Observation of Surface Andreev Bound States of Superfluid ^{3}He by Transverse Acoustic Impedance Measurements

Y. Aoki, Y. Wada, M. Saitoh, R. Nomura, and Y. Okuda

Department of Condensed Matter Physics, Tokyo Institute of Technology, 2-12-1 O-okayama, Meguro, Tokyo 152-8551, Japan

Y. Nagato,[1] M. Yamamoto,[2] S. Higashitani,[2] and K. Nagai[2]

[1]*Information Media Center, Hiroshima University, 1-7-1 Kagamiyama, Higashi-hiroshima 739-8521, Japan*

[2]*Faculty of Integrated Arts and Sciences, Hiroshima University, 1-7-1 Kagamiyama, Higashi-hiroshima 739-8521, Japan*

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Complex transverse acoustic impedance of the superfluid ^{3}He-B was measured at the frequencies of 10 to 80 MHz at 17.0 bar by a cw bridge method. The observed temperature dependence was well explained by the quasiclassical theory with random *S*-matrix model for a diffusive surface. The temperature dependence was influenced by pair breaking and by quasiparticle density of states at the surface, which was drastically modified from the bulk one by the presence of surface Andreev bound states.

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The response of a superfluid to the motion of a transversely oscillating wall is controlled by a normal component. This is how the torsional oscillator measurements provide information about a normal fluid density ρ_n or a superfluid density ρ_s. At low enough temperatures ρ_s reaches 100% and the superfluid completely decouples from the wall. Is this true for an unconventionally paired superfluid when the oscillation frequency is very high? Naively, bulk superfluid should not respond to the motion regardless of the frequency so long as the frequency is lower than the gap energy Δ and the resonance frequencies with collective modes. However, we found that the response of the superfluid ^{3}He-B is quite different from that expected from the bulk superfluid and is strongly affected by the presence of surface Andreev bound states (SABS), which allows low energy excitations of quasiparticles even at low temperatures.

Superfluid states near a wall in non *s*-wave pairing superfluids and superconductors are significantly modified from those in the bulk. Some components of the order parameters are suppressed within several times the coherence length from the wall, and the SABS form there [1–3]. The SABS were observed as a zero-bias conductance peak of the tunneling spectrum in unconventional superconductors and are recognized as their universal feature [4,5]. Superfluid ^{3}He was the first experimentally identified unconventional BCS superfluid. Experimental investigations on its bulk properties have been extensive and provided a sound understanding of the unconventional pairing state. It is a foundation for understanding newly discovered exotic superconductors. But much less is known about the SABS of superfluid ^{3}He due to lack of an appropriate surface probe for the electrically neutral superfluid. Existence of the SABS was indicated by a suppression of superfluidity due to finite size effects of superfluid ^{3}He in narrow pores and a slab geometry [6–8]. Only limited information on the microscopic properties of the SABS was obtained from these measurements.

There have been several transverse acoustic impedance measurements of superfluid ^{3}He [9,10]. Most of them were the measurements of only an imaginary component of the impedance except for a very early experiment by Roach and Ketterson [9]. Their measurement covered a wide frequency range, but the temperature was not low enough to see the whole behavior of the impedance. We investigated a complex transverse acoustic impedance of superfluid ^{3}He-B in a wide frequency range experimentally and theoretically and obtained the entire feature of temperature dependence for the first time. Both the real and the imaginary parts of the transverse acoustic impedance were systematically measured in a frequency range of 9.3 to 77.8 MHz at 17.0 bar in zero magnetic field. The system was in the ballistic regime in this frequency range [11]. The observed temperature dependences are well reproduced by quasiclassical theory with random *S*-matrix model for a diffusive surface. The temperature dependences are governed by pair breaking and by quasiparticle density of states within the gap originating from the SABS.

Transverse acoustic impedance of superfluid ^{3}He was obtained by measuring the change of the quality factor Q and the resonance frequency ω_0 of an ac-cut quartz transducer which was electrically oscillated in a shear mode while immersed in the liquid ^{3}He. We used a cw bridge method to measure the acoustic response of the transducer. The cw bridge was constructed by a quadrature hybrid and a high frequency lock-in amplifier. The resonance shapes were obtained by sweeping the frequency of the cw excitation around the resonance, and Q and ω_0 were determined by fitting these shapes to Lorentzian form. Details of our spectrometer, data analysis, and the accuracy were described in our previous papers [12,13]. In order to measure the transverse acoustic impedance in a wide frequency range, we used two transducers whose fundamental frequencies were 9.3 (transducer 1) and 15.5 MHz (transducer 2), and measured at the 1st and 3rd harmonics of transducer 1 and the 1st, 3rd, and 5th of transducer 2. These transducers had gold electrodes with a coaxial shape. Their surfaces were optically flat but rough in the atomic scale. The sample cell that contained the transducers was mounted on a copper nuclear demagnetization refrigerator. It was made of high quality copper and had sintered silver of 140 m^2. The temperature of liquid ^{3}He was measured by a ^{3}He melting curve thermometer. Data on warming agreed with those on cooling within the experimental accuracy at the sweep rate of 0.1 μK/ min [12]. Superfluid transition temperature T_c was determined as a point where Z' at 9.25 MHz deviates from the value in normal phase. This should agree with the true T_c within 1.5 μK.

The complex transverse acoustic impedance $Z = Z' + iZ''$ is defined as a ratio of the stress tensor of the liquid at the surface Π_{xz} to the wall velocity u_x as $\Pi_{xz} = Zu_x$. Here we assumed the surface is in the xy plane. Z can be written by Q and ω_0 of the transducer as [11]

$$Z' + iZ'' = (\tfrac{1}{4}n\pi Z_q \Delta Q^{-1}) + i(\tfrac{1}{2}n\pi Z_q \Delta\omega_0/\omega_0), \qquad (1)$$

where $Z_q = \rho_q c_q$ is the acoustic impedance of the quartz given by its density ρ_q and velocity c_q, n is the harmonics number of the crystal resonance, and $\Delta\omega_0$ and ΔQ^{-1} represent the changes in ω_0 and Q^{-1} from an unloaded condition. This relation holds under the condition that $Z_q \gg Z$, which is well satisfied in the quartz and the liquid ^{3}He.

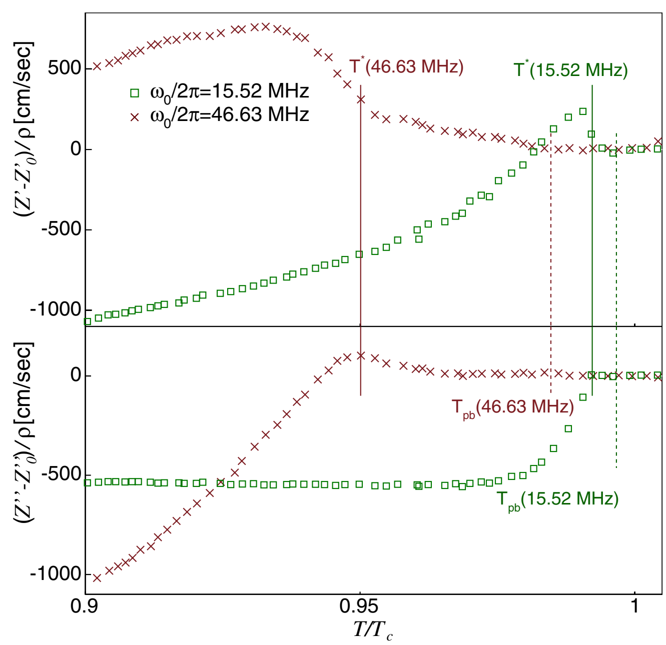

FIG. 1 (color online). Temperature dependence of the complex transverse acoustic impedance around T_c at the frequencies of 15.52 and 46.63 MHz. Pressure of the liquid ^{3}He was 17.0 bar. Broken lines are T_{pb} and solid lines are T^* for each frequency (see text).

Figure 1 shows the temperature dependence of Z' and Z'' measured from the normal liquid values just above T_c, Z'_0, and Z''_0. They are divided by the liquid density ρ. Data down to $0.9T_c$ are shown in the figure at two frequencies of 46.63 and 15.52 MHz to see the behaviors around T_c. Whole temperature dependence is shown in Fig. 2 as points at frequencies of 9.25, 15.52, 27.81, 46.63, and 77.75 MHz. Because data at 77.75 MHz was noisier than others, we could not obtain Z' with good accuracy and plotted only Z'' in the figure. Data above $0.9T_c$ are not shown in order to see others at lower frequencies clearly.

Let us describe the temperature dependence from the high temperature side. At T_c neither Z' nor Z'' deviate from Z'_0 and Z''_0 as if still in the normal state. Z' begins to increase at a temperature defined as T_{pb}, but Z'' still remains Z''_0. T_{pb} agrees with the pair breaking edge temperature [11] of each frequency within 5 μK. Z' has a kink at a temperature defined as T^*, and Z'' shows a small peak at the same temperature T^* and decreases rather steeply with cooling. The peaks were recognized clearly only in the high frequency data in Fig. 2. On further cooling Z' has a maximum and eventually decreases slowly. The slope of the decrease is almost parallel with each other at different frequencies and thus Z' seems to saturate to the values depending on the frequency. The saturation values are larger at higher frequencies by extrapolating them to low

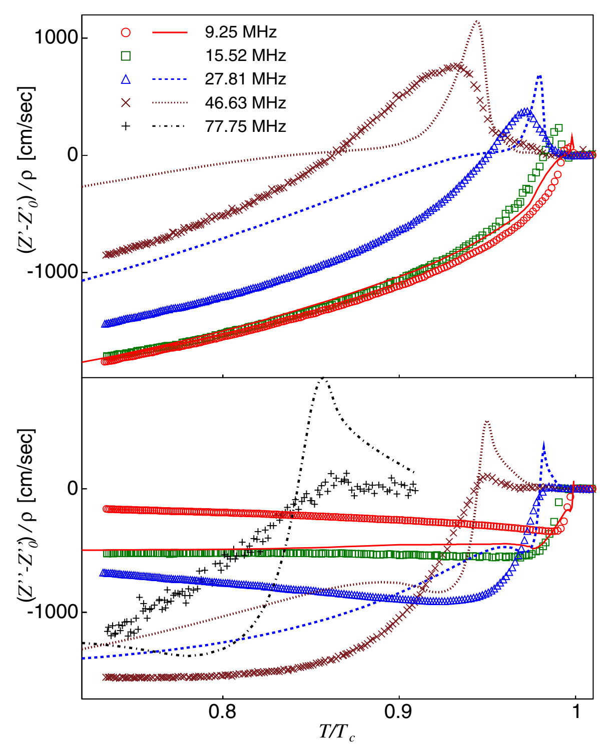

FIG. 2 (color online). Temperature dependence of the complex transverse acoustic impedance at the frequencies of 9.25, 15.52, 27.81, 46.63, and 77.75 MHz. Pressure of the liquid ^{3}He was 17.0 bar. Points are experimental results and lines are theoretical calculations.

temperature. Z'' saturates to constant values within the experimental temperature range and the values are smaller at higher frequencies. T_{pb} and T^* are lower at higher frequencies. The observed features did not appear in the same temperature range at lower pressures where T_c is lower and ^{3}He is in the normal state.

There exist several theoretical calculations of Z assuming $\omega_0 \ll \Delta$ [14–16]. Surface effect was not included in these calculations. The calculations showed that Z' monotonically decreased as ρ_n, which agrees well with the observed Z' at 9.25 MHz except for the region near T_c. However, the observed Z' at higher frequencies did not converge on the calculation even at low temperatures, but tended toward the frequency dependent saturation values. Observed Z'' is 1 order of magnitude larger than the calculations even in the lowest frequency case 9.25 MHz. These facts suggest that we have to take into account the surface effect on Z and to extend the theory to the region $\omega_0 > \Delta$ in order to explain the observation.

Let us consider a system in which liquid ^{3}He fills the $z > 0$ domain and a rough plane wall at $z = 0$ is oscillating in the x direction like $R(t) \propto e^{-i\omega_0 t}$. In this Letter, we use (K, α) to specify a Fermi momentum: K is the component of the Fermi momentum parallel to the wall and $\alpha = \pm$ denotes the direction of the z component.

The origin of momentum transfer between the wall and liquid ^{3}He is the diffusive scattering of ^{3}He quasiparticles by the wall. We treat the diffusive wall effect using the random S-matrix model [17,18]. When one moves to a reference frame moving with the wall, the boundary condition is described by a surface S matrix

$$S_{KQ} = -\left(\frac{1 - i\eta}{1 + i\eta}\right)_{KQ}, \qquad (2)$$

where $K(Q)$ is the parallel component of the incident (scattered) Fermi momentum and η is a Hermite matrix that represents the rough wall effects. Coming back to the rest frame, one finds that the oscillating wall boundary condition is given by a time dependent S-matrix $S_{KQ}e^{i(K-Q)R(t)}$. Using this boundary condition, we can construct a formal solution [17] for the quasiclassical Green's function in the Keldysh space. After taking the average over the wall roughness, we find that the Keldysh quasiclassical Green's function $\check{G}_{\alpha\alpha}(K, z, t, t')$ for the Fermi momentum (K, α) at the wall is given by

$$\check{G}_{\alpha\alpha}(K, 0, t, t') = \check{G}_s + (\check{G}_s + i\alpha)\check{\mathcal{G}}(\check{G}_s - i\alpha), \qquad (3)$$

where $\check{G}_s$ is the Green's function for the specular wall,

$$\check{\mathcal{G}}(K, t, t') = (\check{G}_s^{-1} - \check{\Sigma})^{-1}, \qquad (4)$$

and the surface self energy $\check{\Sigma}$ is given by

$$\check{\Sigma}(K, t, t') = \sum_Q \frac{2W}{\Sigma_{Q'} 1} e^{-i(K-Q)[R(t)-R(t')]}\check{\mathcal{G}}(Q, t, t'). \qquad (5)$$

Since we are dealing with Keldysh Green's functions, the products in the above equations should read $\check{A}\check{B}(t, t') = \int d\bar{t}\check{A}(t, \bar{t})\check{B}(\bar{t}, t')$. In Eq. (5), we have adopted a simple form for the correlation function of random matrix η, i.e., $\overline{\eta^*_{KQ}\eta_{KQ}} = 2W/\Sigma_Q 1$ with W a parameter that specifies the roughness of the wall [17]. One can show that $W = 1$ corresponds to the diffuse surface boundary condition and $W = 0$ corresponds to the specular surface boundary condition. In what follows, we consider only the case with a diffusive surface ($W = 1$), because it is obvious that in case of the specular surface the wall motion will not have any effect on the liquid.

In equilibrium, one can calculate the surface density of states from Eq. (3) [18]. In the inset of Fig. 3, we show the total density of states of the Balian-Werthamer (BW) state at the surface. In the BW state, there exist surface bound states whose energy depends on the polar angle θ of the Fermi momentum. The bound state energy is zero when $\theta = 0$ and increases as a function of θ. In case of the specular surface ($W = 0$), the surface bound states fill up the bulk energy gap. In case of the diffusive surface ($W = 1$), however, the bound state is broadened and its energy saturates when θ increases. As a result, there appears an upper energy edge Δ^* of the bound state band as can be seen in the inset of Fig. 3 [18,19].

Now we treat the effect of wall motion by perturbation theory. Retaining the first order term in $R(t)$ in Eq. (5), we obtain the linear response of the Keldysh Green's function $\delta\check{G}$. The stress tensor at the wall is calculated from the Keldysh part of the Green's function

$$\Pi_{xz} = \sum_{\alpha=\pm}\frac{N(0)}{2}\int_0^{\pi/2}\sin\theta d\theta\int d\phi(K_x\alpha v_K) \times \frac{i}{4}\,\mathrm{Tr}[\delta\hat{G}^K_{\alpha\alpha}(K, t, t)], \qquad (6)$$

where $N(0)$ is the density of states at the Fermi surface,

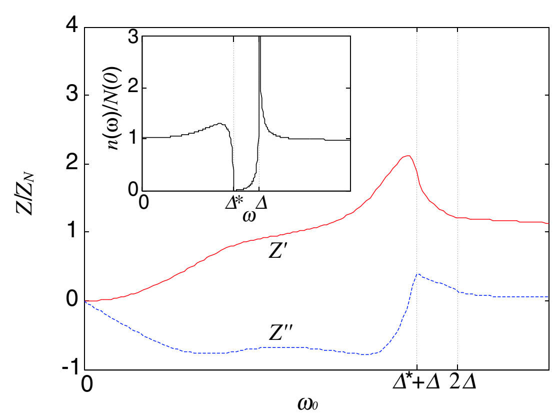

FIG. 3 (color online). Frequency dependence of the transverse acoustic impedance Z of superfluid ^{3}He-B at $T = 0.2T_c$. Z is scaled by its normal state value Z_N. Inset: Quasiparticle density of states at the diffusive surface of the BW state at $T = 0.2T_c$. Δ is the bulk energy gap, and Δ^* is the upper edge of the surface bound state band.

$v_K = v_F \cos\theta$ is the z component of the Fermi velocity, and inside of the parentheses is the momentum current. The acoustic impedance is given by $\Pi_{xz}/\dot{R}$.

For the s-wave pairing superfluid, it is straightforward to obtain an analytical expression for Z [20]. One can show that in the low frequency limit $\omega_0 \to 0$, Z agrees with previous theories [14–16] with the Fermi liquid effects neglected. At higher frequencies, however, Z becomes frequency dependent and shows a jump at the pair breaking edge temperature.

We have calculated numerically the stress tensor given by Eq. (6) and Z of the BW state using the self-consistent order parameter and the quasiclassical Green's functions obtained in Ref. [18]. The calculations have been made within the weak coupling theory. In Fig. 2, we plot the temperature dependence of Z-Z_N with Z_N the theoretical normal state value [15] to compare with the observed Z-Z_0 at 17 bar. Necessary parameters T_c, ρ, v_F were taken from Ref. [11]. It is remarkable that characteristic structures in the observed temperature dependence are recovered by the present calculation in which pair excitations are taken into account but the Fermi liquid effect is not considered. We can find that Z' begins to increase from Z'_N near T_{pb}, and that Z' has a kink and Z'' has a peak near T^*. Below T^*, Z' has a broad peak and decreases to a finite value in the low temperature limit. The origin of the structures becomes clear when we calculate the frequency dependence at a fixed temperature. In Fig. 3, we show the results at $T = 0.2T_c$ because the features of Z are well resolved. We first find that there is no jump at $\omega_0 = 2\Delta$ which was found in s-wave pairing superfluid [20] but a slight change in the slope. Quite interesting is a weak singularity at $\omega_0 = \Delta + \Delta^*$ found both in the real part and the imaginary part. This is a singularity due to the pair excitation of a surface bound state and a propagating Bogoliubov quasiparticle.

The small peak in Z'' at $T = T^*$ has been reported by transverse acoustic impedance measurements [9,10] and by a shear horizontal surface acoustic wave measurement [21]. Kalbfeld *et al.* [10] observed this peak near the imaginary-squashing mode resonance of the longitudinal response. In fact, T^* happens to be close to the temperature $T_{\rm isq}$ at which the imaginary-squashing mode frequency $\sqrt{12/5}\Delta(T_{\rm isq})$ [11] is equal to ω_0; the maximum difference between T^* and $T_{\rm isq}$ is about 30 μK. The order parameter collective mode coupled with the transverse sound [22] will contribute to Z through the Fermi liquid effects. As is known in the normal state [23], however, the transverse sound contribution is obscured by the incoherent particle-hole excitations. The present analysis indicates that the structures in the temperature dependence of Z are not caused by the collective modes but are dominated by the presence of the SABS and T^* is a temperature at which $\Delta + \Delta^*$ is equal to the frequency ω_0. For more quantitative discussion, study of the Fermi liquid effects is, of course, necessary as well as of the strong coupling effects. Since the Fermi liquid effect in the presence of a wall becomes a nonlocal effect, such study with further experiments will enable us to answer the fundamental question of how the momentum of the wall is transferred to the superfluid and how the transverse wave motion develops as it is away from the wall.

In conclusion, the transverse acoustic response of ^{3}He-B was measured in a wide frequency range at 17.0 bar. Entire features in the transverse acoustic impedance were well reproduced by a quasiclassical theory with a random S-matrix model for a diffusive surface. The temperature dependence was influenced by the pair breaking and the SABS. Existence of the density of states within the gap energy at the surface was confirmed experimentally for the first time in superfluid ^{3}He.

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