Tunneling Mechanism of Light Transmission through Metallic Films

F. J. García de Abajo,[1,2] G. Gómez-Santos,[3] L. A. Blanco,[2] A. G. Borisov,[4] and S. V. Shabanov[5]

[1]*Unidad de Física de Materiales CSIC-UPV/EHU, Apartado 1072, 20080 San Sebastian, Spain*
[2]*Donostia International Physics Center (DIPC), Apartado 1072, 20080 San Sebastian, Spain*
[3]*Departamento de Física de la Materia Condensada, Facultad de Ciencias, Universidad Autónoma de Madrid, Cantoblanco, 28049 Madrid, Spain*
[4]*Laboratoire des Collisions Atomiques et Moléculaires, Université Paris-Sud, Bâtiment 351, F-91405 Orsay Cedex, France*
[5]*Department of Mathematics, University of Florida, Florida 32611, USA*

(Received 16 November 2004; published 2 August 2005)

A mechanism of light transmission through metallic films is proposed, assisted by tunneling between resonating buried dielectric inclusions. This is illustrated by arrays of Si spheres embedded in Ag. Strong transmission peaks are observed near the Mie resonances of the spheres. The interaction among various planes of spheres and interference effects between these resonances and the surface plasmons of Ag lead to mixing and splitting of the resonances. Transmission is proved to be limited only by absorption. For small spheres, the effective dielectric constant of the resulting material can be tuned to values close to unity, and a method is proposed to turn the resulting materials invisible.

DOI: 10.1103/PhysRevLett.95.067403 PACS numbers: 78.66.Bz, 42.25.Fx, 42.79.Ci, 73.20.Mf

Substantial efforts have been placed on studying light transmission through perforated metallic films since the discovery by Ebbesen *et al.* [1] of extraordinary transmission through subwavelength periodic hole arrays, leading to such remarkable effects as giant funneling of microwave radiation through metallic micrometer slits [2]. Two distinct mechanisms have been identified that contribute to enhance light transmission: (i) dynamical scattering assisted by surface plasmons (SPs) and (ii) resonant transmission by coupling to propagating modes. (i) Isolated subwavelength holes can support only evanescent modes, but dynamical diffraction assisted by SPs in periodic hole arrays lead to transmission maxima that can beat the severe $1/\lambda^4$ dependence on wavelength λ of the transmission derived by Bethe for single apertures [3], although there is still some controversy regarding the relative relevance of SPs and dynamical diffraction [4–7]. (ii) Slits differ from holes in that the former, however small, supports at least one transmission mode that can couple resonantly to external light to transmit several orders of magnitude more light than impinging directly on the slit aperture [2].

In this Letter we investigate yet another mechanism of enhanced transmission based upon hopping between dielectric inclusions that can sustain localized resonances inside metallic films. We illustrate this concept by studying light transmission through metallic Ag films that contain spherical Si inclusions arranged in square-lattice layers, which belong to the family of periodic arrays of nanoresonators coupled via spatially homogeneous waveguides [8]. The transmission is shown to take large values near the Mie resonances of the spheres, and it is limited only by metal absorption.

We first consider a single layer of inclusions [Fig. 1(a)]. Maxwell's equations are solved rigorously for this geometry using a layer-by-layer version of the Korringa-Kohn-Rostoker multiple-scattering method [9], with lattice sums performed in real space within the metal. The complex, frequency-dependent dielectric functions of Ag and Si are taken from optical data [10].

The resonant modes of the Si spheres can be visualized through their Mie scattering coefficients inside bulk metal [11], showing prominent features in the visible and near-IR domains for a sphere radius $R = 75$ nm [Fig. 1(b)]. The corresponding transmittance spectrum exhibits peaks near those resonances [Fig. 1(c), at normal incidence] that can exceed a transmission $T = 50\%$ at wavelengths near $\lambda = 975$ nm, well above the $T = 13\%$ value expected for a homogeneous Ag film of thickness $2b = 15$ nm (i.e., the combined thickness of the thin burying layers at either side of the spheres). Smaller separations between the sphere and the film boundaries lead to even higher transmission (e.g., $T = 70\%$ for $b = 2.5$ nm). The transmission peaks are redshifted with respect to the resonances in bulk Ag as a result of interaction between sphere modes and vacuum modes. However, the relevant multipoles of the inclusions and the polarization symmetry associated with those peaks can be well ascribed by comparing Figs. 1(b) and 1(c).

The interaction of spherical inclusions along the plane can be important at small separations between the spheres. This is illustrated in Fig. 2, which shows a contour plot of the transmittance T as a function of lattice constant a and wavelength. T has been multiplied by $a^2/\pi R^2$ to obtain transmission cross sections per sphere normalized to the sphere projected area. The $E1$ and $E2$ Mie resonances of Fig. 1(b) show up as two lines of transmission maxima relatively independent of a. However, these two modes interact and repel each other when the spheres are near touching each other (bottom part of the figure).

The SPs of the planar Ag interfaces cannot couple directly to external propagating modes. However, the presence of Si inclusions provides a source of momentum that

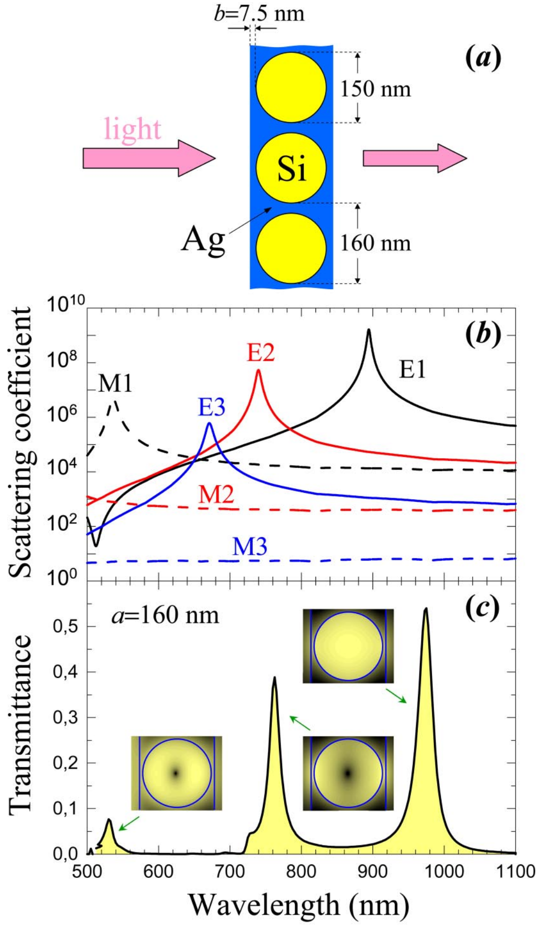

FIG. 1 (color online). (a) Transversal section of a Ag film containing a buried square planar array of 150 nm Si spheres in square-lattice configuration. (b) Square modulus of the Mie scattering coefficients of single 150 nm Si spheres inside bulk Ag for different multipoles ($l = 1, 2, 3$) and polarization modes [electric (E) and magnetic (M)] as a function of free-space light wavelength. (c) Absolute transmittance through the film of (a) under normal incidence for a lattice constant $a = 160$ nm. The insets show maps of the electric field strength in the slab in logarithmic scale for circularly polarized incident light at the peak wavelengths.

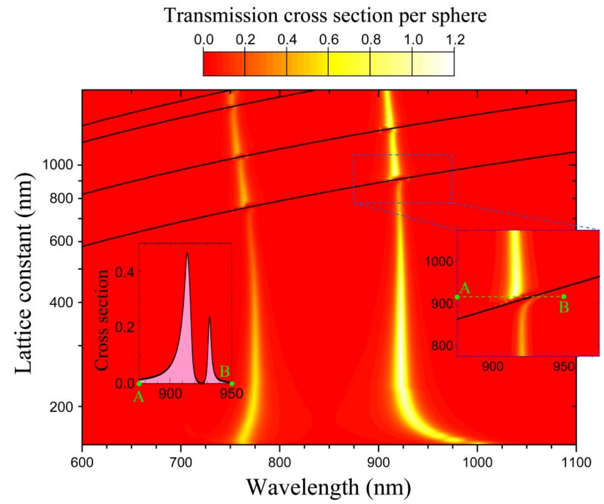

FIG. 2 (color online). Transmittance through the film described in Fig. 1(a) under normal incidence as a function of wavelength and lattice constant. The transmittance is normalized to the projected area of the spheres. The solid curves are given by the condition that the momentum of the planar surface plasmon of Ag matches some reciprocal lattice vector of the sphere lattice. The right inset is a blowup of an interference region, and the left inset shows the cross section along the segment AB.

makes this coupling possible. The coupling increases when the interaction between lattice sites (spheres) via SPs is constructive, which at normal incidence (all lattice sites are in phase) leads to the kinematical condition that SP momenta that match reciprocal lattice vectors of the square lattice of spheres are favored, that is, $\lambda\sqrt{n^2 + m^2} = a\sqrt{\epsilon_{\rm Ag}/(\epsilon_{\rm Ag} + 1)}$ [1], where the surface plasmon dispersion relation has been invoked. The solid curves in Fig. 2 are obtained from this equation for different combinations of integers n and m, and they would lie near regions of transmission enhancement in hole arrays [1], but that transmission mechanism is marginal for our buried spheres. Instead, a rich transmission structure shows up when the noted kinematical condition (solid curves) is at the same wavelength as a Mie resonance [see right inset of Fig. 2]: the discrete, localized Mie modes couple to the continuum of SPs to yield a transmission spectrum exhibiting a point of zero transmission, as it is well known from Fano's theory [12] (see left inset of Fig. 2).

Several layers of spheres can interact in a single film producing splitting and shifting of the Mie resonances, as shown in Fig. 3. The main 975 nm feature of Fig. 1(c) is subdivided here into a series of N peaks in a film containing N layers. The coupling between Mie modes at consecutive layers is the same as in previously introduced photonic molecules that arise from the interaction of Mie modes in neighboring dielectric particles [13]. The tight-binding approach used in that context becomes a natural tool to understand the hopping of light between contiguous spheres in our case. In this spirit, an analogy with electron transmission between localized states in quantum systems can be established by considering a simple tight-binding model [14] in which a resonant mode trapped in each layer is associated with a state $|i\rangle$ (with $i = 1, 2, \ldots, N$). The dynamics is governed by the Hamiltonian

$$H = (\omega_b + {\rm i}\Sigma_s)(|1\rangle\langle 1| + |N\rangle\langle N|) + \sum_{i=2}^{i=N-1} (\omega_b + {\rm i}\Sigma_b)|i\rangle \times \langle i| + \sum_{i=1}^{i=N-1} (V|i\rangle\langle i+1| + {\rm H.c.}),$$

where ω_b and Σ_b are the energy and width of the resonances, and V is the hopping amplitude, which expresses the magnitude of the interaction between spheres. For internal layers the width Σ_b accounts for dissipation into bulk metal. The states $|1\rangle$ and $|N\rangle$ interact with the continuum of light modes on the left and right hand sides of the film, respectively, described by a frequency-dependent self-energy $\Sigma_s(\omega)$. The transmittance in the model is defined by the squared absolute value of the transition amplitude and can be written as [14]

$$T = 4|\Sigma_s(\omega) g_{1,N}(\omega + i0^+)|^2, \quad (1)$$

where $g_{1,N}(z) = \langle 1|(z - H)^{-1}|N\rangle$. The parameters of the Hamiltonian have been chosen to produce the best fit to the transmittance obtained from the numerical solution of Maxwell's equations: $\Sigma_b = -0.002\omega_b$, $V = 0.03\omega_b$, and a linear function coupling to vacuum given by $\Sigma_s(\omega) = -0.005\omega_b - 0.03(\omega - \omega_b)$. Note that radiative coupling via nonvanishing $\mathrm{Im}\{V\}$ can produce changes in the line shapes [15] that lie beyond the scope of this work.

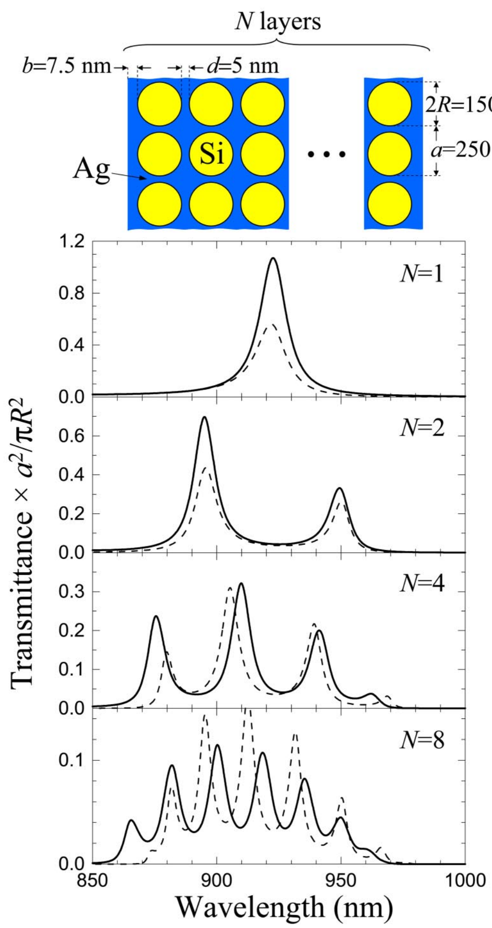

FIG. 3 (color online). Transmittance through Ag films containing different numbers N of buried planar arrays of Si spheres of radius $R = 75$ nm under normal incidence conditions. The planar unit cell is a square of side $a = 250$ nm, which prevents interaction between spheres within each plane. Solid curves are exact solutions of Maxwell's equations. Broken curves are obtained from the tight-binding model explained in the text [Eq. (1)].

The results derived from Eq. (1) are shown in Fig. 3 as dashed curves. Characteristic features of the exact transmittance (solid curves) are well reproduced, including peak splitting, peak asymmetry coming from the energy dependence of Σ_s, and narrowing of the resonances with an increasing number of layers. This model corroborates that the main mechanism producing transmission is tunneling between electromagnetic modes trapped in each layer.

The dielectric function of Ag near the main resonance of Fig. 3 is essentially negative ($\epsilon_{\mathrm{Ag}} = -49 + 0.6i$), so that coupling among the spheres and between the spheres and the film planar surfaces is mediated by evanescent modes. However, the small imaginary part of ϵ_{Ag} plays a leading role in reducing the absolute transmittance ($\mathrm{Im}\{\epsilon_{\mathrm{Si}}\} \ll 1$ [10]). This is made clear in Fig. 4, which has been calculated under the same conditions as in Fig. 3 but setting the imaginary part of ϵ_{Ag} to a positive infinitesimal. All resonances in the figure reach a maximum of 100% transmission, a fact that can be rigorously explained following simple arguments based upon the change by π of the phase of reflected and transmitted resonant components of the field across the resonance [16].

A Mie cavity mode inside a lossless bulk metal must have infinite lifetime because light can neither scape through nor be absorbed by the metal. Therefore, the width of the resonance in the lossless film for $N = 1$ (6.72 nm FWHM, as obtained from Fig. 4) has to originate in the coupling of the $E1$ mode with the continuum of free states

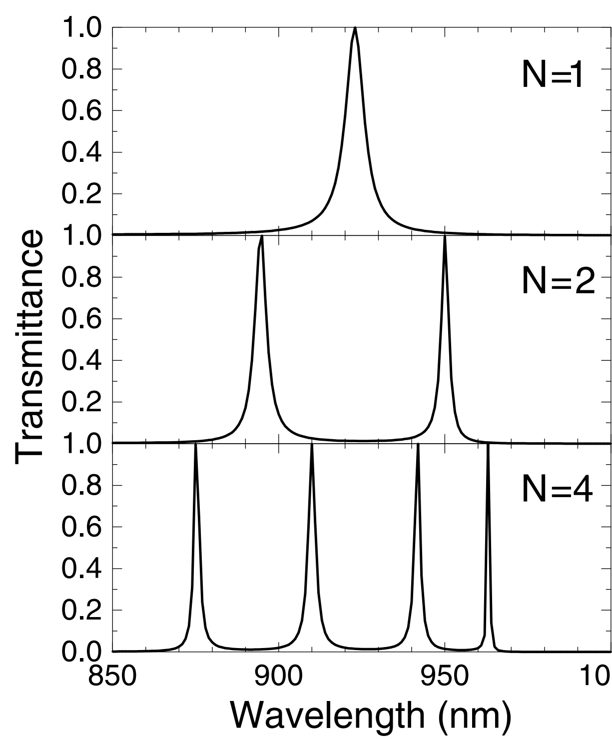

FIG. 4. Absolute transmittance under the same conditions as in Fig. 3, but setting the imaginary part of the dielectric constant of Ag to zero.

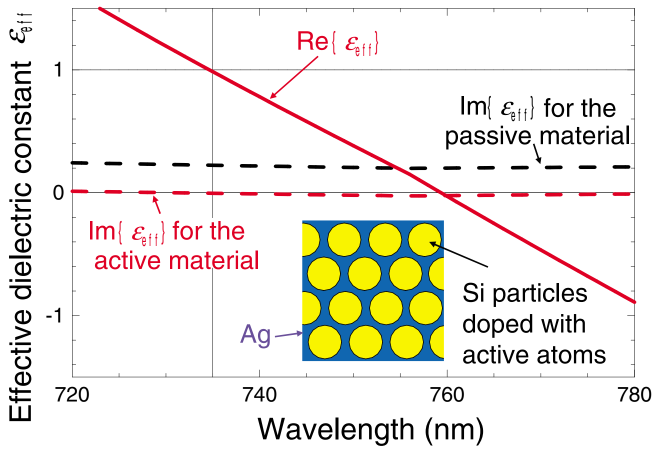

FIG. 5 (color online). Effective dielectric function of an fcc array of Si spherical inclusions in Ag with a filling fraction of Si of 50%. The effective dielectric function is modified when Si is doped with active centers, described by adding a negative imaginary part ($-0.252i$) to its dielectric function. The resulting material is invisible ($\epsilon_{eff} = 1$) at a wavelength of 735 nm.

on either side of the film (leakage into the vacuum). For multilayer systems (e.g., $N = 2$), as a consequence of trace preservation in interacting systems as compared to the noninteracting ones, this broadening has to remain the same (indeed, the sum of the FWHM in the two peaks for $N = 2$ is 6.72 nm as well). This *leakage* contribution to peak broadening adds up to the intrinsic width of the Mie resonance in the lossy bulk metal [4.53 nm FWHM as obtained from the E1 peak in Fig. 1(b)], yielding a total width of 11.25 nm, which compares well with the value of 11.83 nm observed for the actual lossy metallic film ($N =$ 1 solid curve in Fig. 3).

Therefore, our metallic film would be invisible to normally incident radiation at specific resonant wavelengths if the metal was free of absorption. The question arises whether the same mechanism of hopping between dielectric inclusions in metals can be utilized to fabricate solid structures that are invisible to light incident with any angle at some wavelength or, in other words, whether a medium exists exhibiting effective dielectric function $\epsilon_{eff} = 1$ (and magnetic response $\mu_{eff} = 1$ as well, of course). To explore this possibility, we have represented ϵ_{eff} in Fig. 5 for a composite material containing 50% of Si inside spherical inclusions, which are arranged within Ag in an fcc lattice and separated by distances much shorter than the wavelength. ϵ_{eff} is obtained using Maxwell-Garnett theory [17], and we have found qualitatively similar results from rigorous solution of Maxwell's equations for the reflectivity of a surface of the composite material as compared to Fresnel equations for an equivalent homogeneous medium. There is a wavelength at which the real part of ϵ_{eff} is 1, but its imaginary part takes a small but non-negligible value.

We propose to dope Si in such a structure with optically active atoms or molecules that are capable of sustaining an inversion of population under excitation by light of a different wavelength or by electric discharge. The resulting lasing activity can be well represented by a negative imaginary part contributing to the dielectric function of doped Si [18], whose value can be controlled by the doping dose or, in 4-level active atoms, also via the pumping intensity. In particular, when this imaginary contribution to ϵ_{Si} is taken as $-0.252i$, then $\epsilon_{eff} = 1$ at 735 nm (the imaginary part of ϵ_{eff} cancels out exactly), and the material becomes invisible at that wavelength.

In conclusion, we have shown that light transmission through metallic films can be enhanced by coupling to resonances localized in dielectric inclusions, paving the way towards driving light deep inside metals. Light circuits based upon this mechanism can be envisioned. When the inclusions are small compared to the wavelength, the resulting metamaterial can be made invisible (i.e., $\epsilon_{eff} = 1$) if metal absorption is compensated by doping the dielectric with active atoms under inversion population conditions. We have proposed a specific design for such invisible material at wavelengths in the visible, but structures insensitive to microwaves should also be realizable due to smaller absorption in this range and using reradiating active elements in the millimeter scale.

[1] T. W. Ebbesen *et al.*, Nature (London) **391**, 667 (1998); H. F. Ghaemi *et al.*, Phys. Rev. B **58**, 6779 (1998).
[2] Y. Takakura, Phys. Rev. Lett. **86**, 5601 (2001); F. Yang and J. R. Sambles, Phys. Rev. Lett. **89**, 063901 (2002).
[3] H. A. Bethe, Phys. Rev. **66**, 163 (1944).
[4] M. M. J. Treacy, Appl. Phys. Lett. **75**, 606 (1999); Phys. Rev. B **66**, 195105 (2002).
[5] E. Popov *et al.*, Phys. Rev. B **62**, 16 100 (2000).
[6] L. Martín-Moreno *et al.*, Phys. Rev. Lett. **86**, 1114 (2001).
[7] M. Sarrazin *et al.*, Phys. Rev. B **67**, 085415 (2003).
[8] S. G. Tikhodeev *et al.*, Phys. Rev. B **66**, 045102 (2002); A. Christ *et al.*, Phys. Rev. Lett. **91**, 183901 (2003); T. Zentgraf *et al.*, Phys. Rev. Lett. **93**, 243901 (2004).
[9] N. Stefanou *et al.*, Comput. Phys. Commun. **113**, 49 (1998); **132**, 189 (2000).
[10] E. D. Palik, *Handbook of Optical Constants of Solids* (Academic Press, New York, 1985).
[11] J. D. Jackson, *Classical Electrodynamics* (Wiley, New York, 1975).
[12] U. Fano, Phys. Rev. **124**, 1866 (1961).
[13] T. Mukaiyama *et al.*, Phys. Rev. Lett. **82**, 4623 (1999).
[14] E. N. Economou, *Green's Functions in Quantum Physics* (Springer-Verlag, Berlin, 1983).
[15] C. Ropers *et al.*, Phys. Rev. Lett. **94**, 113901 (2005).
[16] A. G. Borisov *et al.*, Phys. Rev. B **71**, 075408 (2005).
[17] J. C. Maxwell-Garnett, Phil. Trans. R. Soc. A **203**, 385 (1904); **205**, 237 (1906).
[18] See V. I. Kopp *et al.*, Phys. Rev. Lett. **86**, 1753 (2001). We overlook the change in Re{ϵ_{Si}} here, as it produces only a shift of the wavelength of invisibility.