Models of Environment and T_1 Relaxation in Josephson Charge Qubits

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A theoretical interpretation of the recent experiments of Astafiev *et al.* on the T_1-relaxation rate in Josephson charge qubits is proposed. The experimentally observed reproducible nonmonotonic dependence of T_1 on the splitting E_J of the qubit levels suggests further specification of the previously proposed models of the background charge noise. From our point of view the most promising is the "Andreev fluctuator" model of the noise. In this model the fluctuator is a Cooper pair that tunnels from a superconductor and occupies a pair of localized electronic states. Within this model one can naturally explain both the average linear $T_1(E_J)$ dependence and the irregular fluctuations.

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Proposals to implement qubits using superconducting nanocircuits have undergone an amazing development during the last years [1–5]. In a Josephson charge qubit (JCQ) information is encoded in the charge states of a Cooper pair box. The JCQ is manipulated by tuning gate voltage and magnetic flux. Both time resolved coherent oscillations in single and coupled JCQ have been recently observed [2,6]. Although decoherence is a severe limitation to the performances of these devices, the dominant source of noise is yet to be identified.

A significant step towards a characterization of the environment in a JCQ has been recently made by Astafiev *et al.* [7] The experimental setup consists of a Cooper pair box connected to a reservoir through a tunnel junction of SQUID geometry with Josephson energy E_J pierced by an external magnetic field. Provided that $E_c \gg E_J \gg T$ (where E_c, E_J, and T are correspondingly the charging energy, Josephson energy, and temperature, $k_B = \hbar = 1$), only two charge states $|0\rangle$ and $|1\rangle$ are relevant and the Hamiltonian of the box reads

$$H_q = -\frac{\delta E_c}{2}\sigma_z - \frac{E_J}{2}\sigma_x, \tag{1}$$

where $\delta E_c = E_c(1 - C_g V_g/e)$, C_g is the gate capacitance, V_g is the gate voltage, and e denotes the electron charge. In the rotated basis $\{|+\rangle, |-\rangle\}$ the Hamiltonian (1) reads

$$H_q = -\frac{E}{2}\rho_z, \qquad \rho_z = \sigma_z\cos\theta + \sigma_x\sin\theta, \tag{2}$$

where $E = \sqrt{\delta E_c^2 + E_J^2}$ and $\theta = \arctan(E_J/\delta E_c)$. One can distinguish the off degeneracy working points ($\theta \approx 0$ and $\delta E_c \gg E_J$) and the degeneracy one ($\theta = \pi/2$ and $\delta E_c = 0$). Astafiev *et al.* [7] measured the energy relaxation rate Γ_1 of the JCQ in a wide range of parameters. Two main features have been observed: (i) Linear increase of Γ_1 with E_J at large E_J, and (ii) Small nonmonotonous fluctuations in the $\Gamma_1(E_J)$ function on this linear background.

We do not believe that the existing experimental information is sufficient to identify a unique interpretation. However, it substantially reduces the range of possibilities. In this Letter we show that some models which have been used to study dephasing in JCQ cannot explain these features. We propose a model where all of them appear naturally.

Many different mechanisms can be responsible for decoherence in JCQ. We will consider three models (see Fig. 1), all based on the idea that the oxide layer close to some metallic reservoir, like one of the leads or gates or Cooper pair box itself, is disordered and thus hosts trapping centers, i.e., localized states for the electrons.

Model I: The reservoir is a normal metal, and electrons can tunnel from any state below the chemical potential to an unoccupied trap above the chemical potential or from an occupied trap to an extended state above the chemical potential. Model II: There is no tunneling between the reservoir and the traps, but an electron in an occupied trap below the chemical potential can be excited into an empty trap. Model III: The reservoir is a superconductor and a Cooper pair can split in such a way that each of the electrons end up at an empty trap (Andreev fluctuator). At the end of the Letter we discuss why we do not believe that dephasing through phonon or photon modes can describe the experimental results.

To describe the qubit interacting with the environment one has to supplement Eq. (1) by the Hamiltonians of the environment, H_E, the interaction H_I, and the tunneling H_T. Regardless of its particular features an environment is coupled to a JCQ through the charge degree of freedom: $\sigma_z = \rho_z\cos\theta - \rho_x\sin\theta$. Since only the term ρ_x in the

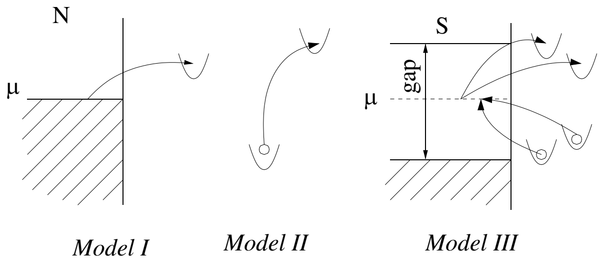

FIG. 1. The three models.

interaction Hamiltonian will change the state of the qubit, the relaxation rate Γ_1 should be proportional to $\sin^2\theta$ if the qubit-environment coupling is weak.

The charge of the qubit affects the environment in two ways. First, the electrostatic interaction shifts the energy of the localized states. Second, the amplitude t of the tunneling between the trap and the reservoir (another trap) depends on the qubits' charge: $t(\sigma_z) = t_0 + \tilde{t}\sigma_z$.

The total Hamiltonian can be written as

$$H = H_q + H_I + H_t + H_E. \tag{3}$$

For the environment Hamiltonian we write

$$H_E^{(\mathrm{I})} = \sum_\alpha \epsilon_\alpha c_\alpha^\dagger c_\alpha + \sum_k \Omega_k f_k^\dagger f_k,$$
$$H_E^{(\mathrm{II,III})} = \sum_\alpha \epsilon_\alpha c_\alpha^\dagger c_\alpha.$$

where c_α, $(c_\alpha^\dagger)$ destroys (creates) an electron in trap α and f_k, $(f_k^\dagger)$ destroys (creates) an electron in the reservoir. (We assume that the superconductor in Model III is always in the ground state.)

H_t describes the tunneling in the absence of the qubit

$$H_t^{(\mathrm{I})} = t_0 \sum_{k,\alpha} (c_\alpha^\dagger f_k + c_\alpha f_k^\dagger), \qquad H_t^{(\mathrm{II})} = t_0 \sum_{\alpha \neq \beta} c_\alpha^\dagger c_\beta,$$
$$H_t^{(\mathrm{III})} = t_0 \sum_{\alpha \neq \beta} (c_\alpha^\dagger c_\beta^\dagger + c_\alpha c_\beta).$$

The coupling of the qubit with the environment is governed by the Hamiltonian

$$H_I = \left(v \sum_\alpha c_\alpha^\dagger c_\alpha + \frac{\tilde{t}}{t_0} H_t \right) \sigma_z. \tag{4}$$

Let us now discuss processes where the qubit, initially prepared in the excited state, releases the energy by exciting the environment (T_1 processes). When the coupling is weak the relaxation rate can be derived by using the Fermi golden rule. If originally the qubit is in the excited state and the bath occupies a state $|i\rangle$ with probability ρ_i the decay rate to the ground state, $\Gamma_\downarrow$, is

$$\Gamma_\downarrow = 2\pi \sum_{i,f} \rho_i^0 |\langle +, f| T e^{-i\int_0^t H_I(t')dt'} |-, i\rangle|^2. \tag{5}$$

Here $|f\rangle$ is the final state of the bath. If the qubit is prepared in the ground state the transition rate to the excited state is at thermal equilibrium $\Gamma_\uparrow = e^{-E/T}\Gamma_\downarrow$. The total relaxation rate is then $\Gamma_1 = \Gamma_\downarrow + \Gamma_\uparrow = (1 + e^{-E/T})\Gamma_\downarrow$. In the limit $E \gg T$ we have $\Gamma_1 \approx \Gamma_\downarrow$.

When using Eq. (5) the exponential has to be expanded to the necessary order. Notice that the second term in Eq. (4) that describes the change in the tunneling amplitude will both flip the qubit (remember that $\sigma_z = \rho_z \cos\theta - \rho_x \sin\theta$) and change the occupation of the trap. Thus, it contributes to Γ_1 already in the first order:

$$\Gamma_1^{(1)} = 2\pi \tilde{t}^2 g(E) \sin^2\theta. \tag{6}$$

Here $g(\omega)$ is the density of states of excitations. The electrostatic interaction term, $\frac{v}{2}\sigma_z c^\dagger c$, does not change the occupation of the localized state, so it contributes only in the second order. Assuming that $(t_0 g_0)^2/E \ll g(E)$, where g_0 is the density of states in the metal, we can write this contribution as

$$\Gamma_1^{(2)} = \frac{2\pi}{E^2} v^2 [t_0 + 2\tilde{t}\cos\theta]^2 g(E) \sin^2\theta. \tag{7}$$

For Model II, the contribution (7) does not appear as long as all traps are coupled equally to the qubit. Then moving one electron from one trap to another will not change the electrostatic potential. Accordingly, for Model II, the v^2 should be interpreted as $\langle v^2 \rangle$ averaged over some scatter of v.

Let us calculate $g(\omega)$ for Models I–III. Consider the density of the localized state

$$\nu(\epsilon) = \bar{\nu} + \delta\nu(\epsilon), \tag{8}$$

where $\bar{\nu}$ is the average value of $\nu(\epsilon)$ (we assume that $\bar{\nu}$ is ϵ independent). The random deviations $\delta\nu(\epsilon)$ are assumed to be small, $\delta\nu(\epsilon) \ll \bar{\nu}$ and only short-range correlated:

$$\langle \delta\nu(\epsilon)\delta\nu(\epsilon') \rangle = A\delta(\epsilon - \epsilon'). \tag{9}$$

For the density of states $\langle g(\omega) \rangle$ averaged over different realizations of the random distribution of trap energies, we have

$$g(\omega)^{(\mathrm{I})} = g_0 \int_{-\omega}^{\omega} d\epsilon \nu(\epsilon), \qquad \langle g(\omega) \rangle^{(\mathrm{I})} = 2\bar{\nu} g_0 \omega,$$
$$g(\omega)^{(\mathrm{II})} = \int_0^\omega d\epsilon \nu(\epsilon)\nu(\epsilon - \omega), \qquad \langle g(\omega) \rangle^{(\mathrm{II})} = \bar{\nu}^2 \omega,$$
$$g(\omega)^{(\mathrm{III})} = \int_0^\omega d\epsilon [\nu(\epsilon)\nu(\omega - \epsilon) + \nu(-\epsilon)\nu(\epsilon - \omega)], \qquad \langle g(\omega) \rangle^{(\mathrm{III})} = 2\bar{\nu}^2 \omega. \tag{10}$$

g_0 denotes the density of states in the metal and we neglect its energy dependence.

In each of the three models $\langle g(\omega) \rangle$ is a linear function of the frequency ω.

This statement may become incorrect when Coulomb interaction between trapped electrons is taken into account. For example in Model II the tunneling amplitude depends exponentially on the distance between the traps. Only traps which are close in space can exchange charge. For such pairs the Coulomb interaction between the traps modifies the density of states to (see Ref. [8]) $\langle g(\omega)\rangle^{(\mathrm{II})} = \bar{\nu}^2(\epsilon_c^{(\mathrm{II})} + \omega)$, where ϵ_c is the energy of the Coulomb interaction between two electrons that occupy the two traps. $\epsilon_c^{(\mathrm{II})}$ can be estimated as $\epsilon_c^{(\mathrm{II})} = e^2/r_t$, where r_t is the typical distance between the traps, i.e., typical tunneling length.

As for Model III, the Coulomb repulsion leads to $\langle g(\omega)\rangle^{(\mathrm{III})} = 2\bar{\nu}^2(\omega - \epsilon_c^{(\mathrm{III})})$. When estimating $\epsilon_c^{(\mathrm{III})}$ we need to take into account the screening provided by the superconductor [9]: each electron trapped near the superconductor creates an image charge and thus forms a dipole moment of the order of er_t. The distance between the two traps is determined by the coherence length ξ of the superconductor. Therefore, $\epsilon_c^{(\mathrm{III})} \sim e^2 r_t^2/\xi^3$.

It is informative to compare the Coulomb energies $\epsilon_c^{(\mathrm{II,III})}$ with the superconducting gap $\Delta \sim \hbar v_F/\xi$, where v_F is the Fermi velocity [10]. Since $e^2/\hbar v_F \approx 1$, we have $\epsilon_c^{(\mathrm{II})}/\Delta \sim \xi/r_t$, and $\epsilon_c^{(\mathrm{III})}/\Delta \sim (r_t/\xi)^2$. It is natural to assume that $r_t \ll \xi$. Therefore, $\Delta \ll \epsilon_c^{(\mathrm{II})}$ and $\Delta \gg \epsilon_c^{(\mathrm{III})}$. When considering JQC one is interested in $\omega \sim E_J \ll \Delta$. We conclude that $\omega \ll \epsilon_c^{(\mathrm{II})}$ and thus $\langle g(\omega)\rangle^{(\mathrm{II})} = \mathrm{const}$. At the same time $\omega \gg \epsilon_c^{(\mathrm{III})}$, $\langle g(\omega)\rangle^{(\mathrm{III})}$ is determined by Eq. (10) and the "Andreev fluctuators" can lead to a linear dependence of Γ_1 on E_J.

Returning now to the Eqs. (6) and (7) we see that the first order contribution is directly proportional to the density of states. The second order term, because of the E in the denominator, does not give a linearly increasing relaxation rate even if the density of states is linear.

We conclude that to get a linear rate from a linear density of states we need some term in the Hamiltonian that gives a contribution to first order.

So far we only considered the average relaxation rate, let us now turn to the fluctuations.

Consider the two-point correlator

$$\begin{aligned}
\langle g(\omega)g(\omega')\rangle_c^{(\mathrm{I})} &= 2Ag_0^2\min(\omega,\omega'),\\
\langle g(\omega)g(\omega')\rangle_c^{(\mathrm{II})} &= 2A\bar{\nu}^2\min(\omega,\omega') + A^2\omega\delta(\omega-\omega'),\\
\langle g(\omega)g(\omega')\rangle_c^{(\mathrm{III})} &= 8A\bar{\nu}^2\min(\omega,\omega') + 4A^2\omega\delta(\omega-\omega').
\end{aligned} \tag{11}$$

Note that from Eqs. (10) and (11) it follows that in Model I $g(\omega)$ is a monotonous function of ω, whereas Models II, III lead to nonmonotonous fluctuations with short-range correlations.

Figure 2 shows for Model III at the optimal point the relaxation rate as function of frequency for a particular realization of the position in energy of the traps. The straight lines represent the result of an ensemble averaging. If the fluctuations are rapid one can instead average, over suitable frequency, windows for a single sample.

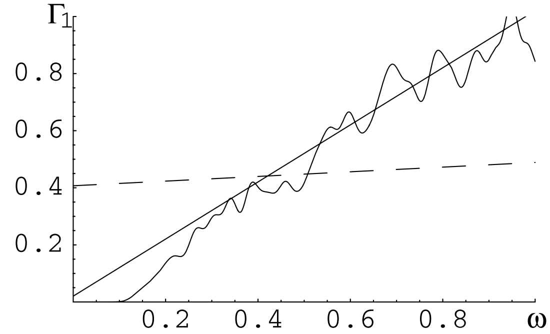

FIG. 2. The relaxation rate as a function of frequency for Model III. The curve shows one typical realization of the disorder while the solid line is the average over realizations. The dashed line shows $\langle\Gamma_1(\omega)^2\rangle$.

The second order term can also give rise to fluctuations in the relaxation rate. Let us focus on Model I at the optimal point ($\cos\theta = 0$). The main source of fluctuations is the v^2 term and for this it follows immediately from Eqs. (7) and (11) that the correlator is

$$\langle\Gamma_1^{(2)}(\omega)\Gamma_1^{(2)}(\omega')\rangle_c^{(\mathrm{I})} = 2Ag_0^2\frac{\min(\omega,\omega')}{\omega^2\omega'^2}. \tag{12}$$

Thus, also in Model I there will be fluctuations, but the peaks will have a different shape, and the correlations are long range. Also, in Models II and III the amplitude of the oscillations increases with increasing E, whereas in Model I it decreases since it only has contributions to second order. This could be a way to distinguish the different models.

For comparison with Fig. 2 we show in Fig. 3, Γ_1 for Model I for the case where $v^2t_0^2/\tilde{t}^2 = 10$ in the units of E.

Above we took the correlation of the levels to be a true δ function. In reality, the levels will be broadened by relaxa-

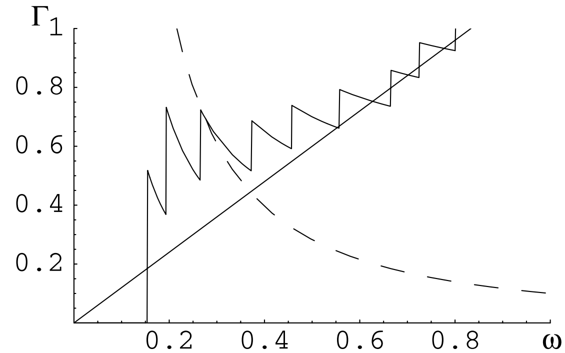

FIG. 3. The relaxation rate as a function of frequency for Model I. The curve shows one typical realization of the disorder while the solid line is the average over realizations. The dashed line shows $\langle\Gamma_1(\omega)^2\rangle$.

tional processes that go beyond our models. To take this into account we can use instead for the correlator some δ-like function $\delta_\sigma(\epsilon)$ with characteristic width σ. For example, one can think about Gaussian, $\delta_\sigma^{(G)} = e^{-\epsilon^2/2\sigma^2}/\sqrt{2\pi}\sigma$ or Lorentzian, $\delta_\sigma^{(L)} = \sigma/\pi(\epsilon^2 + \sigma^2)$ correlations. The δ functions in Eq. (11) are then replaced by the function $\tilde{\delta}_{\tilde{\sigma}}$ which again is a δ-like function; in the Gaussian case $\tilde{\delta}_{\tilde{\sigma}}^{(G)} = \delta_{\sqrt{2}\sigma}^{(G)}$ and for the Lorentzian, $\tilde{\delta}_{\tilde{\sigma}}^{(L)} = \delta_{2\sigma}^{(L)}$.

Note that both phonon and photon radiation could cause a linear frequency dependence of the relaxation rate in two dimensions due to their linear dispersion. However, we do not believe that they can be responsible for the observed resonances. In this case a peak in the Γ_1 as a function of E follows directly from a resonance in the density of states $g(E)$. Let us estimate the frequency of such a resonance assuming that the resonant structure in the density of states arises from quantization of phonon levels in a confined geometry. According to Ref. [7] one such resonance was at a frequency of 30 GHz. The sound velocity is 10^3 m/s and corresponds to a wavelength of 30 nm. While not impossibly small, this appears to be smaller than the typical sizes of >100 nm of the structures in the samples used. On the other hand, the possibility that there could be a coupling to a standing photon mode in the experimental cavity looks more likely. A similar argument, but using the speed of light, gives us a wavelength of 1 cm, which is of the right order of magnitude. Only two samples where measured [7] with slightly different resonant frequencies (20 and 30 GHz), but this could be caused by different position or size of the sample. In view of the fact that the cavity contains the sample and mount as absorbing material and that no special care was taken to create a high Q cavity, it appears unlikely that such a sharp resonance line would be created. This could be tested by introducing some absorbing material into the cavity to see if the resonant peaks will disappear. An alternative way to discriminate between a phonon or photon resonance peak and one created by a resonant fluctuator would be to thermocycle a given sample. If the resonance is caused by some fluctuator, the latter probably would be rearranged by the heating, and thus the peak positions would shift.

In summary, we have argued that dephasing by phonons or photons is unlikely to explain the experimental results although they cannot be ruled out conclusively. A more likely explanation is some resonant fluctuator model. We have discussed three such models; and all of them depend on the effect of the state of the qubit affecting the barrier height to reproduce the linear dependence of the relaxation rate on E.

We think that Model III (Andreev fluctuators) is the most promising for the following reasons. Models II and III allow for rapid, nonmonotonous oscillations of the Γ_1 for large E, whereas Model I (to first order only) will show a steplike monotonous increase of Γ_1. To second order there are nonmonotonous oscillations also for Model I, but they have a different shape. Model II is less likely than Model III because the Coulomb interaction most likely changes the density of states to constant for the relevant range of energies.

To experimentally determine the coupling constants we suggest the following: If one probes the same energy E at different working points (by changing both δE_c and E_J) there should, to first order, be a collapse of the data points if one plots $\Gamma_1/\sin^2\theta$ as a function of E, whereas the terms with $\cos\theta$ in $\Gamma_1^{(2)}$ will cause some deviation. In particular, it seems instructive to plot $[\Gamma(E,\theta)/\Gamma(E,\pi/2) - 1]/\cos\theta = 4(v/E)^2(t_0/\tilde{t} + \cos\theta)$ as a function of $\cos\theta$. From this, one could extract the ratios v/E and $t_0/\tilde{t}$.

We proposed thermocycling as an experimental check for the presence of fluctuators and introduction of some absorbing material in the cavity to rule out photon resonances.

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[1] D. Vion, A. Aassime, A. Cottet, P. Joyez, H. Pothier, C. Urbina, D. Esteve, and M. H. Devoret, Science **296**, 886 (2002).
[2] Y. Nakamura, Yu. A. Pashkin, and J. S. Tsai, Nature (London) **398**, 786 (1999).
[3] Y. Yu, S. Han, X. Chu, S. Chu, and Z. Wang, Science **296**, 889 (2002).
[4] J. M. Martinis, S. Nam, J. Aumentado, and C. Urbina, Phys. Rev. Lett. **89**, 117901, (2002).
[5] I. Chiorescu, Y. Nakamura, C. Harmans, and J. Mooij, Science **299**, 1869 (2003).
[6] Yu. A. Pashkin, T. Yamamoto, O. Astafiev, Y. Nakamura, D. V. Averin, and J. S. Tsai, Nature (London) **421**, 823 (2003).
[7] O. Astafiev, Yu. A. Pashkin, Y. Nakamura, T. Yamamoto, and J. S. Tsai, Phys. Rev. Lett. **93**, 267007 (2004).
[8] A. L. Efros and B. I. Shklovskii, in *Electron-Electron Interactions in Disordered Systems*, edited by A. L. Efros and M. Pollack (Elsevier, New York, 1985).
[9] V. I. Kozub, A. A. Zuzin, Y. M. Galperin, and V. Vinokur, cond-mat/0411379.
[10] We considered here the ''clean'' case, when the electronic mean free path exceeds ξ. Taking disorder into account does not change the final conclusion.