Conserved Charges and Thermodynamics of the Spinning Gödel Black Hole

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We compute the mass, angular momenta, and charge of the Gödel-type rotating black hole solution to five-dimensional minimal supergravity. A generalized Smarr formula is derived, and the first law of thermodynamics is verified. The computation rests on a new approach to conserved charges in gauge theories that allows for their computation at finite radius.

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Black hole solutions in supergravity theories have attracted a lot of interest recently for two main reasons: on the one hand, higher dimensional supersymmetric theories play a prominent role in the effort of unifying gravity with the three microscopic forces; on the other hand, black hole solutions are preferred laboratories to study the effects of quantum gravity.

Among the supersymmetric solutions of five-dimensional minimal supergravity [1], a maximally supersymmetric analogue of the Gödel universe [2] has been found. This solution can be lifted to 10 or 11 dimensions (see also [3]) and has been intensively studied as a background for string and M theory; see, e.g., [4,5].

Black holes in Gödel-type backgrounds have been proposed in [6–10]. Usually, given new black hole solutions, the conserved charges are among the first properties to be studied; see, e.g., [11–13]. Indeed, they are needed in order to check whether these solutions satisfy the same remarkable laws of thermodynamics as their four-dimensional cousins [14,15]. The computation of the mass, angular momenta, and electric charge of the Gödel black holes is an open problem, mentioned explicitly in [7] with partial results obtained in [16], because the naive application of traditional approaches fails. The aim of this Letter is to solve this problem for the five-dimensional spinning Gödel-type black hole [7] and to derive both the generalized Smarr formula and the first law.

Analytic expression for the charges.—In odd spacetime dimensions $n = 2N + 1$, the Einstein-Maxwell Lagrangian with Chern-Simons term and cosmological constant reads

$$L[g, A] = \frac{\sqrt{-g}}{16\pi}[R - 2\Lambda - F_{\mu\nu}F^{\mu\nu}] - \frac{2\lambda}{16\pi(N+1)\sqrt{3}}\epsilon^{\gamma\alpha\beta\cdots\mu\nu}A_\gamma F_{\alpha\beta}\cdots F_{\mu\nu}. \tag{1}$$

The bosonic part of $n = 5$ minimal supergravity corresponds to $\Lambda = 0$, $\lambda = 1$. The fields of the theory are collectively denoted by $\phi^i \equiv (g_{\mu\nu}, A_\mu)$. Consider any fixed background solution $\bar{\phi}^i$. The equivalence classes of conserved $(n-2)$-forms of the linearized theory for the variables $\varphi^i \equiv \phi^i - \bar{\phi}^i = (h_{\mu\nu}, a_\mu)$ can be shown [17] to be in one-to-one correspondence with equivalence classes of field dependent gauge parameters $\xi^\mu([\varphi], x)$, $\epsilon([\varphi], x)$ satisfying

$$\begin{cases} \mathcal{L}_\xi \bar{g}_{\mu\nu} = 0, \\ \mathcal{L}_\xi \bar{A}_\mu + \partial_\mu \epsilon = 0, \end{cases} \tag{2}$$

on-shell, i.e., when evaluated for solutions of the linearized theory. Conserved $(n-2)$-forms are considered equivalent if they differ on shell by the exterior derivative of an $(n-3)$-form, while field dependent gauge parameters are equivalent if they agree on shell. If $n \geq 3$ and under reasonable assumptions on the background $\bar{g}_{\mu\nu}$, the equivalence classes of solutions to the first equation of (2) are classified by the field independent Killing vectors $\bar{\xi}^\mu(x)$ of the background $\bar{g}_{\mu\nu}$ [18]. For the backgrounds that we consider below, the second equation will then also be satisfied by taking $\epsilon = 0$. It is then straightforward to show that the system (2) admits only one more equivalence class of solutions characterized by $\xi^\mu = 0$, $\epsilon = c \in \mathbb{R}$.

One then computes the weakly vanishing Noether currents

$$S^\mu_{\xi,\epsilon} = \frac{\delta L}{\delta g_{\mu\nu}}(2\xi_\nu) + \frac{\delta L}{\delta A_\mu}(A_\rho \xi^\rho) + \frac{\delta L}{\delta A_\mu}\epsilon, \tag{3}$$

associated with gauge transformations. For second order theories, the $(n-2)$-forms $k_{\xi,c}[\varphi;\phi] = k^{[\mu\nu]}_{\xi,c}(d^{n-2}x)_{\mu\nu}$ are defined through the formula

$$k^{[\mu\nu]}_{\xi,c} = \frac{1}{2}\varphi^i \frac{\partial S^\mu_{\xi,c}}{\partial \phi^i_\nu} + \left[\frac{2}{3}\partial_\lambda \varphi^i - \frac{1}{3}\varphi^i \partial_\lambda\right]\frac{\partial^S S^\mu_{\xi,c}}{\partial \phi^i_{\lambda\nu}} - (\mu \leftrightarrow \nu), \tag{4}$$

with $(d^{n-p}x)_{\mu_1\ldots\mu_p} = \frac{1}{p!(n-p)!}\epsilon_{\mu_1\ldots\mu_n}dx^{\mu_{p+1}}\ldots dx^{\mu_n}$ and $\partial^S\phi^j_{\mu\rho}/\partial\phi^i_{\lambda\nu} = \delta^j_i\delta^\lambda_{(\mu}\delta^\nu_{\rho)}$. The forms $k_{\xi,c}[\varphi;\bar{\phi}]$ are closed, $dk_{\xi,c}[\varphi;\bar{\phi}] = 0$, whenever $\bar{\phi}$ satisfies the equations of motion, φ the linearized equations of motion, and (ξ, ϵ) the system (2). For the solutions $(\bar{\xi}, 0)$, one can write $k_{\bar{\xi},0} = k^{\rm grav}_{\bar{\xi}} + k^{\rm em}_{\bar{\xi}} + \lambda k^{\rm CS}_{\bar{\xi}}$. The gravitational contribution, which depends only on the metric and its deviations, coincides with the Abbott-Deser expression [19] and, if one computes in the Hamiltonian framework, with the expression derived in the Regge-Teitelboim approach [20]. Using the Killing equation, it can be written in the Iyer-Wald form [21]:

$$k^{\rm grav}_{\bar\xi}[h;\bar g] = -\delta K^K_{\bar\xi} - \bar\xi\cdot\Theta, \tag{5}$$

where

$$K^K_{\bar\xi} = (d^{n-2}x)_{\mu\nu}\frac{\sqrt{-g}}{16\pi}[D^\mu\bar\xi^\nu - (\mu\leftrightarrow\nu)] \tag{6}$$

is the Komar $(n-2)$-form and

$$\Theta = (d^{n-1}x)_\mu\frac{\sqrt{-\bar g}}{16\pi}(\bar D_\sigma h^{\mu\sigma} - \bar D^\mu h). \tag{7}$$

Here and below, $\delta g_{\mu\nu} = h_{\mu\nu}$, $\delta A_\mu = a_\mu$, and, after the variation, (g, A) are replaced by $(\bar g, \bar A)$. We also assume that the variation leaves $\bar\xi$ unchanged. After dropping a d exact form and using (2), the electromagnetic contribution becomes

$$k^{\rm em}_{\bar\xi}[a;\bar A,\bar g] = -\delta Q^{\rm em}_{\bar\xi,0} - \bar\xi\cdot\Theta^{\rm em}, \tag{8}$$

where

$$Q^{\rm em}_{\bar\xi,c} = (d^{n-2}x)_{\mu\nu}\frac{\sqrt{-g}}{4\pi}(F^{\mu\nu}(\bar\xi^\rho A_\rho + c)), \tag{9}$$

$$\Theta^{\rm em} = (d^{n-1}x)_\mu\frac{\sqrt{-\bar g}}{4\pi}(\bar F^{\alpha\mu}a_\alpha). \tag{10}$$

The Chern-Simons term contributes as

$$k^{\rm CS}_{\bar\xi}[a,\bar A] = -\frac{N(d^{n-2}x)_{\mu\nu}}{4\sqrt{3}\pi} \times \epsilon^{\mu\nu\sigma\alpha\beta\cdots\gamma\delta}a_\sigma\bar F_{\alpha\beta}\cdots\bar F_{\gamma\delta}(\bar A_\rho\xi^\rho). \tag{11}$$

For the solution (0, 1) of (2) corresponding to the conserved electric charge, we get, up to a d exact term,

$$k_{0,1}[a,h;\bar A,\bar g] = -\delta(Q^{\rm em}_{0,1} + \lambda J), \tag{12}$$

$$J = \frac{(d^{n-2}x)_{\mu\nu}}{4\pi\sqrt{3}}\epsilon^{\mu\nu\sigma\alpha\beta\cdots\gamma\delta}A_\sigma F_{\alpha\beta}\cdots F_{\gamma\delta}. \tag{13}$$

Consider a path γ in solution space joining the solution ϕ to the background $\bar\phi$. Let $\tilde\phi$ be a point on the path and $\tilde\varphi$ a tangent vector at this point. Because $k_{\xi,c}[\tilde\varphi,\tilde\phi]$ is closed if (2) holds with $\bar\phi$ replaced by $\tilde\phi$, it follows that

$$K_{\bar\xi,c} = \int_\gamma k_{\bar\xi,c}[d_V\phi;\phi], \tag{14}$$

with $d_V\phi$ a one-form in field space, is closed when integrated along a path γ in solution space as long as (2) holds for all solutions along the path [22] (see also [23]). Explicitly, if the path is parametrized by $\phi^{(s)}$ for $s\in[0,1]$, we have

$$K_{\bar\xi,c} = \int_0^1 k_{\bar\xi,c}[\varphi^{(s)};\phi^{(s)}], \tag{15}$$

with $\varphi^{(s)} = \frac{d}{ds}\phi^{(s)}$. Whenever two $(n-2)$-dimensional closed hypersurfaces S and S' can be chosen as the only boundaries of an $(n-1)$-dimensional hypersurface Σ, the charges defined by

$$Q_{\bar\xi,c} = \oint_S K_{\bar\xi,c} \tag{16}$$

do not depend on the hypersurfaces S used for their evaluation. Furthermore, the integrability conditions satisfied by $k_{\bar\xi,c}[d_V\phi;\phi]$ imply, in the absence of topological obstructions, that these charges do not depend on the path, but only on the initial and the final solutions [24].

Mass, angular momenta, and electric charge of Gödel black holes.—We now assume $n=5$, $\Lambda=0$, $\lambda=1$. The Gödel-type solution [1,3] to the field equations is given by

$$\begin{aligned}\bar{ds}^2 &= -(dt + jr^2\sigma_3)^2 + dr^2\\ &\quad + \frac{r^2}{4}(d\theta^2 + d\psi^2 + d\phi^2 + 2\cos\theta d\psi d\phi),\\ \bar A &= \frac{\sqrt{3}}{2}jr^2\sigma_3,\end{aligned} \tag{17}$$

where the Euler angles (θ,ϕ,ψ) belong to the intervals $0\le\theta\le\pi$, $0\le\phi\le2\pi$, $0\le\psi<4\pi$, and where $\sigma_3 = d\phi + \cos\theta d\psi$. It is the reference solution with respect to which we will measure the charges of the black hole solutions of [7] that we are interested in. These latter solutions can be written as

$$\begin{aligned}ds^2 &= \bar{ds}^2 + \frac{2m}{r^2}\left(dt - \frac{l}{2}\sigma_3\right)^2 - 2mj^2r^2\sigma_3^2\\ &\quad + [k(r)-1]dr^2, \quad A = \bar A,\\ k^{-1}(r) &= 1 - \frac{2m}{r^2} + \frac{16j^2m^2}{r^2} + \frac{8jml}{r^2} + \frac{2ml^2}{r^4}.\end{aligned} \tag{18}$$

They reduce to the Schwarzschild-Gödel black hole when $l=0$, whereas the five-dimensional Kerr black hole with equal rotation parameters is recovered when $j=0$.

For the charges defined through (14) and (16), we choose to integrate over the surface S defined by $t = \text{const} = r$, while the path $\gamma{:}(g^{(s)}, A^{(s)})$ interpolating between the background Gödel-type universe $(\bar g,\bar A)$ and the black hole (g, A) is obtained by substituting (m, l) by (sm, sl) in (18), with $s\in[0,1]$.

Because $A^{(s)}_\mu = \bar A_\mu$ for all s, the mass

$$\mathcal{E} \equiv \oint_S K_{\partial/\partial t,0} \tag{19}$$

of the black hole comes from the gravitational part only,

$$\begin{aligned}\mathcal{E} &= -\left[\oint_S K^K_{\partial/\partial t}\right]^g_{\bar g} - \int_0^1 ds\oint_S\frac{\partial}{\partial t}\Theta[h^s;g^s]\\ &= \frac{3\pi}{4}m - 8\pi j^2m^2 - \pi jml.\end{aligned} \tag{20}$$

Unlike the five-dimensional Kerr black hole [11,13], the mass of which is recovered for $j=0$, we also see that the rotation parameter l brings a new contribution to the mass with respect to the Schwarzschild-Gödel black hole.

Note that the integral over the path is really needed here in order to obtain meaningful results, because the naive application of the Abbott-Deser, Iyer-Wald, or Regge-Teitelboim expressions gives as a result

$$\mathcal{E}^{\text{naive}} = \oint_S k_{\partial/\partial t,0}[g - \bar{g}, \bar{g}] = 8\pi m^2 j^4 r^2 + O(1), \quad (21)$$

which, as pointed out in [16], diverges for large r. A correct application consists in using these expressions to compare the masses of infinitesimally close black holes, i.e., black holes with $m + \delta m$, $l + \delta l$ as compared to black holes with m, l. Indeed, $\oint_S k_{\partial/\partial t,0}[\delta g, g] = \delta\mathcal{E}$, with $\mathcal{E}$ given by the right-hand side of (20), which is finite and r independent as it should since $dk_{\partial/\partial t,0}[\delta g, g] = 0$. Finite mass differences can then be obtained by adding up the infinitesimal results. This procedure is, for instance, also needed if one wishes to compute in this way the masses of the conical deficit solutions [25] in asymptotically flat $(2 + 1)$-dimensional gravity.

Because our computation of the mass does not depend on the radius r at which one computes, one can consider, if one so wishes, that one computes inside the velocity of light surface. Similarly, if one uses this method to compute the mass of de Sitter black holes, one can compute inside the cosmological horizon, and problems of interpretation, due to the fact that the Killing vector becomes spacelike, are avoided.

The expression for the angular momentum

$$J^{\phi} \equiv -\oint_S K_{\partial/\partial t,0} \quad (22)$$

reduces to

$$J^{\phi} = \left[\oint K^K_{\partial/\partial t}\right]^g_{\bar{g}} + \left[\oint Q^{\text{em}}_{\partial/\partial t,0}\right]^{A,g}_{\bar{A},\bar{g}} = \frac{1}{2}\pi m l - \pi j m l^2 - 4\pi j^2 m^2 l, \quad (23)$$

while the angular momenta for the other three rotational Killing vectors [7] vanish.

The electric charge picks up a contribution from the Chern-Simons term and is explicitly given by

$$\mathcal{Q} \equiv -\oint_S K_{0,1} = [Q^{\text{em}}_{0,1} + \lambda J]^{g,\bar{A}}_{\bar{g},\bar{A}} = 2\sqrt{3}\pi j m l. \quad (24)$$

In particular, it vanishes for the Schwarzschild-Gödel black hole.

Generalized Smarr formula and first law.—Consider a stationary black hole with Killing horizon determined by $\xi_H = k + \Omega^H_a m^a$, where k denotes the timelike Killing vector, Ω^H_a the angular velocities of the horizon, and m^a the axial Killing vectors, and let $\mathcal{E} = \oint_S K_{k,0}$, $J^a = -\oint_S K_{m^a,0}$. As in [24], the generalized Smarr relation then follows directly from the identity

$$\oint_S K_{\xi_H,0} = \oint_H K_{\xi_H,0}, \quad (25)$$

where H is a $(n-2)$-dimensional surface on the horizon. Indeed, the definition of ξ_H and the charges imply

$$\mathcal{E} - \Omega^H_a J^a = \oint_H K_{\xi_H,0}. \quad (26)$$

Because $A^{(s)}_\mu = \bar{A}_\mu$, the right-hand side becomes

$$\oint_H K_{\xi_H,0} = -\left[\oint_H K^K_{\xi_H}\right]^g_{\bar{g}} - \left[\oint_H Q^{\text{em}}_{\xi_H,0}\right]^{g,\bar{A}}_{\bar{g},\bar{A}} + \oint_H C_{\xi_H;\gamma},$$
$$C_{\xi_H;\gamma} = -\int_0^1 ds\, \xi_H \cdot \Theta[h^{(s)}; g^{(s)}]. \quad (27)$$

Now, $-\oint_H K^K_{\xi_H}[g] = \frac{\kappa\mathcal{A}}{8\pi}$, where κ is the surface gravity and $\mathcal{A}$ the area of the horizon, while $-[\oint_H Q^{\text{em}}_{\xi_H,0}]^{g,\bar{A}}_{\bar{g},\bar{A}} = \Phi_H \mathcal{Q}$, where $\Phi_H = -(\xi_H \cdot A)$ is the corotating electric potential, which is constant on the horizon [13,15]. We thus get

$$\mathcal{E} - \Omega^H_a J^a = \frac{\kappa\mathcal{A}}{8\pi} + \Phi_H \mathcal{Q} + \oint_H K^K_{\xi_H}[\bar{g}] + \oint_H C_{\xi_H;\gamma}. \quad (28)$$

In order to apply this formula in the case of the black hole (18), we have to compute the remaining quantities. The radius r_H and the angular velocities Ω^H_ϕ and Ω^H_ψ are solutions of

$$\left[\frac{\partial \xi^2}{\partial \Omega^\phi}\right]_{r_H,\Omega^H_a} = 0, \qquad \left[\frac{\partial \xi^2}{\partial \Omega^\psi}\right]_{r_H,\Omega^H_a} = 0, \qquad [\xi^2]_{r_H,\Omega^H_a} = 0. \quad (29)$$

Defining for convenience $\alpha = (1 - 8j^2 m)(1 - 8j^2 m - 8jl - 2m^{-1}l^2)$ and $\beta = 1 - 8j^2 m - 4r_H^2 j^2 + 2ml^2 r_H^{-4}$, we find

$$r_H^2 = m - 4jml - 8j^2m^2 + m\sqrt{\alpha}$$
$$\Omega^H_\phi = 4\frac{j + mlr_H^{-4}}{\beta}, \qquad \Omega^H_\psi = 0.$$

The electric potential is given by $\Phi_H = -(\xi_H \cdot \bar{A}) = -\frac{\sqrt{3}}{2} j r_H^2 \Omega^\phi_H$. The area and surface gravity of the horizon are

$$\mathcal{A} = 2\pi^2 r_H^3 \sqrt{\beta}, \qquad \kappa = \frac{2m\sqrt{\alpha}}{r_H^3\sqrt{\beta}}. \quad (30)$$

For the Gödel-Schwarzschild black hole, we recover the results of [7,16]

$$r_H^2 = 2m(1 - 8j^2m), \qquad \mathcal{A} = 2\pi^2\sqrt{8m^3(1 - 8j^2m)^5},$$
$$\Omega^H_\phi = \frac{4j}{(1 - 8j^2m)^2}, \qquad \kappa = \frac{1}{\sqrt{2m(1 - 8j^2m)^3}}.$$

Using

$$\oint_H K^K_{\xi_H}[\bar{g}] = -\pi j^2 r_H^4 - \pi j^3 r_H^6 \Omega^H_\phi, \quad (31)$$

$$\oint_H C_{\xi_H;\gamma} = \frac{\pi m}{4} - 4\pi j^2 m^2 - \pi jml + 2\pi j^2 m r_H^2, \quad (32)$$

together with the explicit expressions for all the other quantities, one can verify that the generalized Smarr formula (28) reduces, indeed, to an identity.

We can also compare with the generalized Smarr formula derived for asymptotically flat black holes in five-dimensional supergravity [13]: for the Gödel-type black hole (18) we get

$$\frac{2}{3}\mathcal{E} - \Omega_a \mathcal{J}^a - \frac{\kappa\mathcal{A}}{8\pi} - \frac{2}{3}\Phi_H \mathcal{Q} = -\frac{2\pi}{3} jm(2jm + l). \quad (33)$$

The right-hand side, which vanishes when $j = 0$, describes the breaking of the Smarr formula for asymptotically flat black holes due to the presence of the additional dimensionful parameter j. This is somewhat reminiscent to what happens for Kerr-AdS black holes [12]. In the latter case, different values of the cosmological constant Λ describe different theories because Λ appears explicitly in the action. Even though this is not the case for j, we have also taken j here as a parameter specifying the background because all charges have been computed with respect to the Gödel background.

As for Kerr-AdS black holes, the spinning Gödel black hole satisfies a standard form of the first law. Indeed, using the explicit expressions for the quantities involved, one can now explicitly check that the first law

$$\delta\mathcal{E} = \Omega_a \delta\mathcal{J}^a + \Phi_H \delta\mathcal{Q} + \frac{\kappa}{8\pi}\delta\mathcal{A} \quad (34)$$

holds. As pointed out in [12], the validity of the first law provides a strong support for our definitions of total energy and angular momentum. Furthermore, in the limit of vanishing j, we recover the usual expressions for five-dimensional asymptotically flat black holes.

Discussion.—In the case of the nonrotating Gödel black hole, $l = 0 = \mathcal{J}^\phi = \mathcal{Q}$, the parametrization $M^* = 2m - 16j^2m^2$, $\beta^* = \frac{2j}{1-8j^2m}$ suggested by the analysis of [26] allows one to write a nonanomalously broken Smarr formula of the form $\frac{2}{3}\mathcal{E}^* = \frac{\kappa\mathcal{A}}{8\pi}$, where $\mathcal{E}^* = \frac{3\pi}{8}M^*$, with κ and $\mathcal{A}$ unchanged. With $\mathcal{E}^*$ as the energy and β^* the fixed parameter characterizing the Gödel background, the first law is, however, not satisfied.

A way out, in the case $l = 0$, is to consider the Killing vector $k' = (1 + \beta^{*2}M^*)^{-2/3}\frac{\partial}{\partial t}$. The associated energy is $\mathcal{E}' \equiv \oint_S K_{k'} = \frac{3\pi}{8}M^*(1 + \beta^{*2}M^*)^{-2/3}$. The first law now holds and, in addition, with κ' defined with respect to k', so does the nonanomalously broken Smarr formula $\frac{2}{3}\mathcal{E}' = \frac{\kappa'\mathcal{A}}{8\pi}$. Furthermore, it turns out that the prefactor acts as an integrating factor and the first law is verified for variations of both M^* and β^*.

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