Nonlinearly Induced Relaxation to the Ground State in a Two-Level System

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We report the experimental demonstration of a nonlinear process in a two-level system, in which the amplitude of the excited state decays, transferring irreversibly a large fraction of its energy to the ground state, while shedding a part of it into radiation states. The experiments where preformed in a nonlinear optical waveguide, supporting two or three modes. The process is general, and is expected to occur in other nonlinear few level systems such as nonlinear quantum wells and Bose-Einstein condensates.

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The two-level system has been a favorite canonical model in many areas in physics, yielding predictions that could be understood on the most intuitive level. In the absence of external interactions, there are no transitions between the states and any initial distribution of amplitude is maintained. The introduction of nonlinearities could induce population transfer between the states, even without external perturbations. Consider, for example, an optical waveguide that supports two guided modes. The optical waveguide is completely equivalent to a quantum well (see Fig. 1), supporting a discrete set of orthogonal eigenmodes which are solutions of the Schrödinger equation:

$$\left(\frac{\partial^2}{\partial x^2} + k_0^2 n^2(x)\right)u(x) = \beta^2 u(x), \tag{1}$$

where k_0 is the free-space wave number, $n(x)$ is the refractive-index profile, and β is the propagation constant of eigenmode $u(x)$. In this analogy, the potential is replaced by the refractive-index structure and time with propagation coordinate z. The handiness and simple control of experimental parameters available in the optical waveguide system makes it particularly suitable for experimental studies of nonlinear effects. Silberberg and Stegeman [1] studied the nonlinear interaction between two eigenmodes and showed that the nonlinear coupling leads to periodic power exchange between them. In general, the lowest order mode (ground state) is stable even for a strong nonlinearity, while the higher mode tends to oscillate strongly, periodically exchanging much of its energy with the ground state. As discussed below, this description changes dramatically when the continuum of radiation modes (Fig. 1) is included in the interaction. In particular, it is possible to achieve an efficient irreversible transfer of excitation to the ground state. In the analogous quantum well system, this process will therefore result in an energy relaxation of the system through the nonlinear interaction. We report here the first experimental observation of such a process in a nonlinear optical waveguide.

Stability of nonlinear bound states was investigated in general terms [2], and later in the context of Bose-Einstein condensates (BEC) and guided waves [3–5]. Soffer and Weinstein [6] have studied the nonlinear Schrödinger equation with two bound states and a continuum of unbound states. They pointed out the process of *ground-state selection* (GSS), in which the excited bound state becomes unstable at arbitrarily small power, thereby transferring its energy into the ground state while shedding a small portion of its energy into the continuum modes. The GSS process ends with the two-level system reaching and staying in its nonlinearly-stable ground state. This description has been later generalized to include systems with more than two bound states [7]. It was shown that energy could be transferred from excited bound states into the ground state via the process of (partially degenerate) four wave mixing. This process could occur for those excited states failing to fulfill a stability criterion [7]. In a multimoded system with many bound states, it follows from this criterion that only those bound states having their eigenvalues lying higher than about half of the potential-well height may become unstable, thereby taking part in the GSS process.

The process of GSS is general [6,7], and might be relevant to various nonlinear physical systems supporting a discrete set of bound states. A possible example is the collective excitations of Bose-Einstein condensates [8,9], shown to be nonlinearly coupled [10–13]. The interaction

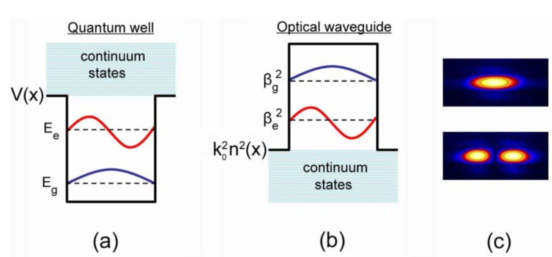

FIG. 1 (color online). Analogy between the quantum well system (a) and the optical waveguide system (b). $V(x)$ is the well's potential, x is a spatial coordinate, and $E_{g,e}$ are the energy eigenvalues of the ground and excited states. $n(x)$ is the refractive-index profile and $\beta_{g,e}$, are the propagation constants eigenvalues of the ground and excited states. (c) Intensity modal shapes of the ground (top) and excited (bottom) states of the waveguide.

of the condensate with the noncondensate cloud [14] could result in GSS-related effects in the condensate dynamics, such as relaxation to the ground state. Experiments performed in photorefractive slab waveguides supporting many spatial modes [15], and in $Al_xGa_{1-x}As$ slab waveguides supporting two nondegenerate polarization states [16], have demonstrated related effects which may also be interpreted in terms of the GSS process. An example for an applicative implication of the GSS process could be all-optical switching devices based on nonlinear waveguide junctions [17].

The experimental system we used for demonstrating GSS is an $Al_xGa_{1-x}As$ slab waveguide, onto which a 1D waveguide is defined by etching into the top layer of the sample [see Fig. 2(a)]. The experimental setup is shown in Fig. 2(b). We used a synchronously pumped optical parametric oscillator, operating at a wavelength of 1.53 μm, with a repetition rate of 80 MHz and pulse width of about 120 fsec. The sample length was limited to 10 mm due to pulse dispersion, as discussed below.

We begin with a study of a 16 μm wide waveguide supporting two guided modes. In order to achieve a controlled excitation we imaged a π phase step onto the input facet of the waveguide [Fig. 2(b)]. Shifting the step position about the waveguide's center resulted in a change in the modal content of the excitation. The pure modal fields were extracted from measurements of the field distribution

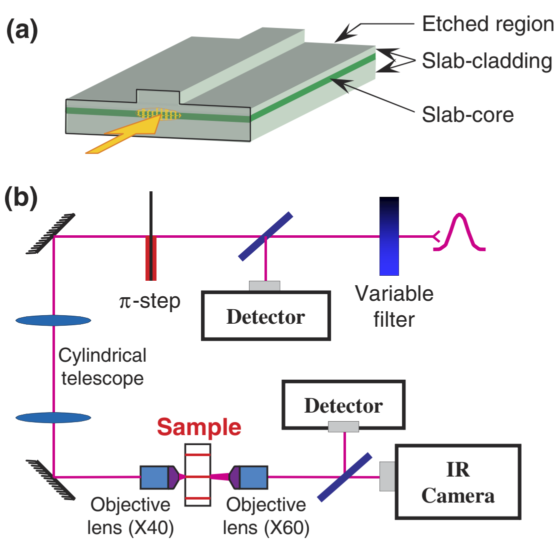

FIG. 2 (color online). Experimental setup. (a) The $Al_xGa_{1-x}As$ slab-waveguide system used in our experiments. The light is vertically confined inside a high index layer (the slab core), sandwiched between two low-index clad layers. An effective 1D waveguide is defined by etching into the top clad layer of the slab waveguide. Top-cladding, core, and bottom-cladding widths of the slab waveguide are 1.5 μm, 1.5 μm, and 4 μm, respectively, producing a vertical index step of 0.03. (b) Experimental system. Using the cylindrical telescope, the horizontal width of the beam is tuned so that it best matches that of the excited waveguide mode.

at the output (see Fig. 3), and any output distribution was then expanded in term of these eigenmodes, by fitting the modal content to the measured intensity profile. In the following, we shell refer to the fraction of intensity carried by each mode, relative to the overall intensity of the excitation, as I_1 and I_2, implying that $I_1 + I_2 = 1$.

The measurement results are shown in Fig. 4. Two sets of measurements are shown, corresponding to two initial modal distributions. Figures 4(a) and 4(d) show plots of I_1 and I_2 at the waveguide output, as a function of the output average power. In the case of $I_1 = 0.1$ (90% of the power initially in mode 2), shown in Fig. 4(a), increasing the power results in the transfer of energy from mode 2 to mode 1, thereby increasing I_1 until reaching a value of 0.47. Increasing the initial value of I_1 to 0.2 [Fig. 4(d)], results in $I_1 = 0.75$ at highest power level. At $I_1 < 0.07$, the effect was hardly observed (results not shown), suggesting a "seed" is needed for the process to initiate. Figures 4(b) and 4(e) show the intensity profile at the waveguide output for low and for highest power level, corresponding to the cases shown in Figs. 4(a) and 4(d), respectively. Figs. 4(c) and 4(f), show corresponding photographs of the output patterns for low and high power. We have verified that these observations did not result from nonlinear loss of the higher mode, by measuring input and output power at each measurement point. We conclude that GSS has been demonstrated in our double-moded waveguide, resulting in the increase of the ground-state content (I_1) at high power.

We also investigated GSS in a 30 μm wide, three-mode waveguide. Modal analysis was not conducted for this case; still, the qualitative results shown in Fig. 5 clearly demonstrate the GSS effect. We launched a narrow beam into the waveguide, exciting a combination of modes 1, 2, and 3 with their relative content determined by the input beam position and inclination with respect to the waveguide center. Two sets of measurements are shown, with different modal content of the excitation at the waveguide input. Figures 5(a) and 5(b) show intensity profile and photographs at the waveguide output, for the first set, at

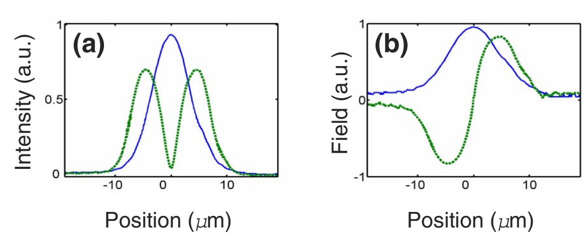

FIG. 3 (color online). Experimentally obtained modes of the double-moded waveguide used in our experiments. (a) Intensity distribution of mode 1 (solid line) and mode 2 (dashed line). (b) Corresponding modal-field distributions. Waveguide width is 16 μm, with an index-step of 0.0007 obtained by etching 0.83 μm deep into the top cladding of the slab-waveguide sample.

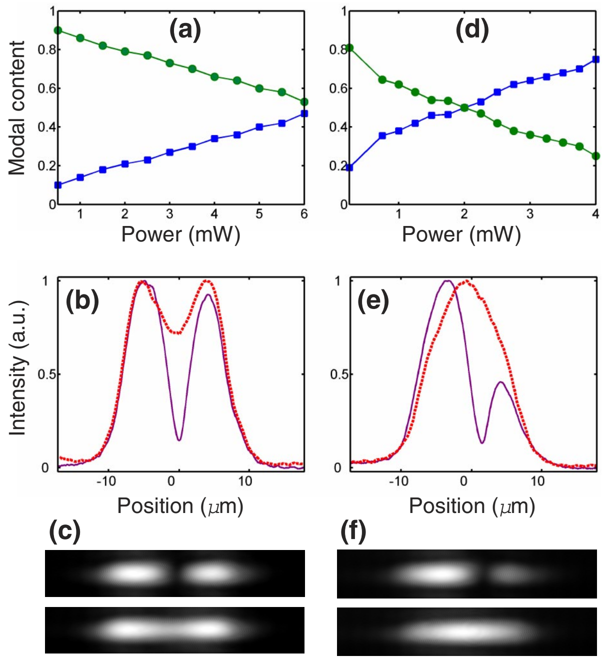

FIG. 4 (color online). Experimental results for the double-moded waveguide of Fig. 3. Two sets of measurements are shown, of the same waveguide, under slightly different launching conditions (π-step position), expressed by a different modal content of the excitation at the waveguide input. (a), (b), (c): first set, (d), (e), (f): second set. (a), (d): the modal content at the waveguide output, expressed in terms of the intensity fractions of mode 1 (squares) and mode 2 (circles), as a function of the output (average) power. The intensity fraction of mode 1 in the linear case is about 0.1 and 0.2, for set 1 and set 2, respectively, growing to about 0.45 and 0.75 at maximal power, in the process of ground-state selection. (b), (e): cross sections of the intensity profile at the waveguide output, for low power (i.e., linear, in solid line) and maximal power (dashed line). (c), (f): photographs of the intensity measured at the waveguide output, for low power (upper panel) and maximal power (lower panel).

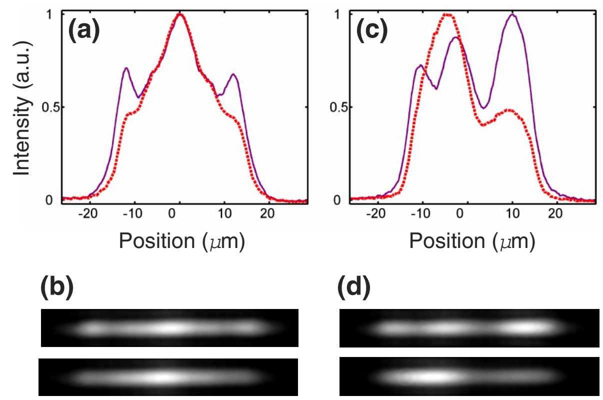

FIG. 5 (color online). Experimental results for a three-moded waveguide. Two sets of measurements are shown, of the same waveguide, under different launching conditions, expressed in a different modal content of the excitation. (a), (b): first set, (c), (d): second set. (a), (c): Cross sections of the intensity at the waveguide output, for low power (i.e., linear, solid line) and high power (dashed line). At low power, the intensity profile exhibits three lobes, indicating substantial mode 3 content. At high power the three lobes disappear, indicating reduced mode 3 content, after being transformed into lower waveguide modes. (b), (d): Photographs of the intensity measured at the waveguide output, for low power (upper panel) and high power (lower panel). Sample parameters are the same as that of the double-moded waveguide, except of the waveguide width being 30 μm.

low and high power. At low power, the intensity profile exhibits three lobes, indicating substantial mode 3 content in the excitation. At high power the two outer lobes disappear almost completely, clearly indicating reduced mode 3 content, after being transformed into lower waveguide modes. The effect measured in the second set [Figs. 5(c) and 5(d)] is even more pronounced, as the three lobes appearing at low power completely disappear as power increases, while the intensity at the waveguide output possesses now two lobes, indicating a mixed mode 1 and mode 2 excitation, with very small mode 3 contribution. Again, we ruled out possible nonlinear loss mechanisms that might interfere with the interpretation of the results shown in Fig. 5. We conclude that mode 3 may indeed participate in the generalized GSS process, thereby transferring its energy to lower-lying stable modes.

We note that sample length was limited to 10 mm, since our pulse broadens temporally by a factor of 3 by the time it reaches the sample output, thereby reducing its peak intensity by the same factor with respect to the input. This relatively short propagation distance forced us to use high peak powers at the input, deviating somewhat from the assumptions made in the theoretical analysis [6,7]. At the highest peak powers the levels are high enough to introduce some deformation of the waveguide linear potential, resulting in modification of the waveguide modes. We have measured this deformation and found it to be rather small. Nevertheless, at the highest power levels [Figs. 4(a) and 4(d)] the results represent a rough estimate of the actual modal content. Simulations using beam propagation methods (BPM) (not shown) [18] accurately reproduced our experimental results. We have observed in these simulations a trade-off between power levels and propagation length, proving the observed effect to be an accumulative one, as in [7]. Furthermore, when the density of the continuum states was reduced (by narrowing the width of the simulation sheet), an inhibition of the GSS process was observed, consistent with the description of [6,7]. We conclude, then, that the effect observed in out experiments origins primarily from the GSS process.

In view of these findings, one might consider a nonlinear multimode waveguide as a ‘‘modal cooler,’’ which transfers energy in the process of GSS, from high waveguide modes into the ground state, while radiating some energy to carry away the resulting difference in entropy. This, however, would work only for double-moded waveguides,

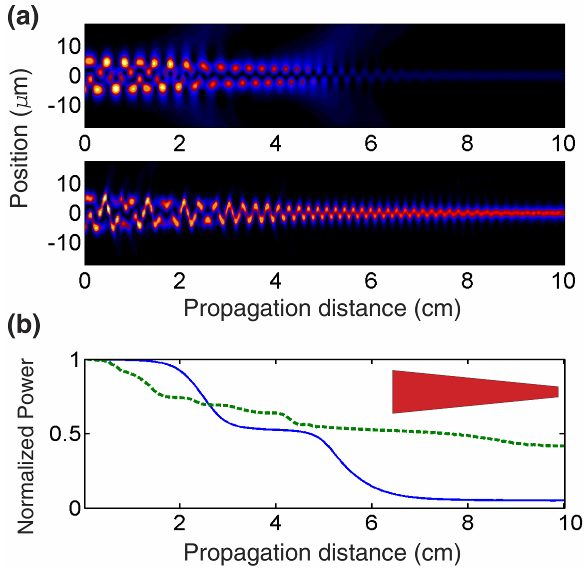

FIG. 6 (color online). Simulation results of a tapered waveguide exhibiting a cascaded GSS process (illustrated in inset). (a) Low-power (upper panel) and high-power (lower panel) propagation in the waveguide. (b) Waveguide power, normalized to the input power, as a function of propagation distance, for low power (solid line) and high power (dashed line). Initial waveguide width is 25 μm (with three guided modes), gradually decreasing to 3 μm (where the waveguide is single moded). Modal intensity fraction at the input is 0.05, 0.475, and 0.475 for modes 1, 2, and 3, respectively. At low power, 95% of the power is radiated out of the waveguide in two steps, corresponding to the widths in which the tapered-waveguide loses each guided mode. At 1500 Watt, 42% of the input power reaches the waveguide end, being occupied completely in the ground state. Index step is 0.001 and overall taper length is 10 cm.

since already in a three-mode waveguide mode 2 may become stable [7], as demonstrated above, and therefore will not relax to the ground state. One approach to overcome this is to use a tapered waveguide, in which the number of guided modes gradually decreases with its width [see inset of Fig. 6(b)]. At low power, the field in such a waveguide will suffer large radiation losses each time the waveguide losses another guided mode. At high power, the unstable nonlinear bound states (i.e., those having their propagation constants lying in the lower half of the spectrum) will get gradually depopulated into the stable states, before becoming nonguided by the waveguide narrowing. Figure 6 shows simulation results of such a device, starting as a three-moded waveguide, and getting narrower until being highly single moded. At low power, the waveguide field suffers high radiation losses, while at high power the waveguide transmission grows from 0.05 to 0.42. We note however that in using such a scheme, this process becomes less and less efficient as the number of guided linear modes initially supported by the waveguide grows higher.

In summary, we have reported the first experimental observation of ground-state selection, in two- and three-moded $Al_xGa_{1-x}As$ waveguides, and discussed the idea of implementing cascaded relaxation to the ground state using a nonlinear tapered waveguide. We strongly believe that better understanding of this fundamental effect is in need, in the optical waveguide system as well as in other systems (e.g., BEC).

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