Curvilinear Motion of Multivortex Laser-Soliton Complexes with Strong and Weak Coupling

Nikolay N. Rosanov,* Sergey V. Fedorov,† and Anatoly N. Shatsev‡

Research Institute for Laser Physics, St. Petersburg 199034, Russia

(Received 10 December 2004; published 27 July 2005)

We reveal the existence of stable dissipative soliton complexes with curvilinear motion of their center of mass. This type of motion results from the field distribution asymmetry and is well pronounced for asymmetric complexes of laser solitons with strong coupling. We present results of numerical simulations of such complexes in a model of wide-aperture lasers or laser amplifiers with saturable gain and absorption. The complex consists of a pair of strongly coupled vortex solitons weakly coupled with a number of other vortex solitons. Similar complexes are expected to exist in different spatially distributed nonlinear dissipative systems, including schemes with discrete dissipative solitons.

DOI: 10.1103/PhysRevLett.95.053903 PACS numbers: 42.65.Tg, 05.45.Yv, 42.55.Ah

It is common knowledge that the space homogeneity results in the law of straight-line motion of center of mass of mechanical systems, in accordance with the first Newton's law and special relativity. This fundamental law is also valid for a wide class of different fields including their multidimensional localized particlelike structures—solitons [1,2]. However, it could be violated even in homogeneous systems under conditions of dissipation, in the presence of energy sinks and sources. In the latter case, the energy balance provides extreme robustness of dissipative solitons, or autosolitons, known in very different fields of science and nature, such as biology, chemistry, plasma physics, semiconductors, nonlinear optics, and lasers [3,4]. This brings up the fundamental question: Is the curvilinear motion of dissipative solitons possible? Because of the recent progress in generation of dissipative optical solitons [5–7] we restrict ourselves here to the consideration of optical structures only.

Because of symmetry reasons, the trajectory of a moving dissipative soliton with symmetric field distribution cannot be curvilinear, and the trajectory curvature is connected with asymmetry of this distribution [8]. In complexes with a small overlapping of solitons, their interaction is weak and can be treated by the perturbation approach [9]; an important point is that the solitons maintain their individuality in the coupled state. However, asymmetric rotating laser structures have also been found [10] and studied [11,12] that cannot be interpreted as weakly coupled states of individual solitons. Contrary to the case of conservative multihumped structures studied recently [2,13], these dissipative structures are stable and their rotation takes place even in the absence of vortices [10,11]. However, because of their distinct asymmetry, strongly coupled vortex solitons are particularly promising for realization of curvilinear motion. To determine the type—strong or weak—of interaction of dissipative solitons, we use here the quantitative criterion for determination of the type based on the analysis of the topology of energy flows [14].

In this Letter, we demonstrate, for the first time, the curvilinear motion of center of mass of laser-soliton complexes, on the scale comparable with the complexes' sizes. With this goal, we introduce a new class of asymmetric multivortex structures where a "core" formed by a pair of strongly coupled solitons is weakly coupled with a number of other solitons—"satellites." Because of extreme robustness of dissipative solitons, the whole nonstationary structure is localized and stable, despite the difference in angular velocities of the core and satellites; see also [15].

We consider simultaneously two $(2+1)$-dimensional schemes where laser solitons have been predicted; see [4] and references therein. The first is a wide-aperture laser, with a large Fresnel number, containing an intracavitary saturable absorber. We assume that the scheme also includes an anisotropic element, which introduces sufficiently high polarization losses for one of the two field polarization components. The relaxation times of both gain and absorption are taken to be small as compared with the field lifetime in the empty cavity t_c—the case of the class A laser. Under these assumptions and in the mean-field approximation, the field evolution is governed by the complex Ginzburg-Landau equation for scalar electric field envelope E [4,11]

$$\frac{\partial E}{\partial t} = (i + d)\nabla_\perp^2 E + f(|E|^2)E, \qquad (1)$$

$$f(|E|^2) = -1 + \frac{g_0}{1 + |E|^2} - \frac{a_0}{1 + b|E|^2}, \qquad (2)$$

where t is the dimensionless time normalized by t_c, $\nabla_\perp^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2$ is the transverse Laplacian with the dimensionless Cartesian transverse coordinates $\boldsymbol{r}_\perp = (x, y)$ normalized by the width of the effective Fresnel zone, d is the effective diffusion coefficient assumed to be small $0 < d \ll 1$, and the nonlinear function $f(I)$ of the field intensity, $I = |E|^2$, describes the nonlinear resonant and linear nonresonant gain and absorption. Neglecting frequency detunings, the function $f(I)$ turns to be real, g_0 and a_0 are real small-signal gain and absorption, respectively, and the parameter b is the ratio of the saturation intensities for gain and loses. Nonresonant losses are scaled to unity by

 © 2005 The American Physical Society

time normalization. The second scheme described by the same governing Eq. (1) is an optical amplifier, also with saturable gain and absorption; in this scheme the evolution variable t has the sense of longitudinal coordinate z. We use the standard definitions of averaged Poynting vector $\boldsymbol{S}_\perp = \mathrm{Im}(E^*\nabla_\perp E)$ and of the transverse coordinates of the structure's center of mass $\boldsymbol{R}_\perp = \int \boldsymbol{r}_\perp |E|^2 d\boldsymbol{r}_\perp / \int |E|^2 d\boldsymbol{r}_\perp$.

In polar coordinates (r, φ) the electric field envelope for a single soliton has the form $E(r, \varphi, t) = A(r)\exp(im\varphi - i\nu t)$. The spectral parameter ν plays the role of eigenvalue with discrete spectrum and presents the nonlinear shift of frequency—for laser scheme and of propagation constant—for amplifier. Here we consider complexes of vortices with topological charges $m = \pm 1$. The parameters used in our numerical simulations are $a_0 = 2$, $b = 10$, $0.04 < d < 0.15$, $g_0 = 2.108$ and 2.11. Below we present the results of such simulations based on the splitting method and the algorithm of the fast Fourier transform (see, e.g., [4,10–12,15]).

Stable localized complexes of coupled laser vortex solitons are found by the numerical solution of Eq. (1). In Fig. 1(a) we present the instantaneous intensity distribution for a strongly coupled pair of vortices with opposite topological charges, $m_1 = 1$, $m_2 = -1$. Stable strongly coupled pairs with these charges never have symmetric forms, but the asymmetric structure is extremely robust, rotating with the period $T = 410$. Its center of mass moves along a circle with the same period (rotation of the direction of pair orientation and revolution of the center of mass are synchronized). As one can see from Fig. 1(c), the Poynting vector averaged over the transverse section, $\langle \boldsymbol{S}_\perp \rangle = \int \boldsymbol{S}_\perp d\boldsymbol{r}_\perp$, is tangent to the trajectory of the center of mass. The field of the strongly coupled pair of vortices with the same topological charges, $m_1 = m_2 = -1$, has central symmetry. Thus its center of mass is motionless, and the pair rotates around the center with the period $T \approx 840$. However, when we add to this structure (a core) an additional soliton with the same charge (a satellite), then, after a certain transient process, the complex becomes evidently asymmetric with very different periods of rotation of the core ($T_{\mathrm{core}} \approx 900$) and satellite ($T_{\mathrm{sat}} \approx 4400$); see Fig. 1(b). This complex is robust and survives after small perturbations introduced in simulations. Because of the central symmetry of the core the whole structure is reproduced after rotation at the angle $\alpha = \pi/(T_{\mathrm{sat}}/T_{\mathrm{core}} - 1)$; see Fig. 1(d), during the time interval $T_1 = \frac{1}{2}(T_{\mathrm{core}}^{-1} - T_{\mathrm{sat}}^{-1})^{-1}$. In the case of core without central symmetry this

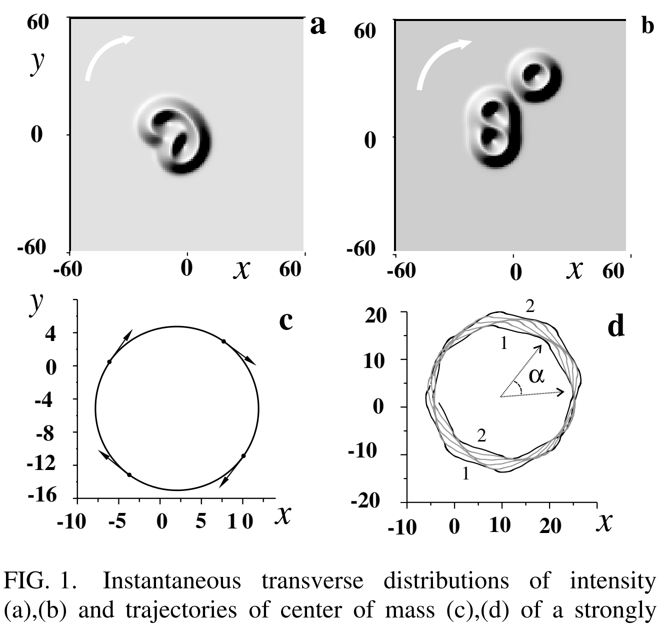

FIG. 1. Instantaneous transverse distributions of intensity (a),(b) and trajectories of center of mass (c),(d) of a strongly coupled rotating pair of two vortices with opposite charges [(a),(c), $d = 0.06$)] and the core-one satellite structure [(b),(d), $d = 0.12$; 1: the first revolution of the satellite, $t = (73\,710, 78\,110)$; 2: the sixth revolution, $t = (95\,310, 99\,710)$]. Arrows in (a),(b) indicate direction of structure rotation, and arrows in (c) indicate directions of averaged Poynting vector, $\langle \boldsymbol{S}_\perp \rangle$, in different moments; $g_0 = 2.108$.

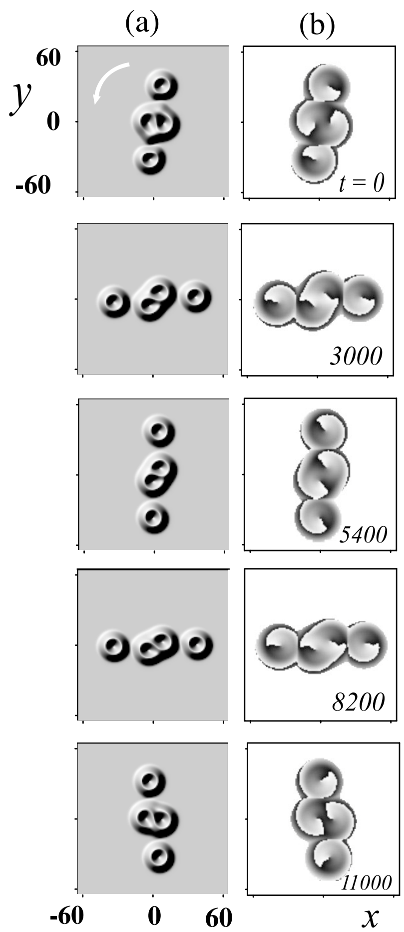

FIG. 2. Transverse distributions of intensity (a) and phase (b) for a core-two satellite structure. The rotation period is 1100 for the core and 11 000 for the satellites. The arrow indicates rotation direction. Phase images are scaled from $-\pi$ (black) to π (white); $d = 0.12$, $g_0 = 2.108$.

interval would be equal to $2T_1$; cf. the coincidence of the hour (satellite) and minute (core) hands in a standard watch and in a watch with the minute hand continued symmetrically around the center. There is also translational motion of the center of mass with extremely small velocity [Fig. 1(d)].

Because of nonlinear interaction between the two rotations and to the well-known phenomenon of synchronization of rhythms, $T_{\rm core} = T_0/n_{\rm core}$, $T_{\rm sat} = T_0/n_{\rm sat}$, where T_0 is the minimum period of the whole structure rotation at the angle multiple to 2π, and the value $T_{\rm core}/T_{\rm sat}$ should be a rational number within a certain synchronization band: $T_{\rm core}/T_{\rm sat} = n_{\rm sat}/n_{\rm core}$, with integer $n_{\rm core}$ and $n_{\rm sat}$. Then the steady-state trajectory of the center of mass closes, with a neglectably slow translational motion, and consists of an integer number of segments with identical shape, but with orientation differing by the angle α. In the case of Fig. 1(d), $n_{\rm core} = 49$ and $n_{\rm sat} = 10$; we present there only half of the whole period T_0. Note that the period $T_0 =$ 44 000 characterizes also the duration of the transient process of the synchronization, whereas the duration of complex formation is about $T_{\rm core} \ll T_0$. For improper initial conditions the satellite, e.g., moves off instead of coupling to the core. Nevertheless, the complexes are robust and exist in a fairly wide range of the system parameters ($0.036 < d < 0.15$, $2.097 < g_0 < 2.117$). After the transient period, the numbers $n_{\rm sat}$ and $n_{\rm core}$ for formed complexes do not depend on the initial conditions in our simulations. However, these numbers depend heavily on the system parameters. Varying the diffusion coefficient d, we have found, e.g., complexes with $n_{\rm sat} = 1$ and $n_{\rm core}$ from 4 to 9.

In the next complex found, there are two satellites revolving around the core with the period $T_{\rm sat} \approx 11\,000$, their coupling with the core being weak, Fig. 2. The period of the core rotation is now $T_{\rm core} \approx 1100$; the topological charges of all 4 vortices are the same ($m = 1$), and the rotation is counterclockwise. To demonstrate the character of vortices coupling, we present in Fig. 3 the topological structure of energy flows found by the method described in [14–16].

One can see that the core is surrounded by two common limit cycles, which is the evidence of strong coupling of vortices in the core [14]. In contrast, the satellites keep their three individual limit cycles, and there are two saddles and one node between the core and each of the satellites, which shows the weak type of their coupling. The distance between the satellites is sufficiently large to exclude their direct interaction. However, each satellite "feels" the opposite one through the mediation of the core. It means that a satellite disturbs the core, and the opposite satellite interacts with the disturbed core. Therefore, the relative position of the satellites is not arbitrary. When we shift the position of one of them, then with time the satellites restore their symmetric opposite positions, as shown in Fig. 4. This confirms the stability of such structures. Important is also that, due to this

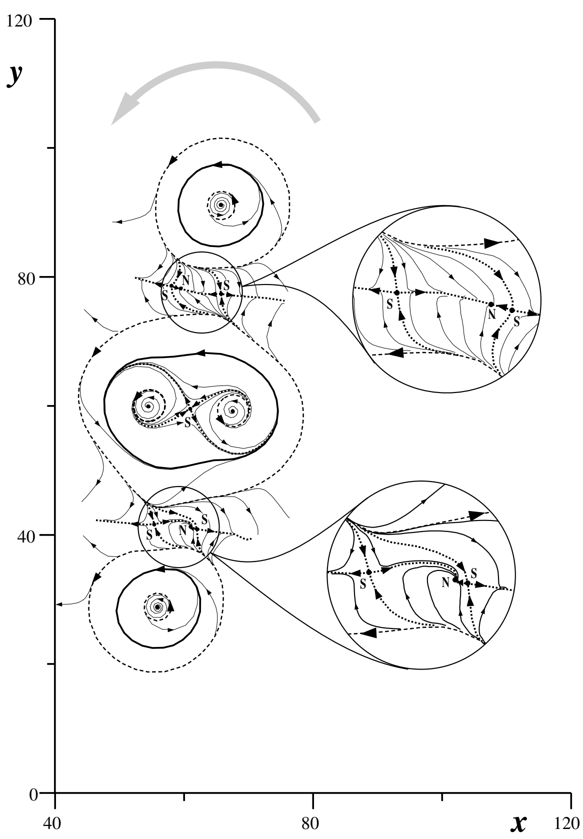

FIG. 3. Energy flows for the stable localized structure given in Fig. 2, at $t = 0$. Closed trajectories correspond to limit cycles. Two insets represent the vicinity of the weak overlapping of the core and satellites, and include nodes N and saddles S with separatrices given by dashed curves. The large arrow indicates rotation direction.

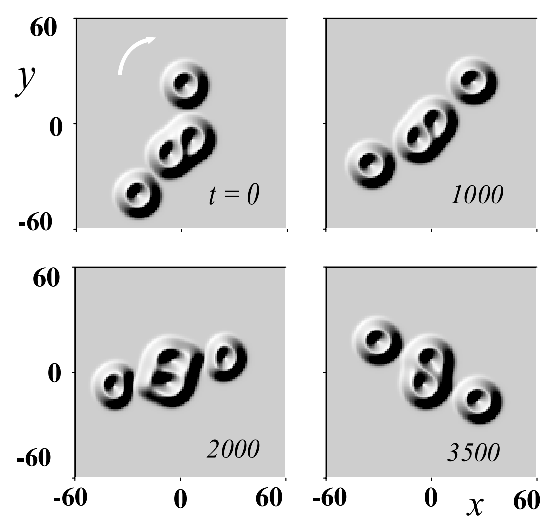

FIG. 4. Dynamics of restoration of opposite positions of two satellites after their initial perturbation. Parameters are $d = 0.15$ and $g_0 = 2.108$.

symmetry, the center of mass of this complex is practically motionless.

Under the finite-difference approximation of Eq. (1) the scheme corresponds to the two-dimensional array of single-mode waveguides, each waveguide possessing nonlinear (saturable) gain and absorption, for both the cavity and the cavityless variants. One-dimensional versions of similar schemes have been studied recently [17]. Much richer is the variety of solitonic structures in two-dimensional arrays. If we restrict ourselves by field distributions with slowly varying amplitudes of modes (from one to another waveguide), then, e.g., our results demonstrate the existence of rotating dissipative solitons in discrete systems, under the above restrictions. Possible also are solitons with sharp phase changes on one discretization step that have no counterparts for the continuous model of Eq. (1).

To conclude, we have presented the first evidence of the curvilinear motion for the center of mass of dissipative optical soliton complexes. This feature has been discovered for a wide class of asymmetric vortex structures with the strong type of coupling, as confirmed by the analysis of energy flows. The origin of this feature is the asymmetry of energy sinks and sources in the complexes in nondegenerate systems (without symmetry to the Galilean transformation) [4,8]. It is possible to show that in the linear limit when function f in Eq. (1) is replaced by constant $f_0 = f(0)$, this equation has solutions in the form of asymmetric Gaussian beams whose centers move along hyperbolas (curvilinearly) in the case $d > 0$; their trajectories are in line when $d = 0$ or the beam envelope has an axis of symmetry. A soliton complex moves in line if the field envelope has an axis of symmetry and the ''longitudinal'' asymmetry (between the complex front and back parts along the direction of motion coinciding with the symmetry axis). Next, important is whether the field distribution has the ''transverse'' symmetry with respect to the direction of motion. Complexes without such symmetry rotate, and their center of mass motion is curvilinear. Rotation of complexes averages the front-back asymmetry and reduces the velocity of in-line motion practically down to zero. Though the center of mass is determined by the intensity distribution only, the curvature of its trajectory is connected with the field ''phase degree of freedom.'' Note also that these complexes are not solid: Their different parts can rotate with different angular velocities. The field nonlinear nature manifests itself in synchronization of and certain ratios between the rotation periods and the form of the trajectories of the center of mass. This type of soliton motion opens up new possibilities for two-dimensional information storage and parallel signal processing applications. We believe that these results will initiate experimental efforts to observe different types of laser solitons in semiconductor wide-aperture VCSELs with a saturable absorber and in analogous amplifiers, arrays of active waveguides, and active photonic crystals.

This work was supported by the Russian Foundation for Basic Research, Grants No. 04-02-16605 and No. 04-02-81014 Bel.

*Electronic address: nrosanov@yahoo.com
†Electronic address: sfedorov@sf3997.spb.edu
‡Electronic address: anshat@yahoo.com

[1] G. I. Stegeman and M. Segev, Science **286**, 1518 (1999).
[2] Yu. S. Kivshar and G. P. Agrawal, *Optical Solitons* (Academic Press, Amsterdam, 2003).
[3] B. S. Kerner and V. V. Osipov, *AutoSolitons* (Kluwer, Dordrecht, 1994).
[4] N. N. Rosanov, *Spatial Hysteresis and Optical Patterns* (Springer, Berlin, 2002).
[5] S. Barland *et al.*, Nature (London) **419**, 699 (2002).
[6] V. B. Taranenko, I. Ganne, R. J. Kuszelewicz, and C. O. Weiss, Phys. Rev. A **61**, 063818 (2000).
[7] E. A. Ultanir, G. I. Stegeman, D. Michaelis, Ch. H. Lange, and F. Lederer, Phys. Rev. Lett. **90**, 253903 (2003).
[8] N. N. Rosanov, Opt. Spectrosc. (USSR) **96**, 405 (2004).
[9] K. A. Gorshkov and L. A. Ostrovsky, Physica (Amsterdam) **3D**, 428 (1981).
[10] N. N. Rosanov, A. V. Fedorov, S. V. Fedorov, and G. V. Khodova, Opt. Spectrosc. (USSR) **79**, 795 (1995).
[11] S. V. Fedorov, N. N. Rosanov, A. N. Shatsev, N. A. Veretenov, and A. G. Vladimirov, IEEE J. Quantum Electron. **39**, 197 (2003).
[12] N. N. Rosanov, S. V. Fedorov, and A. N. Shatsev, Opt. Spectrosc. (USSR) **95**, 843 (2003).
[13] A. S. Desyatnikov and Yu. S. Kivshar, Phys. Rev. Lett. **88**, 053901 (2002).
[14] N. N. Rosanov, S. V. Fedorov, and A. N. Shatsev, Sov. Phys. JETP **98**, 427 (2004).
[15] N. N. Rosanov, S. V. Fedorov, A. N. Shatsev, and N. A. Loiko, Opt. Spectrosc. (USSR) **97**, 88 (2004).
[16] N. N. Rosanov, in *Dissipative Solitons*, edited by N. Akhmediev and A. Ankiewicz, Lecture Notes in Physics Vol. 661 (Springer, Berlin, 2005), p. 101.
[17] E. A. Ultanir, G. I. Stegeman, and D. N. Christodoulides, Opt. Lett. **29**, 845 (2004).