Comment on "Ambiguities in the Up-Quark Mass"

In a recent Letter, Creutz [1] argued that instanton effects in quantum chromodynamics lead to an additive ambiguity in the definition of the light-quark masses and that this "calls into question the acceptability of attempts to solve the strong CP problem via a vanishing mass for the [up] quark." Here we show that, contrary to this claim, the instanton effects discussed in [1] actually enhance (rather than interfere with) the viability of the $m_u = 0$ solution of the strong CP problem.

To better understand the role of these instanton effects, we must treat the CP violating phase θ explicitly; this was not done in [1]. We begin by setting the coefficient of the topological term $F\tilde{F}$ in the Lagrangian to zero; we then identify θ as the phase of the determinant of the light-quark mass matrix m. Instanton effects on the renormalization of m can be accounted for explicitly via an extra term [2–4] in the renormalization-group equation for m,

$$a\frac{d}{da}m = \gamma(g)m + c(g)a^{n_f-2}(\det m^\dagger)(m^\dagger)^{-1}, \qquad (1)$$

where a is the short-distance cutoff, n_f is the number of light flavors, and $c(g) = c_0(8\pi^2/g^2)^6 e^{-8\pi^2/g^2}[1 + O(g^2)]$; $c_0 = 0.048$ for $n_f = 3$. Although the coefficient $c(g)$ in Eq. (1) is found via an instanton calculation, the m dependence of this term is fixed by the transformation properties of m under the chiral flavor group $\mathrm{SU}(n_f) \times \mathrm{SU}(n_f)$.

If we left multiply Eq. (1) by m^{-1}, take the trace, and use $d\det m = (\det m)\,\mathrm{Tr} m^{-1} dm$, we find

$$a\frac{d}{da}\det m = n_f\gamma(g)\det m + c(g)a^{n_f-2}(\det m^\dagger m)\,\mathrm{Tr}(m^\dagger m)^{-1}. \qquad (2)$$

From Eq. (2), we see that the nonperturbative contribution to the renormalization of $\det m$ is always real. Thus, if $\det m$ vanishes at any particular scale a, it is real at all scales. If we can explain why $\det m = 0$ at any one scale, then we will have solved the strong CP problem.

Models with $\det m = 0$ at a high scale have been proposed, and involve spontaneous breaking of a "horizontal" or "family" symmetry; a general analysis of this class of models was given in [4]. Nonperturbative contributions to $\mathrm{Re}\det m$ actually *improve* the status of these models [2–4], because a nonzero up-quark mass at $1/a \sim \Lambda_{\rm QCD} \sim 1$ GeV is generated by instanton effects at shorter distances. This is different from the superficially similar Kaplan-Manohar mechanism [5], which takes $m_u = 0$ at $1/a \sim \Lambda_{\rm QCD}$, and relies on higher-order effects in chiral perturbation theory to simulate $m_u \neq 0$; this is problematic for several reasons [6].

A different class of models posits that CP violation is spontaneous (e.g., [7]); in these models, $\mathrm{Im}\det m$ is automatically zero at a high scale, but $\mathrm{Re}\det m$ is not. Equation (2) tells us that instanton effects do not contribute to the renormalization of $\mathrm{Im}\det m$. Thus, nonperturbative generation of a nonzero up-quark mass also enhances the viability of this class of solutions to the strong CP problem.

We conclude that the effects discussed in [1] are beneficial, rather than detrimental, to all versions of the $m_u = 0$ solution of the strong CP problem.

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Mark Srednicki*
Department of Physics
University of California
Santa Barbara, California 93106, USA

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*Electronic address: mark@physics.ucsb.edu

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