

Quantization of the Diagonal Resistance: Density Gradients and the Empirical Resistance Rule in a 2D System

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We have observed quantization of the *diagonal* resistance, R_{xx} , at the edges of several quantum Hall states. Each quantized R_{xx} value is close to the difference between the two adjacent Hall plateaus in the *off-diagonal* resistance, R_{xy} . Peaks in R_{xx} occur at different positions in positive and negative magnetic fields. Practically all R_{xx} features can be explained *quantitatively* by a 1%/cm electron density gradient. Therefore, R_{xx} is determined by R_{xy} and unrelated to the diagonal resistivity ρ_{xx} . Our findings throw an unexpected light on the empirical resistivity rule for 2D systems.

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The quantum Hall effects [1–3] in a two-dimensional electron system (2DES) in a high magnetic field (B) are characterized by plateaus quantized to integer or fractional values of the resistance quantum, h/e^2 , in the off-diagonal resistance, R_{xy} , with concomitantly occurring vanishing values in the diagonal resistance R_{xx} . A multitude of integer (IQHE) and fractional (FQHE) quantum Hall effects have been discovered over the past two decades, pointing to a wide variety of underlying single-particle and many-particle electronic states. In all cases, resistance quantization is the purview of R_{xy} , while vanishing resistance occurs solely in R_{xx} .

In general, the Hall resistance, R_{xy} , is a monotonic function of B following a somewhat irregular sequence of slightly broadened steps. The magnetoresistance, R_{xx} , has the appearance of the derivative of R_{xy} with respect to B . In fact, a phenomenological relationship, $R_{xx} \propto B \times dR_{xy}/dB$, called the resistivity rule, is so closely observed in experiment [4–6] that one suspects a deep fundamental relationship. While there exists a somewhat complex model based on density fluctuations that can account for the general features [7], the origin of the empirical resistivity rule continues to be enigmatic.

Recent experiments on ultrahigh quality specimens performed at very low temperatures in the second Landau level [8,9] have yielded data that seem to contradict the resistivity rule. Instead of rising in a stairlike fashion from the IQHE through the FQHE plateaus to the next IQHE, R_{xy} switches back and forth several times between FQHE and IQHE values (Fig. 1). This phenomenon is called the reentrant IQHE or RIQHE. At present we have only an *ad hoc* understanding of its origin [10,11]. As a result of the RIQHE, R_{xy} is no longer monotonic and the empirical resistivity rule now implies regions of negative R_{xx} , which

is contrary to simple inspection. It is unclear whether another relationship holds in this region.

In examining the relationship between R_{xx} and R_{xy} in this filling factor regime, we discovered, at temperatures of 6 mK, regions of *quantized diagonal resistance*, R_{xx} . The quantized values of R_{xx} are very close to the differences of the two adjacent Hall plateaus in R_{xy} , and we can explain them in terms of a slight, unintentional electron density gradient. Furthermore, we can explain *quantitatively* the complete R_{xx} trace, including its asymmetry in B field directions, as being the result of such a density gradient. In turn, the resistivity rule derives trivially from such a gradient picture. In this way, R_{xx} is just a reflection of R_{xy} measured at two slightly different densities, and the diagonal resistivity, ρ_{xx} , has practically no bearing on R_{xx} . We discuss the implication of these astonishing findings.

Our experiments were carried out in a demagnetization-dilution refrigerator combination described in Ref. [9]. The lowest temperature (T) in our experiment was ~ 6 mK. The 2DES is confined in a 30 nm wide GaAs/Al_xGa_{1-x}As quantum well that is delta doped on both sides of the well at a setback distance of 100 nm. The wafer was grown under rotation to minimize electron density gradients. The specimen consists of a 4 mm \times 4 mm square with 8 diffused indium contacts as seen in the inset of Fig. 2. The electron density is $n = 3 \times 10^{11}$ cm⁻² and the mobility is $\mu = 31 \times 10^6$ cm²/Vs. Lock-in techniques with ~ 7 Hz were utilized to measure R_{xx} and R_{xy} at an excitation current of 1 nA.

Figure 1 shows our data on R_{xx} and R_{xy} at $T \sim 9$ mK. The top trace in light gray shows the Hall resistance, R_{xy} , in the negative B field direction ($B -$). R_{xy} in the positive B field direction ($B +$) is virtually identical to this R_{xy} and not shown. The R_{xy} trace shows clear FQHE plateaus at

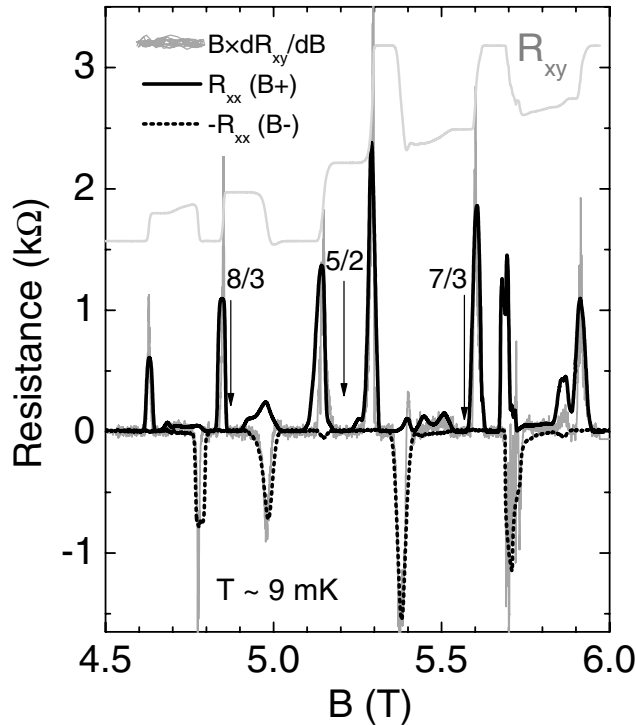


FIG. 1. Comparison of $B \times dR_{xy}/dB$ (noisy gray trace), $R_{xx}(B+)$ (black line), and $-R_{xx}(B-)$ (dotted line). The Hall resistance in the reversed B field direction is plotted as the light gray trace. Arrows mark the positions of prominent FQHE states.

$\nu = 5/2, 7/3, 8/3$. Four RIQHE states are apparent. At these positions R_{xy} , for a finite B -field region, assumes an IQHE value. The bottom part of Fig. 1 shows R_{xx} data. $R_{xx}(B+)$ is plotted in black showing the expected deep minima at the positions of plateaus in R_{xy} and sharp spikes at their edges. However, in this case the spikes are concentrated on the sides at which R_{xy} is rising sharply, whereas the sides associated with sharp drops in R_{xy} are almost absent in R_{xx} . Figure 1 also shows $R_{xx}(B-)$ data but plotted as $-R_{xx}(B-)$, in the negative y direction, for later comparisons. As in $R_{xx}(B+)$ we observe the required minima and the sharp spikes at their flanks. In this case, however, the spikes are concentrated on the sides at which R_{xy} is dropping sharply.

To examine the resistivity rule in the second Landau level, we show in Fig. 1 $R_{diff} = B \times dR_{xy}/dB$, as a gray trace, multiplied by a constant $c = 0.003$ for comparison with R_{xx} . There is a very strong resemblance between the traces. The positive parts of $c \times R_{diff}$ are well reflected in $R_{xx}(B+)$, whereas the negative going sections of $c \times R_{diff}$ closely match the inverted $R_{xx}(B-)$ trace. At first such an asymmetry appears puzzling but will make sense in the context of the following R_{xx} quantization and the edge channel picture.

In Fig. 2, we show $R_{xx}(B+)$ as a black trace taken at $T \sim 6$ mK. The main features are similar to what was observed at $T \sim 9$ mK in Fig. 1. However, at 6 mK, when the Hall resistance varies abruptly, R_{xx} assumes a flat top. This is

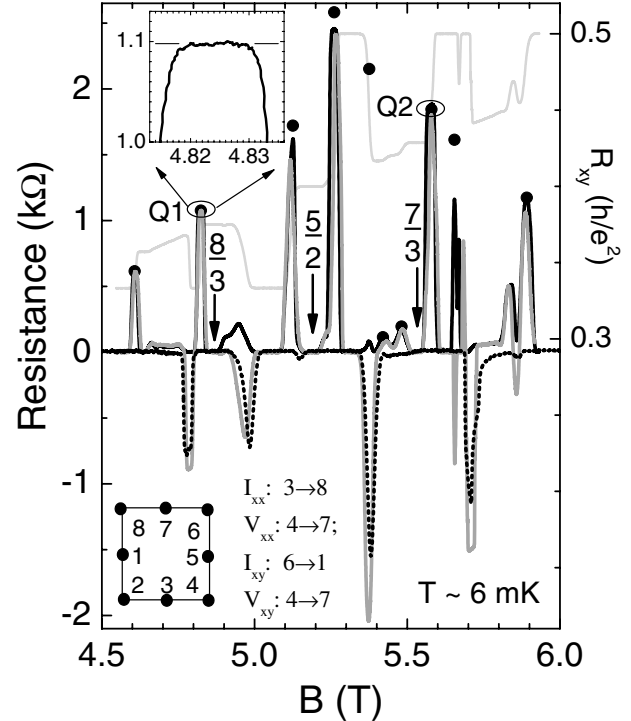


FIG. 2. Main plot: $R_{xx}(B+)$ (black trace), $-R_{xx}(B-)$ (dotted line), and $\Delta R_{xy} = R_{xy}(n) - R_{xy}(n - 0.005n)$ (gray curve), where n is electron density. The temperature is ~ 6 mK. Arrows mark the major FQHE states. The bottom inset shows contact configuration for R_{xx} and R_{xy} measurement. In the top inset, the region marked as Q1 is magnified to show the quantization of R_{xx} .

clearly seen in the upper inset, which shows a magnification of the region Q1, near $\nu = 8/3$. A similar quantization is observed in the region marked Q2 at a B field ~ 5.58 T and close to $\nu = 7/3$. R_{xy} (thin light gray trace), on the other hand, shows no anomaly, but varies monotonically between plateaus. The observed R_{xx} plateaus appear only for $T < 9$ mK and their width increase with decreasing T .

The R_{xx} plateau values are very close to the difference between the values of the two adjacent quantum Hall plateaus in R_{xy} . For example, around $\nu = 8/3$, $R_{xx} \cong 1097 \Omega$, while $\Delta R_{xy} = R_{xy}(\nu = 8/3) - R_{xy}(\nu = 3) = h/e^2 \times (3/8 - 1/3) \sim 1076 \Omega$. The solid dots in Fig. 2 represent the value of ΔR_{xy} calculated from the adjacent R_{xy} pairs. It is apparent that $R_{xx} \cong \Delta R_{xy}$ holds for all well-developed peaks, even for those between adjacent FQHE states, such as between $\nu = 2 + 2/5$ and $2 + 3/8$ and between $\nu = 2 + 3/8$ and $2 + 1/3$.

Having made this discovery in the second Landau level, we tested the relationship more generally and found that $R_{xx} \cong \Delta R_{xy}$ also holds for the well-developed peaks in the lowest Landau level. Figure 3 shows R_{xx} (black) and R_{xy} (light gray) traces for $2 > \nu > 1$. The solid dots represent ΔR_{xy} . Their position, in many cases, matches closely the height of the strong peaks in R_{xx} , although, in this filling factor range, we did not observe quantization of R_{xx} .

Plateaus in R_{xx} have been previously observed in macroscopically inhomogeneous 2D systems [12–19]. In fact, the phenomenology of these data is reminiscent of the content of Figs. 1 and 2. For example, in Si MOSFETs with spatially varying electron densities Syphers and Stiles [12] observed R_{xx} quantization and strong peaks in R_{xx} in one B -field direction, but practically vanishing R_{xx} in the opposite field direction. In all cases, the observations are best explained in the edge channel picture. At the boundary between two regions of different filling factors ν_1 and ν_2 ($\nu_1 > \nu_2$), a total of $\nu_1 - \nu_2$ edge channels are being reflected. This leads to a voltage drop between these two regions equivalent to $R_{xx} = h/e^2 \times (1/\nu_2 - 1/\nu_1)$. From the similarities between our data and those reported in the literature, we deduce that a very small, unintentional density gradient exists in our 2D specimens. The width $\Delta B \sim 0.01$ T of the Q1 translates directly into a difference of $\sim 0.5\%$ in density between the requisite voltage probes (V_{xx} : $4 \rightarrow 7$, lower inset Fig. 2). Having a distance of about 5 mm, this represents an $1\%/cm$ density gradient.

Since the R_{xx} plateaus seem to be caused by a density gradient, one wonders whether other R_{xx} features of Fig. 2 follow. To test this, we assume a density n_1 at voltage probe 4 and a density n_2 at voltage probe 7 and calculate the difference $\Delta R_{xy} = R_{xy}(n_1) - R_{xy}(n_2) \equiv R_{xy}(n) - R_{xy}(n + \Delta n)$ with $\Delta n = n_2 - n_1$. This is performed numerically, by subtracting the 6 mK R_{xy} trace from the same trace with a B -field axis compressed by $2\Delta n/(n_1 + n_2) \cong 0.5\%$. The resulting data are plotted as a thick gray trace in Fig. 2. The resemblance with R_{xx} is striking. The positive part of ΔR_{xy} reproduces $R_{xx}(B+)$ perfectly, almost over the entire region. The negative part of ΔR_{xy} , on the other hand, closely matches the inverted $R_{xx}(B-)$. Most importantly, this procedure creates the quantized R_{xx} plateaus of Fig. 2. In fact, as in the second Landau level, ΔR_{xy} (dotted line in Fig. 3) for a density difference of 0.5% also reproduces R_{xx} between $\nu = 1$ and $\nu = 2$ quantitatively. An exception arises around $B \sim 11$ T, where the ΔR_{xy} peak exceeds R_{xx} by about a factor of 2.

It is surprising that a simple modeling based on density inhomogeneity can produce R_{xx} almost perfectly from R_{xy} . In fact, a density gradient can explain the origin of the entire resistivity rule $R_{xx} = c \times R_{\text{diff}}$ simply in terms of ΔR_{xy} . A small density difference Δn between R_{xx} voltage probes creates at any given B a difference in Hall voltage $\Delta V_{xy}(n, B) = I \times [R_{xy}(n, B) - R_{xy}(n + \Delta n, B)] \cong I \times dR_{xy}(n, B)/dn \times \Delta n = I \times dR_{xy}(n, B)/dB \times B/n \times \Delta n = I \times B \times dR_{xy}(n, B)/dB \times (\Delta n/n)$. The latter equality holds since $dB/B = dn/n$. Therefore, $\Delta R_{xy} = \Delta V_{xy}/I = B \times dR_{xy}(n, B)/dB \times (\Delta n/n) = c \times R_{\text{diff}}$. Since $R_{xx} = c \times R_{\text{diff}}$ is borne out in experiments, the resistivity rule can simply be derived from a density gradient. Most importantly, the adjustable parameter, c , is now determined as $c = \Delta n/n$. The value $c = 0.003$ we used in Fig. 1 is in fair agreement with the density gradient of 0.005 , consid-

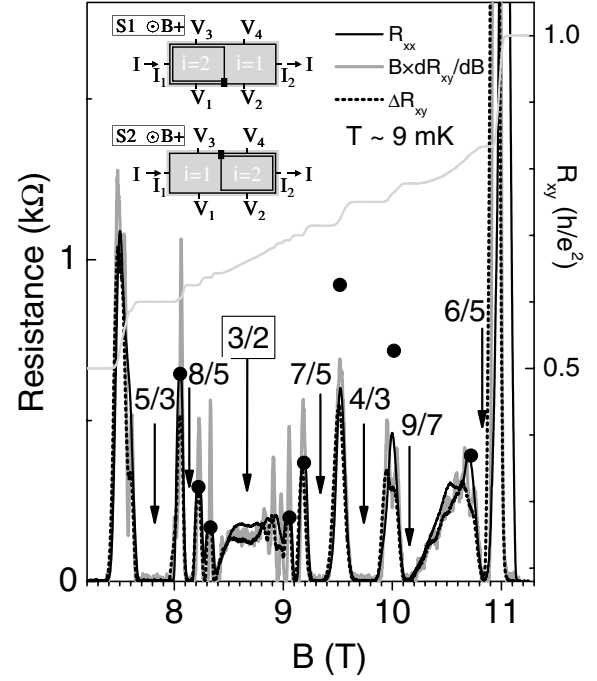


FIG. 3. Magnetoresistance in the first Landau level around $\nu = 3/2$: $R_{xx}(B+)$ (black trace), $B \times dR_{xy}/dB$ (gray curve), and $\Delta R_{xy} = R_{xy}(n) - R_{xy}(n + 0.005n)$ (dotted line). The inset shows the edge model with density inhomogeneity. See text for details.

ering that the two sets of data were taken in two different cooldowns and with different illumination history.

Before discussing the implication of our findings, we address the B -field polarity dependence seen in Figs. 1 and 2. This is best accomplished in the edge channel picture. The inset of Fig. 3 shows two idealized sample configurations (S1 and S2) with current probes I_1, I_2 and voltage probes V_1 through V_4 . Each sample contains two different quantum Hall regimes caused by density inhomogeneity. For simplicity, we adopt QHE states $i = 1$ and $i = 2$. We first emphasize that the edge channel structure does not depend on current direction, but only on the direction of the B field, which is pointing into the plane. The whole specimen is encircled by the $i = 1$ edge state, whereas only a part of it contains a second $i = 2$ edge channel. The edge channels represent equipotentials. When an external current is applied, so-called “hot spots” [20] develop (shown as black squares), at which the potential along the edge suddenly jumps. The $V_1 \rightarrow V_2$ voltage drop in S1 reflects a quantized value of R_{xx} although for a different set of ν 's than seen in Fig. 2. The opposite side of the specimen (V_3, V_4) is in the zero-resistance state [21]. If we reverse the ordering of $i = 1$ and $i = 2$, as in S2, the hot spot switches sides and occurs between V_3 and V_4 while $V_1 - V_2$ vanishes.

When the B field is swept through a normal stretch of monotonically rising R_{xy} values, the region of higher quantum number always remains on the same side of the region

with lower quantum number. Therefore, the hot spot remains on the same side of the sample and the density gradient is observed in R_{xx} on that side only. As the B field is swept through anomalous regions of R_{xy} , in which R_{xy} suddenly *drops* (as in the vicinity of RIQHE states) the spatial order of quantum numbers is *reversed* (as shown in S2), the hot spot switches sides, the voltage drop appears in $V_3 - V_4$, and $V_1 - V_2$ enters the zero-resistance state. Therefore, there are no R_{xx} features observed between probes V_1 and V_2 . They rather appear on the opposite side of the sample. With a limited number of good contacts to our sample, we could not test this switching to the opposite side. However, B -field reversal has the same effect as reversal of the spatial sequence of quantum numbers. Therefore, under reversed B field, we observe at the V_1, V_2 contacts only features in R_{xx} when R_{xy} experiences an anomalous, sudden *drop*. For normal sudden *rises* in R_{xy} and this B -field direction the hot spot resides on the opposite side of the sample, and, hence, $V_1 - V_2$ shows the zero-resistance state and no spike.

The edge channel model, combined with the existence of a density gradient of $\sim 1\%/cm$, explains all our observations in R_{xx} : quantization of R_{xx} , R_{xx} as a reflection of ΔR_{xy} , as well as the appearance of selected peaks in R_{xx} depending on the B -field direction. We emphasize that almost all features of R_{xx} are explained *quantitatively* in terms of ΔR_{xy} caused by a slight density gradient. This explains trivially the resistivity rule in our specimen, not only in the second Landau level with its particularities, but also in the lowest Landau level. Therefore, in our experiments, R_{xx} does not provide any information on ρ_{xx} , but is only a reflection of R_{xy} .

The irrelevance of ρ_{xx} to R_{xx} is rather disturbing. R_{xx} has been used to study many QHE properties from the size of the energy gap, to two-parameter scaling, to the observation of critical points and quantum phase transitions [1,3,22,23]. To the degree to which interpretations of the data in terms of R_{xx} , ΔR_{xy} , or $c \times B \times dR_{xy}/dB$ lead to identical physics, our results do not impact such interpretations based on R_{xx} . While we are unable to review all previous usages of R_{xx} , it appears unlikely that ρ_{xx} is irrelevant to all of them. Furthermore, the interpretation of our data rests on the edge channel picture, which is expected to hold well only in the nondissipative regime. It is, hence, surprising that even the data of Fig. 3 can be interpreted in terms of this model. While it has been shown that the transport at the Fermi level involves all electronic levels below E_f [24], it is not obvious that such a relationship holds globally in the presence of macroscopic inhomogeneities, such as a gradient.

All our data were taken below 20 mK and the gradient model holds for the entire temperature range. In a different, high mobility quantum well sample, this model holds at a high temperature of 1.2 K. In fact, the resistivity rule has been pursued to temperatures as high as 100 K [6], and one

wonders whether the same density gradient is responsible or whether another phenomenon enters. If the latter were the case, one wonders how the resistivity rule can hold without modification or transition for this entire temperature range; starting at low temperatures with a density gradient as its origin, to the highest temperature, with a totally different origin, probably related to ρ_{xx} .

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- [1] *The Quantum Hall Effect*, edited by R. E. Prange and S. M. Girvin (Springer-Verlag, New York, 1990).
 - [2] T. Chakraborty and P. Pietilainen, *The Quantum Hall Effects*, Springer Series in Solid State Sciences Vol. 85 (Springer-Verlag, Berlin, 1995).
 - [3] *Perspectives in Quantum Hall Effects*, edited by S. Das Sarma and A. Pinczuk (Wiley, New York, 1996).
 - [4] A. M. Chang and D. C. Tsui, *Solid State Commun.* **56**, 153 (1985).
 - [5] H.L. Stormer *et al.*, *Solid State Commun.* **84**, 95 (1992).
 - [6] T. Rötger *et al.*, *Phys. Rev. Lett.* **62**, 90 (1989).
 - [7] S.H. Simon and B.I. Halperin, *Phys. Rev. Lett.* **73**, 3278 (1994), and references therein.
 - [8] J.P. Eisenstein *et al.*, *Phys. Rev. Lett.* **88**, 076801 (2002).
 - [9] J.S. Xia *et al.*, *Phys. Rev. Lett.* **93**, 176809 (2004).
 - [10] N. Shibata and D. Yoshioka, *J. Phys. Soc. Jpn.* **72**, 664 (2003).
 - [11] M. O. Goerbig, P. Lederer, and C. M. Smith, *Phys. Rev. B* **69**, 115327 (2004).
 - [12] D.A. Syphers and P.J. Stiles, *Phys. Rev. B* **32**, 6620 (1985).
 - [13] M. Büttiker, *Phys. Rev. B* **38**, 9375 (1988).
 - [14] S. Washburn *et al.*, *Phys. Rev. Lett.* **61**, 2801 (1988).
 - [15] S. Komiyama *et al.*, *Surf. Sci.* **229**, 224 (1990).
 - [16] S.I. Dorozhkin *et al.*, *JETP Lett.* **52**, 652 (1990).
 - [17] A.M. Chang and J.E. Cunningham, *Phys. Rev. Lett.* **69**, 2114 (1992).
 - [18] R. Merz *et al.*, *Phys. Rev. Lett.* **70**, 651 (1993).
 - [19] R.J. Haug, *Semicond. Sci. Technol.* **8**, 131 (1993).
 - [20] R.W. Rendell and S.M. Girvin, *Phys. Rev. B* **23**, 6610 (1981).
 - [21] F. Fischer *et al.*, *Physica (Amsterdam)* **22E**, 108 (2004).
 - [22] H.P. Wei *et al.*, *Phys. Rev. Lett.* **61**, 1294 (1988).
 - [23] H.W. Jiang *et al.*, *Phys. Rev. Lett.* **71**, 1439 (1993).
 - [24] A.M.M. Pruisken, in Ref. [1], Chap. 5.