Quantum Many Particle Systems in Ring-Shaped Optical Lattices

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(Received 27 January 2005; published 1 August 2005)

In the present work we demonstrate how to realize a 1D closed optical lattice experimentally, including a *tunable* boundary phase twist. The latter may induce "persistent currents," visible by studying the atoms' momentum distribution. We show how important phenomena in 1D physics can be studied by physical realization of systems of trapped atoms in ring-shaped optical lattices. A mixture of bosonic and/or fermionic atoms can be loaded into the lattice, realizing a generic quantum system of many interacting particles.

DOI: 10.1103/PhysRevLett.95.063201 PACS numbers: 39.25.+k, 05.60.−k, 67.57.Hi

Studies of one-dimensional systems constitute an intense research activity both in experimental and theoretical physics. They are particularly interesting mainly because quantum effects are strongest at low dimensionality and peculiar phenomena emerge. Prominent examples are the spin-charge separation in Luttinger liquids [1], one-dimensional persistent currents in mesoscopic rings [2], and transmutation of quantum statistics [3]. Most of the approximate schemes working in higher dimensions break down in 1D. Only for a restricted class of model Hamiltonians can physical properties be obtained analytically through powerful techniques such as the Bethe ansatz [4] or conformal field theory [5]. For more generic 1D systems, numerical analysis is the standard route to extract physical information. Degenerate atoms in optical lattices could constitute a further tool for the investigations [6], thus rediscovering Feynman's ideas [7] suggesting that an ideal system with a "quantum logic" can be used to study open problems in quantum physics. Precise knowledge of the model Hamiltonian, manipulation of its coupling constants, and possibility of working with controllable disorder are some of the great advantages of atomic systems in optical lattices compared with solid state devices to experimentally realize Feynman's ideas. The upsurge of interest of the scientific community has been remarkable, and some perspectives disclosed by trapped-atom "labs" have been already explored: the observation of the superfluid-Mott insulator quantum phase transition [8], the analysis of the Tonks-Girardeau regime in strongly interacting bosons [9], and the physical realization of a 1D chain of Josephson junctions [10] were relevant achievements for condensed matter physics. Atoms are conventionally trapped either with magnetic fields or far off resonance laser beams. For our purposes the magnetic trapping potential has a parabolic symmetry. Laser light interacts with the atomic induced dipoles creating attractive or repulsive potentials depending on the sign of the detuning Δ from resonance [11]. This can be used to create different potentials for different atoms, but with a single tunable laser beam. Notice that no light absorption occurs in creating the potential; therefore, the medium can be considered transparent to the laser.

So far only optical lattices with open boundary conditions (OBC) have been realized. This constitutes an experimental limitation, since a variety of studies for finite 1D lattices with periodic boundary conditions (PBC) exists in the literature. In the same way as Gaussian laser beams are useful to produce open optical lattices, we shall take advantage of the rotational symmetry of Laguerre-Gauss (LG) laser modes to produce closed optical lattices. LG beams, obtained experimentally making use of computer generated holograms [12], have already been used in the field of ultracold atoms [13]. A LG mode with frequency ω, wave vector k, and amplitude E_0 propagating along the z axis can be written in cylindrical coordinates (r, φ, z) as [14] $E(r,\varphi) = E_0 f_{pl}(r) e^{il\varphi} e^{i(\omega t - kz)}$, $f_{pl}(r) = (-1)^p \times \sqrt{\frac{2p!}{\pi(p+|l|)!}} \xi^{|l|} L_p^{|l|}(\xi^2) e^{-\xi^2}$, $\xi = \sqrt{2} r/r_0$, where r_0 is the waist of the beam, and $L_p^{|l|}$ are Laguerre functions $L_m^n(x) := (-)^m d^m/dx^m [L_{n+m}(x)]$, $L_{n+m}(x)$ being Laguerre polynomials. The numbers p and l label the radial and azimuthal quantum coordinates, respectively. The lattice modulation is obtained by interference of a LG beam with a plane wave $E_0 e^{i(\omega t - kz)}$: in the far field, the interferogram is periodic in φ with l wells. For even l a perfect 1D ring with $L = l$ lattice sites is obtained. By reflecting the combined beam (LG beam plus plane wave) back on itself one achieves confinement also along z. Indeed, a series of disk shaped traps are obtained. We point out that tunneling between the disks t_z can be made much weaker than the corresponding tunneling within each ring t_ϕ adjusting r_0/λ (i.e., focusing the LG beam). Such a parameter depends monotonously only on L; for $L \gtrsim 15$, $t_z/t_\phi \lesssim 10^{-2}$ can be achieved with $r_0/\lambda \sim 100$. The resulting lattice potential (see Fig. 1) is described by

$$V_{\rm latt} = 4E_0^2[1 + f_{pl}^2 + 2f_{pl}\cos(l\varphi)]\cos(kz)^2. \quad (1)$$

Note that, different from [13], here we need the laser frequency to be tuned below the atomic resonance since we want to trap atoms into the ring. For example, with a laser intensity of $I = 5\,\mathrm{W/cm^2}$ and $\Delta = -10^6$ MHz the potential wells would be separated by a barrier of $\sim 5\ \mu$K much larger than the chemical potential of a standard condensate (whose temperature can reach few nK); with these parameters the scattering rate is $\ll 1$ photon/sec. It is worth noting that, because of the relation: $L_m^{|n-m|}(r) \leftrightarrow H_n[(x-y)/\sqrt{2}]H_m[(x+y)/\sqrt{2}]$, LG modes can be realized also from Hermite-Gauss modes (modulo a $\pi/2$ phase change). Such a "mode converter," realized experimentally in [15], can switch from an open to a closed lattice potential with the same periodicity and L. As we shall discuss further, this device might be useful in the experiments.

We have just illustrated how to realize an optical lattice with PBC. Now we show how to twist them. We apply an external, cone-shaped magnetic field $\mathbf{B} = B_\varphi \mathbf{e}_\varphi + B_z \mathbf{e}_z$. In this way the atomic magnetic dipoles μ_{m_F} experience a field varying along the ring, eventually equipping the periodic lattice by a twist factor: $\Psi \to e^{i\phi_{m_F}}\Psi$ at each winding, Ψ being a generic wave function. The phase factor $\phi_{m_F} = m_F \pi \cos\theta$, with $\tan\theta = B_\varphi/B_z$, is the analog of the Berry phase [16] of the two state system corresponding to the Zeeman splitting of the hyperfine atomic ground states; the role of time is played by the angle φ. We can adjust ϕ_{m_F} using an additional laser beam (with a suitable frequency), relying on the ac Stark shift: $A_E(m_F)$, where the function A_E depends on the intensity of the laser and on the Clebsch-Gordan coefficient of the electric dipole matrix element [17]. The resulting phase twist is $\Phi_\sigma \doteq A_E(m_F) + m_F \pi \cos\theta$, where $\sigma = m_F$. Whereas boundary twists induced by a magnetic field piercing the ring are "symmetric," $\Phi_+ \equiv \Phi_-$, our protocol realizes 1D models with a tunable Φ_σ, thus opening the way to novel investigations discussed below.

For OBC, Φ_σ can be "gauged away" completely from the system. In contrast, the boundary phase cannot be eliminated for closed loops and alters the phase diagram of the system [18]. In fact, Φ_σ emerges from the sum of site-dependent phases causing an increase of the velocity field ($\propto$ to the tight binding amplitude t) that, in absence of dissipation, may set a persistent current. Therefore, different regions in the phase diagram are identified depending on the dynamical response of the system by perturbing Φ_σ. The effect is reflected in the curvature of the $\mathcal{N}$-particle energy levels E_n with respect to the phase twist: $\rho_\sigma = L^2 \Sigma_n p_n [E_n(\Phi_\sigma) - E_n(0)]/(\mathcal{N} t \Phi_\sigma^2)$, where $p_n = e^{-\beta E_n}/Z$ are the Boltzmann weights. For (spinless) bosons $\rho_+ = \rho_- = \rho$ is proportional to the superfluid fraction. Persistent currents are studied analyzing the charge stiffness $D_c \propto \rho_+ + \rho_-$ (for electrons, it is the zero frequency conductivity or Drude weight); a nonvanishing D_c sets a persistent current, visible by releasing the condensate for a time much longer than the typical atomic oscillation period in the lattice wells. Then the spatial distribution of the condensates $|\Psi(\mathbf{r} = \mathbf{k}t, t)|$ is indicative of the initial atomic momentum distribution $|\Psi(\mathbf{k}, 0)|$ [19]; in particular, the phase difference between atoms trapped in different sites produces characteristic interference patterns in the released condensates. In Fig. 2 we show such a pattern for condensates released from the potential of Fig. 1 in mean-field approximation (see also Fig. 3). Supercurrent/superfluid fractions can be studied looking at the response of the system under imprinting of a dynamical phase $\alpha_d(j, \sigma)\delta\tau$ to the atomic wave functions, flashing the atoms with an additional Gaussian laser beam (which can be much closer to resonance than those creating the potential) with a waist larger than the LG mode, and with φ-dependent intensity. The time $\delta\tau$ must be too short to induce atomic motion by absorption during the pulse.

The case $\Phi_+ = -\Phi_-$ is useful to study the spin stiffness $D_s \propto \rho_+ - \rho_-$ indicating long range spin correlations in the system (for charged particles D_s would be proportional to the inverse bulk spin susceptibility [18]). Generic values of $\Phi_+ \neq \pm\Phi_-$ can be seen also as a result of certain correlated-hopping processes (on the untwisted models) [20] and correspond to more exotic cases that, as far as

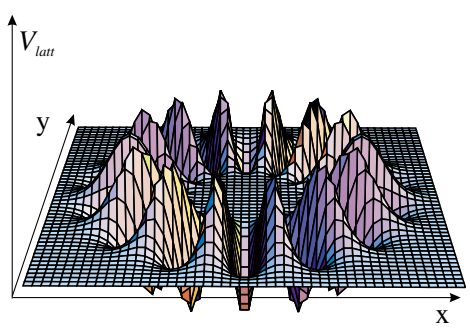

FIG. 1 (color online). The optical potential resulting from the interference of a plane wave with an LG mode with $L = 14$, $p = 0$. For $p \neq 0$ the potential is virtually unaltered.

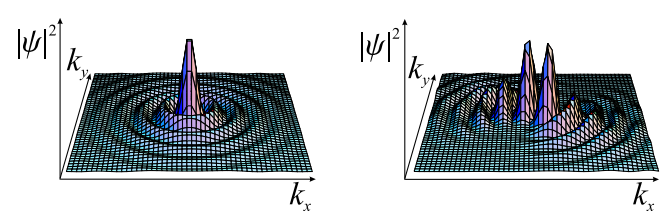

FIG. 2 (color online). Interference pattern for condensates released by the lattice (1), obtained resorting to the analog of light diffraction from a circular grating [19]. The figures show the square of the order parameter $|\psi(k_x, k_y)|^2 = |\psi_0(k_x, k_y)\Sigma_{j=0}^{L-1}\cos\{i[k_x\cos(2\pi j/L) + k_y\sin(2\pi j/L) + \phi_j]\}|^2$. On the left $\phi_j = 0$ for all the condensates; on the right the atoms move along the ring with velocity $\propto \nabla\phi$; the interference pattern reflects a loss of matter at the trap center caused by centrifugal effects.

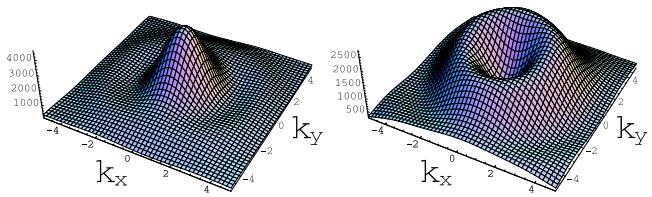

FIG. 3 (color online). The zero temp. momentum distribution for fermions with Hubbard dynamics is presented: $|\Psi(k_x, k_y)|^2 \propto |w(k_x, k_y)|^2 \Sigma_{i,j} e^{i\mathbf{k}\cdot(\mathbf{x}_i - \mathbf{x}_j)} \Sigma_{k_\phi} e^{ik_\phi(\phi_i - \phi_j)} \langle n_{k_\phi} \rangle$; $\mathcal{N}/L = 32/16$ ($\mathcal{N}/L = 1$ is the less advantageous case to discern the effects of Φ at finite size, since the metal-insulator transition strongly suppresses D_c [34]); $\langle n_{k_\phi} \rangle$ is calculated perturbatively at second order in U/t. For $\Phi = 0$ (left), $k_\phi = \{-\pi(\mathcal{N} - 1)/L \ldots \pi(\mathcal{N} - 1)/L\}$. For $\Phi \neq 0$, $k_\phi = \{-\pi(\mathcal{N} - 1)/L + \Phi/L \ldots \pi(\mathcal{N} - 1)/L + \Phi/L\}$; the asymmetry in $|\Psi(k_x, k_y)|^2$ is due to the offset of $\langle n_{k_\phi} \rangle$ caused by Φ.

we know, have not been realized yet in physical systems. To be specific, we consider $\mathcal{N}$ fermions described by the Hubbard model with particle-density modulated kinetic energy

$$H_{\mathrm{Hub}} = -\sum_{j,\sigma} \mu_{j,\sigma} N_{j,\sigma} - \sum_{j,\sigma} [\tilde{t}_j(\sigma) c^\dagger_{j+1,\sigma} c_{j,\sigma} + \mathrm{H.c.}] + U \sum_j N_{j,+} N_{j,-} \tag{2}$$

$$\tilde{t}_j(\sigma) = t \exp\left\{ i\gamma_j(\sigma) + i \sum_l [\alpha_{j,l}(\sigma) N_{l,-\sigma} + A_{j,l}(\sigma) N_{l,\sigma}] \right\}, \tag{3}$$

where $c_{j,\sigma}$'s are fermionic operators, and $N_{l,\sigma} := c^\dagger_{l,\sigma} c_{l,\sigma}$. $U = \pi b_s \int dx |w(x)|^4/m$ and $t = \int dx w(x)[-\frac{1}{2m}\nabla^2 + V_{\mathrm{latt}}]w(x+a)$ [b_s, a, and $w(x)$ indicates the scattering length, the lattice spacing, and Wannier functions, respectively] play the role of the Coulomb and hopping amplitudes, respectively; $\mu_{j,\sigma}$ is of the order of the Bloch band separation [21]; the site dependence can be achieved by tuning the magnetic confinement out of the symmetry axis of the optical ring. For the model (2) in a closed lattice, (3) can be gauged away everywhere but at the boundary; therefore, (2) and (3) is equivalent to the ordinary Hubbard model, but with twisted boundary conditions (BC) [20]. The phase twist is $\Phi_\sigma := \phi(\sigma) + \phi^{(1)}_{+-}(\sigma)\mathcal{N}_{-\sigma} + \phi^{(1)}_{++}(\sigma)(\mathcal{N}_\sigma - 1)$, where $\phi^{(1)}_{+-}(\sigma) = \Sigma^L_{j=1} \alpha_{j,m}(\sigma)$, $\phi(\sigma) = \Sigma^L_{j=1}[\gamma_j(\sigma) + A_{j,j}(\sigma)]$, $\phi^{(1)}_{++}(\sigma) = \Sigma^L_{\substack{j=1 \\ j\neq m-1,m}} A_{j,m}(\sigma) + A_{m,m-1}(\sigma) + A_{m-1,m+1}(\sigma)$. Hence, loading the Hubbard model into the twisted ring effectively leads to the physical realization of the model (2) and (3). To point out the effects of U (smearing of the Fermi distribution with algebraic singularity at k_F) in the persistent current, $|\Psi(\mathbf{k})|^2$ is calculated for the Hubbard ground state at small U/t, and with $\Phi_\sigma = \phi(\sigma)$, $\phi(+) = \phi(-) = \phi$ (see Fig. 3).

The proposed setups could be used to study several issues in one-dimensional systems.

(I) The concept of conformal invariance plays a central role in $1+1$-dimensional critical phenomena: universality is characterized by a single parameter, the conformal anomaly c. The physical meaning of c resides in the concept of Casimir energy, namely, the variation of the vacuum energy density to a change in the BC. For PBC it was shown [5] that the finite size correction to the bulk ground state energy is related to c: $\mathcal{E}_{\mathrm{PBC}} - \mathcal{E}_{\mathrm{bulk}} = -\pi c v/6L$; resorting the modular invariance this correction should be visible in the specific heat of the system, at low temperature: $C(T) = \pi c L T/3v$, for each collective mode of the system; the speed of sound v can be extracted from the dispersion curve, at small k: $v = \Delta\mathcal{E}/\Delta k$, for sufficiently large L (for the XXZ model, numerical analysis suggests that $L \gtrsim 15$ [22]). Except for integrable models, it is hard to measure or even have numerical estimates of c in solid state systems [5,23]. With the presented setups for highly controllable loaded models these measurements can be done with unique accuracy. Both $C(T)$ and $\Delta\mathcal{E}/\Delta k$ can be measured following the techniques employed by Cornell *et al.* [24]. To discern finite size effects in $C(T)$ the PBC to OBC converter discussed above could be a valid tool. Indeed, the finite size correction to $\mathcal{E}_{\mathrm{bulk}}$ for OBC is also proportional to c, but with a *different* coefficient [5]. Then: $cv = \frac{8L}{\pi}(\mathcal{E}_{\mathrm{PBC}} - \mathcal{E}_{\mathrm{OBC}}) + \mathcal{F}_S$, where $\mathcal{F}_S$ is the bulk limit of the surface energy that, being nonuniversal, can be fixed by performing the measurements for different L (mimicking a "finite size scaling analysis"). Remarkably, both the energies $\mathcal{E}_{\mathrm{PBC}}$ and $\mathcal{E}_{\mathrm{OBC}}$ might be accessible measuring the second moment of the velocity of the released condensate [25].

(II) A general model we can engineer in the ring-shaped lattice is

$$H = H_{\mathrm{BH}} + H_{\mathrm{Hub}} + H_I, \tag{4}$$

where H_{BH} is the Bose-Hubbard Hamiltonian [8] and H_I describes a density-density, fermion-boson interaction [26]. By tuning Δ within the fine structure of the fermionic atoms, a spin dependence can be inserted in the hopping amplitude of the Hubbard model: $t \to t_\sigma$. At $\mathcal{N}/L = 1$ and $t_\sigma \ll U$ the Hubbard ring effectively accounts for the physical realization of the twisted XXZ model with anisotropy $\gamma = (t_+^2 + t_-^2)/(2t_+t_-)$ and external field $h = 4\Sigma_\sigma \sigma t_\sigma^2/\mu_\sigma$ [21]. Loading quantum systems described by Hamiltonians of the type (4) in lattices with twisted BC could serve to study charge and spin stiffness in physical systems with tunable interaction and/or disorder. For example, a mixture of ^{87}Rb and ^{40}K atoms constitutes an ideal system to check the recent experimental evidence suggesting that the supersolid order [27] would be effectively favored by the insertion of fermionic degrees of freedom into homogenous bosonic systems. The off-

diagonal long range order manifests in superfluid currents. Jumps between nonvanishing supercurrents should reveal the existence of the supersolid phase [28]. This should be accompanied by a macroscopic occupation in the condensate at a nonvanishing wave vector $[\sim \pi/(na),\ n \geq 2]$ signaling the charge-density-wave instability [29]. The two condensates should be traced in the interference fringes.

It was proved that exactly solvable twisted Hubbard/XXZ rings [18,30] are equivalent to untwisted models for particles with intermediate statistics; this results in modifications of the exponents of the (low energy) correlation functions [20]. The spatial profile of the latter might be detected by photoassociation techniques, as suggested in [31]. (III) Another interesting issue we can study is the conjecture [32] that Poisson *or* Wigner-Dyson level-statistics manifest in that the thermal Drude weights have qualitatively different slopes for integrable (smooth algebraic temperature-decrease, universal behavior of $D(T)/D(0)$) *or* nonintegrable (sharp, nonuniversal suppression of $D(T)/D(0)$) systems. Because of the precise knowledge of the model-Hamiltonian under analysis, we can address the problem directly in a physical system. For example, we could consider ^{40}K pure-XXZ rings with twisted BC for different L's; using the Feshbach resonance one could tune $b_s \sim 2a$; the resulting XXZ model with next-nearest neighbor density-density interaction is nonintegrable (another way is to destroy the integrability introducing disorder into the ring by site-dependent h_j). In short: integrability can be switched on and off by tuning the Feshbach resonance (or adjusting the energy offsets h_j). The presence of persistent currents can be detected along the lines described above (see Figs. 2 and 3). Numerical investigations for the XXZ model suggest that the effect should be visible for $T/L \gtrsim 0.1\gamma$ [33].

In summary, we have suggested a number of protocols to realize closed rings of many quantum particles, by optical means. This is possible by employing slight variations and combinations of techniques already developed within the current experimental activity in atomic physics. We have discussed how several open problems in condensed matter physics can be enlightened by such a setup. We finally observe that the clockwise/anticlockwise currents in a few-wells-ring constitutes a controllable two state-system analogous to the flux qubit realized by a SQUID. As the current is neutral, the corresponding decoherence rate is much lower compared to solid state devices (charged currents). Information transfer could be mediated by an induced-dipole-dipole atomic interaction.

We thank A. Cappelli, M. Inguscio, G. Falci, R. Fazio, and H. Frahm for support and discussions.

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