i\pi\varphi)$. In contrast to the edges of quantum Hall systems, particles are not chiral; i.e., they can propagate in both directions "right" or "left" and $\mu_{1\pm}$ refers to the electrochemical potential of the fermions $\Psi_{1\pm}$ in the wire 1. We will choose $\mu_{1+} = eV$ and $\mu_{1-} = 0$.

To first order in $H_{\rm tun}$, the current $I_\varphi(t) = -e\langle \dot{N}_{1+}\rangle$, where N_{1+} is the number of right-moving electrons in the wire 1, takes the form $I_\varphi(t) = (ie/\hbar)\int dt' \theta(t - t')\,\text{Tr}\{\rho_0[\partial_t N_{1+}(t), H_{\rm tun}(t')]\}$, where $\rho_0 = e^{-\beta H_0}/\text{Tr}e^{-\beta H_0}$, in the interaction representation $O(t) = e^{iH_0 t} O e^{-iH_0 t}$, and $N_{1+} = \int dx \Psi^\dagger_{1+}(x)\Psi_{1+}(x)$. Again, this approach is appropriate to evaluate the magnetic-flux-dependent part of the current in the wire 1 because the latter can be treated perturbatively in Γ_{0+} and Γ_{L+}. Expressing $\partial_t N_{1+}(t)$ as a function of $[N_{1+}(t), H_{\rm tun}(t)]$ hence gives rise to $I_\varphi \propto -\sqrt{T_0 T_L}[e^{2i\pi\varphi}\text{Im}X_{L0}(\omega) + \text{H.c.}]_{\omega = eV/\hbar}$, where $X_{ij}(\omega)$ is the Fourier transform of $X_{ij}(t) = -i\theta(t)\langle[B_i(t), B^\dagger_j(0)]\rangle$ with $B_i = \Psi_{1+}(x = i)\Psi^\dagger_{2+}(x = i)$ and $i, j = 0, L$ or vice versa. The response function $X_{L0}(t)$ can be extracted resorting to standard bosonization techniques at finite temperature by simply analytically continuing [21,22] $\tau \to it$:

$$X_{L0}(t) = \theta(t)\prod_{j=1}^{2}\frac{1}{\sinh^{(\gamma_j+1)}[(\pi/L_{Tj})(L - u_j t + i m_j \delta)]}\frac{a^{2\gamma_j}}{2\pi^2} \times \left(\frac{\pi}{L_{Tj}}\right)^{2\gamma_j+1}\frac{1}{\sinh^{\gamma_j}[(\pi/L_{Tj})(L + u_j t - i m_j \delta)]}, \tag{6}$$

where $u_j = v_F/g_j$ is the plasmon velocity of each wire, the thermal length L_{Tj} is precisely defined as $\hbar u_j \beta$, $\gamma_j = -1/2 + (g_j + g_j^{-1})/4$, δ is a positive infinitesimal, and again $m_j = \pm$ for $j = 1$ and $j = 2$, respectively. We have implicitly considered the situation where $u_1 \approx u_2 = u$, assuming that the interaction strength between electrons is of the same order in magnitude in each wire; the relevant thermal length reads $L_T = \hbar u \beta$. The required Fourier transform may be calculated by contour integration and the poles are at $t = \frac{L}{u} \mp i\delta$, which asserts that the involved wave packets propagate at the plasmon velocity.

Note that even though the previous environmental picture is not completely exact, i.e., ignores this small renormalization effect of the electron velocity, this will only slightly renormalize the inverse of the dephasing length in Eq. (5) via an overall prefactor equal to v_F/u. The contributions from the two poles hence give rise to

$$I\varphi \propto \frac{e^2 V}{h}\sqrt{T_0 T_L}\cos(2\pi\varphi) \times \frac{a^{2\gamma_1+2\gamma_2}[\pi/(LL_T)]^{\gamma_1+\gamma_2}}{\sinh^{\gamma_1}(2\pi L/L_T)\sinh^{\gamma_2}(2\pi L/L_T)}; \tag{7}$$

we have extracted the lowest order contribution in V implying $V \to 0$. At relatively high temperatures $L \gg L_T$, the result can be approximated as $G_\varphi = dI_\varphi/dV \propto (e^2/h)\sqrt{T_0 T_L}\cos(2\pi\varphi)a^{2\gamma_1+2\gamma_2}e^{-(2L/L_\phi)}/[(LL_T)^{\gamma_1+\gamma_2}]$; the *exact* dephasing length in one dimension reads

$$L_\phi^{-1} = \frac{\pi}{2\hbar u\beta}\sum_j\left(\frac{g_j + g_j^{-1}}{2} - 1\right) = l_\phi^{-1}\frac{v_F}{u}. \tag{8}$$

First, it is important to bear in mind that the preceding Nyquist noise result is in quite good agreement with the exact Luttinger-type calculation. Second, it is also crucial to establish the clear physical origin of dephasing in one dimension. Since the motion of electrons is purely ballistic, we can define the dephasing time as $\tau_\phi = L_\phi/u$. Recall that for weak interactions, equating $g_1 = g_2 = g$ and introducing the well-known formula $g = 1 - Ua/(\pi\hbar v_F)$, we extract $\tau_\phi^{-1} \propto T(Ua/\hbar v_F)^2$. We note some agreement with the dephasing times of Refs. [3,13].

τ_ϕ as the electron fractionalization time.—At this step, it is certainly relevant to observe that $\tau_\phi = \hbar\beta/[-\pi + \pi(g + g^{-1})/2]$ is completely equivalent to the electron fractionalization time τ_{Fc} that we have built up in an earlier work [12]. The fractionalization time τ_{Fc} has been precisely identified as follows. If one injects a right-moving electron in a 1D wire at the point $x = 0$ at the time $t = 0$, it is well established [10,11] that this will fatally decompose into two counterpropagating modes, namely, a charge $Q_+ = (1 + g)/2$ (normalized to $-e$) state going to the right at the plasmon velocity and a charge $Q_- = (1 - g)/2$ state going to the left at the same velocity. Note that such a fractionalization scheme reproduces nicely the properties (damping) of the exact electron Green's function [12] and hence those of the TDOS. In Ref. [12] we have defined τ_{Fc} as the time needed for the propagator of the countergoing mode Q_- to vanish at the position of the right-going mode $x \approx u\tau_{Fc}$; at the time τ_{Fc}, the overlap between the two fractional wave packets is negligible and the electron wave function gets clearly fractionalized. For weak U, we find good agreement with Ref. [14]. In Fig. 1, an electron (a hole) which tunnels from the wire 1 (2) to the wire 2 (1) at $x = 0$ gets subject to this fractionalization phenomenon producing the dephasing of electronic interferences. It is essential that the wires are sufficiently long such that the reservoir leads attached at

the extremities of each wire will not hinder the electron fractionalization mechanism emerging at $x = 0$ (see Fig. 1); the length d of each wire must satisfy $d \gg L$.

Quantum limit.—So far we have considered only the relatively high-temperature limit $L \gg L_T = \hbar u \beta$. Now, we would like to briefly comment on the (opposite) quantum limit $\beta^{-1} \to 0$. From Eq. (7), we can easily extract $I_\varphi = (e^2/h) V \sqrt{T_0 T_L} \cos(2\pi\varphi)(a/L)^{2\gamma_1+2\gamma_2}$. Compared to the case of free electrons, one can notice an extra small power-law suppression as a function of the distance between the two point contacts. Nevertheless, if L is not too large compared to the lattice spacing a, one should observe visible electronic interferences when T goes to zero. We may recover this result by applying the Nyquist environmental approach. In the quantum limit, it is easy to show that [4] $\langle \mathcal{K}_j (\frac{L}{v_F})^2 \rangle \to -2 r_j \ln(\omega_F L/v_F)$, where $\omega_F = v_F/a$. Now, using $r_j = 2\gamma_j$, we recover the expression of I_φ established above from the (exact) Luttinger theory.

Electrons with spin.—One can extend the Luttinger theory developed above to the case of electrons with spin. Here, the electron spectrum will exhibit both spin-charge separation and chiral decomposition from the charge sector [11,12]; the crucial point being that the spin propagates at the Fermi velocity v_F, whereas the fractional charge wave packets propagate at the charge plasmon velocity $u > v_F$. Hence this will produce four relevant poles at $t = (L/u) \pm i\delta$ and $t = (L/v_F) \pm i\delta$, which for *weak* interactions give the leading contribution

$$I\varphi \propto \frac{e^2 V}{h} \sqrt{T_0 T_L} \cos(2\pi\varphi) a^{\gamma_1+\gamma_2} L^2 \frac{1}{\sinh^{\gamma_1/2}(2\pi L/L_T)} \times \left(\frac{1}{L L_T}\right)^{1+[(\gamma_1+\gamma_2)/2]} \times \frac{1}{\sinh^{\gamma_2/2}(2\pi L/L_T) \sinh[(\pi/L_T)(L - \frac{v_F}{u} L)]}. \quad (9)$$

Assuming $g_1 = g_2 = g$, the dephasing length obeys [23]

$$L_\phi^{-1} = [\pi/(2\hbar u \beta)] \left(\frac{g + g^{-1}}{2} - g\right), \quad (10)$$

and $\tau_\phi^{-1} \propto T(Ua/\hbar v_F)$; spin-charge separation accents dephasing compared to the spinless case. In Ref. [12], when computing the fractionalization time τ_{Fs} for electrons with spin, we have omitted some relevant terms in [13] $\mathcal{O}(u - v_F)$; when keeping those terms, we check $\tau_\phi = \tau_{Fs}$.

Conclusion.—We have shed some light on the possibility to reveal the electron fractionalization mechanism occurring in one dimension via a well-defined geometry composed of two *weakly coupled* quantum wires. We have shown that the dephasing time related to the suppression of the Aharonov-Bohm oscillations at finite temperature is the electron fractionalization time. We envision to extend this work to different geometries and, in particular, to strongly coupled quantum wires. Finally, Ref. [24] suggests to revisit dephasing for two coupled chiral LLs.

We are grateful to M. Büttiker, I. Gornyi, A. Mirlin, and D. Polyakov for corrective discussions. This work was supported by CIAR, FQRNT, and NSERC.

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