Hawking Radiation in an Electromagnetic Waveguide?

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It is demonstrated that the propagation of electromagnetic waves in an appropriately designed waveguide is (for large wavelengths) analogous to that within a curved space-time—such as around a black hole. As electromagnetic radiation (e.g., microwaves) can be controlled, amplified, and detected (with present-day technology) much easier than sound, for example, we propose a setup for the experimental verification of the Hawking effect. Apart from experimentally testing this striking prediction, this would facilitate the investigation of the trans-Planckian problem.

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Introduction.—One of the major motivations behind the idea of black hole analogues (“dumb holes,” see [1]) is the possibility of an experimental verification of the Hawking effect [2]. Apart from testing one of the most striking theoretical predictions of quantum field theory under the influence of external conditions, such an experiment would enable us to investigate the impact of ultrahigh energy or momentum degrees of freedom (trans-Planckian problem) on the lowest-order Hawking effect and its higher-order corrections (with respect to the small ratio of Hawking temperature over Planck scale) by means of an analogue system. In view of the close relation between the Hawking effect and the concept of black hole entropy, these investigations are potentially relevant for the black hole information paradox, etc.

The analogy between sound waves in moving fluids and scalar fields in curved space-times established in [1] can (in principle) be used to simulate a horizon in liquid Helium [3] or in Bose-Einstein condensates [4], for example, (see also [5]). However, measuring the Hawking effect in those systems goes along with serious difficulties [6]. The main problem is the detection of sound waves corresponding to the realistically very low Hawking temperature.

On the other hand, electromagnetic radiation is much easier to control, to amplify, and to detect with present-day technology and one might hope to exploit this advantage. Optical black hole analogues have been discussed for highly dispersive media which support the phenomenon of slow light [7] and for ordinary nondispersive dielectrics [8]. However, it turns out that a horizon for slow light does not emit Hawking radiation [9], whereas an experimental realization by means of ordinary dielectrics is in principle possible but very challenging. In the following, we shall propose an alternative setup in order to circumvent this difficulty.

Waveguide.—Let us consider the propagation of electromagnetic waves in the waveguide in Fig. 1. The capacitances C are realized by the parallel conducting plates at the bottom of each unit separated by insulating slabs with the dielectric permittivity ε and thickness δz. The inductances L are generated by the remaining space of the $\Delta x \times \Delta y \times \Delta z$ cells (magnetic permeability μ) with the enclosing walls also being conductors. For simplicity, and in order to avoid leakage and transverse modes, etc., we assume the following hierarchy of dimensions of the waveguide and the wavelength λ of the propagating electromagnetic waves:

$$\delta z \ll \delta x \ll \Delta x \sim \Delta z \ll \Delta y \ll \lambda. \qquad (1)$$

In this limit, the waveguide possesses a large slow down and we can omit Maxwell's supplement $\dot{\boldsymbol{D}}$ in Ørsted-Ampère's law $\nabla \times \boldsymbol{H} = \boldsymbol{j} + \dot{\boldsymbol{D}}$, i.e., $\oint d\boldsymbol{r} \cdot \boldsymbol{H} = I + \frac{d}{dt}\int d\boldsymbol{S} \cdot \boldsymbol{D}$, in the upper region of the waveguide (i.e., the surface integral over $S = \Delta y \times \Delta z$). The conditions (1) also ensure that the energy of the waves is basically confined to the waveguide.

Hence, the combination of Ørsted-Ampère's and Faraday's law $\nabla \times \boldsymbol{E} = -\dot{\boldsymbol{B}}$, i.e., $\oint d\boldsymbol{r} \cdot \boldsymbol{E} = -\frac{d}{dt}\int d\boldsymbol{S} \cdot \boldsymbol{B}$, gives the induction law $\Delta U = \frac{d}{dt} LI$ for the effective coils with a possibly time-dependent inductance $L(t)$ given by $L = \mu \Delta x \Delta z / \Delta y$ for a long coil $\Delta y \gg \Delta x, \Delta z$. Denoting the voltage impressed on the nth capacitor by U_n, the

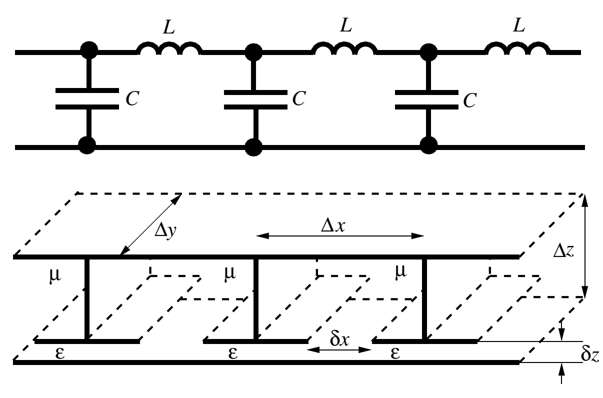

FIG. 1. Circuit diagram and sketch of the waveguide.

above condition implies for the nth coil $U_{n+1} - U_n = \frac{d}{dt} L_n I_n$. By means of Gauss' law $\boldsymbol{\nabla} \cdot \boldsymbol{D} = \varrho$, i.e., $\oint d\boldsymbol{S} \cdot \boldsymbol{D} = Q$, and the conditions (1), we obtain for the nth capacitor $Q_n = C_n U_n$ with Q_n denoting its charge and $C_n = \varepsilon \Delta x \Delta y / \delta z$ its possibly time-dependent capacitance. In the following [10] we shall assume that all the inductances are constant and equal ($L_n = L = \text{const}$) whereas the capacitances are generally time-dependent $C_n(t)$. In analogy to the electromagnetic field in vacuum, we may introduce effective potentials A_n which automatically satisfy the induction law $U_{n+1} - U_n = \frac{d}{dt} L_n I_n$ via

$$U_n = L \frac{dA_n}{dt}, \qquad I_n = A_{n+1} - A_n. \tag{2}$$

Then Kirchoff's law $\frac{d}{dt} Q_n = \frac{d}{dt} C_n U_n = I_n - I_{n-1}$ (i.e., charge conservation) implies the equation of motion

$$\frac{d}{dt} L C_n \frac{d}{dt} A_n = A_{n+1} - 2A_n + A_{n-1}. \tag{3}$$

Effective geometry.—In the continuum limit (i.e., one wavelength involves many units $\lambda \gg \Delta x$), the above equation of motion approaches the wave equation

$$\left(\frac{\partial}{\partial t} \frac{1}{c^2} \frac{\partial}{\partial t} - \frac{\partial^2}{\partial x^2} \right) A = 0, \tag{4}$$

with the space-time dependent velocity of propagation

$$c = \frac{\Delta x}{\sqrt{LC}} = \sqrt{\frac{\delta z}{\epsilon \mu \Delta z}} \ll c_0. \tag{5}$$

If we arrange $c^2(t, x)$ according to

$$c^2(t, x) = c^2(x + vt), \tag{6}$$

with a constant velocity v and transform into the comoving frame ($x \to x + vt$), the wave equation becomes

$$\left[\left(\frac{\partial}{\partial t} + v \frac{\partial}{\partial x} \right) \frac{1}{c^2} \left(\frac{\partial}{\partial t} + v \frac{\partial}{\partial x} \right) - \frac{\partial^2}{\partial x^2} \right] A = 0. \tag{7}$$

Unfortunately, in $1 + 1$ dimensions, the Maxwell equations are trivial and the scalar field is conformally invariant—which prevents the introduction of an effective geometry (in $1 + 1$ dimensions). On the other hand, the waveguide is not really $1 + 1$ dimensional, the y dimension just does not contribute due to $\lambda \gg \Delta y$, see [11]. Taking into account this "silent" y dimension, the above wave equation allows for the identification of an effective metric via $\Box_{\text{eff}} A = \partial_\mu (\sqrt{g_{\text{eff}}} g_{\text{eff}}^{\mu\nu} \partial_\nu A) / \sqrt{g_{\text{eff}}} = 0$ in the $2 + 1$ dimensional Painlevé-Gullstrand-Lemaître form [12]

$$g_{\text{eff}}^{\mu\nu} = \begin{pmatrix} 1 & v & 0 \\ v & v^2 - c^2 & 0 \\ 0 & 0 & -c^2 \end{pmatrix}. \tag{8}$$

Hence, the propagation of electromagnetic waves within the waveguide is equivalent to that in a curved space-time as described by the above metric. Note that the scenario under consideration is similar to the supersonic domain wall discussed in [13].

As one would expect, the metric describes a horizon at $v^2 = c^2$ with a surface gravity corresponding to the gradient of the propagation velocity c (since $v = \text{const}$) and, therefore, a Hawking temperature of

$$T_{\text{Hawking}} = \frac{\hbar}{2\pi k_B} \left| \frac{\partial c}{\partial x} \right|_{v^2 = c^2} \tag{9}$$

As demonstrated in [9], the (classical) wave equation is not enough for the prediction of Hawking radiation—the (quantum) commutation relations have to match as well. If we start from the effective action of the waveguide

$$\mathfrak{A}_{\text{eff}} = \int dt \sum_n \frac{1}{2} (C_n U_n^2 - L_n I_n^2) \to \frac{L \Delta x}{2} \int dt dx \left(\frac{1}{c^2} \left[\frac{\partial A}{\partial t} \right]^2 - \left[\frac{\partial A}{\partial x} \right]^2 \right), \tag{10}$$

and perform the usual canonical quantization procedure, we indeed obtain the correct commutation relations for the Hawking effect—i.e., the conversion of the zero-point oscillations (with energy $\hbar\omega/2$) to real photons by the external conditions $C_n(t)$.

Switching.—In the following we present a microscopic model for the controlled change of the capacitances $C_n(t)$. Let us consider an insulating material such as a semiconductor at very low temperatures whose dielectric properties are mainly governed by a large number of independent localized electrons (say, one per lattice site) which can be described in terms of their three lowest levels a, b, and c. In the usual rotating wave and dipole approximation, the dynamics of the corresponding single-particle amplitudes ψ_a, ψ_b, and ψ_c, is determined by the effective Lagrangian

$$\mathfrak{L}_{\text{RWA}} = i\psi_a^* \dot{\psi}_a + i\psi_b^* \dot{\psi}_b + i\psi_c^* \dot{\psi}_c - \Delta\omega \psi_c^* \psi_c + [\kappa \mathcal{E}(t) \psi_b^* \psi_c + \Omega(t) \psi_a^* \psi_b + \text{H.c.}]. \tag{11}$$

Here $\Delta\omega = \omega_c - \omega_b > 0$ denotes the energy difference between the excited states c and b and the associated dipole moment κ describes the coupling to a small and slow electric test field $\mathcal{E}(t)$. The ground state a is supposed to be strongly localized (small dipole moment) and hence does not couple significantly to $\mathcal{E}(t)$—whereas the states b and c are delocalized and thus do couple to $\mathcal{E}(t)$. However, the ground state a couples to a fast and strong Laser field tuned to the frequency of transition from the ground state a to the first excited state b (but neither from b to c nor from a to c) with the Rabi frequency $\Omega(t)$.

Since the test field $\mathcal{E}(t)$, and thus the induced disturbance ψ_c, are small (linear response $\psi_c \ll \psi_a, \psi_b$), the dynamics for ψ_a and ψ_b decouple and depend on $\Omega(t)$ only, i.e., we may control the occupation amplitudes ψ_a and ψ_b by means of the external laser beam. Furthermore, in the adiabatic regime, where the rate of change of $\mathcal{E}$ and Ω are much smaller than $\Delta\omega$, we can integrate out (i.e., average over) the microscopic degrees of freedom $\psi_{a,b,c}$ in the Lagrangian in Eq. (11) arriving at the contribution to the effective Lagrangian for the test field $\mathcal{E}(t)$

$$\mathfrak{L}_{\text{eff}} = |\psi_b(t)|^2 \frac{\kappa^2}{\Delta\omega} \mathcal{E}^2(t). \tag{12}$$

Obviously, this corresponds to a varying dielectric [14] permittivity $\varepsilon(t)$ and thus capacitance $C_n(t)$ which can be controlled (nonlinear optics) by the laser field, i.e., its Rabi frequency $\Omega(t)$. Note that, in contrast to resonant phenomena such as electromagnetically induced transparency, this effective Lagrangian is valid for all adiabatic frequencies (i.e., far below $\Delta\omega$). In the adiabatic limit, the above effect is equivalent to the quadratic Stark shift (since there is no degeneracy) in stationary perturbation theory. For nonadiabatic frequencies, one must replace $\Delta\omega$ by the detuning between the frequency of the test field $\mathcal{E}(t)$ and the $b \leftrightarrow c$ transition frequency. Assuming one electron (i.e., three-level system) per lattice site with a dipole moment κ corresponding to a length scale of order of the lattice spacing (a few Ångströms), a near-optical energy gap of order $\mathcal{O}(1\text{ eV}) \simeq \mathcal{O}(10^{15}\text{ Hz})$ would satisfy the adiabaticity condition with $\Omega \simeq \mathcal{O}(10^{10}\text{ Hz})$, cf. [15], and still generate a large dielectric permittivity ε.

Discussion.—As we have observed in the previous considerations, it is possible (under the assumptions and approximations made) to increase the capacitances in a controlled way by shining a laser beam on the material—ideally without generating dissipation and noise, etc. The small residual energy absorbed in any real material (due to impurities, etc.) creating high-frequency phonons, for example, must be carried away by cooling the waveguide before it may get converted into low-frequency electromagnetic waves traveling along the waveguide.

Note that a decoherence-free modulation of $\varepsilon(t)$ requires the dipole moment for the transition $a \leftrightarrow b$ to be small and the laser beam to contain a large number of photons—such that it can be described classically as an external field, i.e., stimulated emission/absorption only. In contrast, the spontaneous decay back to ground state a is potentially associated with dissipation, noise and decoherence—and hence the lifetime of the excited state b should be much longer than all other time scales relevant for the effective geometry.

Fortunately, one can simulate a black hole horizon (c decreases $\leftrightarrow$ ε increases) by illuminating the material in its ground state a. The white hole horizon occurs at the transition back to the ground state a and one has to make sure that the two horizons are far apart such that the noise generated by the white hole horizon does not propagate to the black hole horizon and prevent the detection of the Hawking radiation. The dispersion relation following from Eq. (3) reads (for $k_y = 0$ [11])

$$\omega^2 = \frac{4}{LC}\sin^2\left(\frac{k\Delta x}{2}\right) \approx c^2k^2 - \frac{\Delta x^4}{12LC}k^4, \tag{13}$$

and thus the group and phase velocities are (for moderate frequencies [16]) less or equal to $c^2 = \Delta x^2/(LC)$; i.e., we have a subluminal dispersion relation—which is typical for a lattice. As a result, disturbances from the white hole horizon (with moderate frequencies) cannot propagate to the black hole horizon.

In order to answer the main question of whether it will be possible to actually measure the Hawking effect, one has to estimate the Hawking temperature. According to Eq. (9) the Hawking temperature is basically determined by the characteristic time scale on which c changes. For our microscopic model, this switching time is related to the Rabi frequency Ω, which must be much smaller than the optical or near-optical frequencies of the three-level system such as $\Delta\omega$ for the rotating wave approximation to apply. Nevertheless, with strong laser pulses, it is possible to pump a large number of electrons in a semiconductor from the ground state into an excited state in 10–100 picoseconds—whereas the (spontaneous) decay back to the ground state can be much slower [15]. In this case, the order of magnitude of the Hawking temperature could be 10–100 mK, which is a really promising value since there already exist amplifiers and detectors (for microwaves) with a noise temperature of order 10 mK, see, e.g., [17]. Of course, one should cool down the apparatus below that temperature.

One advantage of the present proposal is that it allows for large velocities, say $c_0/10 - c_0/100$ (in contrast to Bose-Einstein condensates [4], for example, with a sound speed of order mm/s). Furthermore, there are no walls in relative motion with respect to the medium, which could lead to the Miles [18] instability (via momentum transfer). With the above values, the thermal wavelength of the Hawking radiation would be of order millimeter—hence the minimum size Δx representing the analogue of the Planck length (knee wavelength) should be smaller than that. Consequently, the laser would have to illuminate a slab with a thickness way below 1 mm.

The power of the radiation can be inferred from the (1 + 1 dimensional) energy-momentum tensor (in the comoving frame)

$$\frac{dE}{dt} = T^1_0 = \frac{\pi}{12\hbar}(k_B T_{\text{Hawking}})^2, \tag{14}$$

where we can get an order of magnitude between 10^{-14} and 10^{-16} W, which vastly exceeds the threshold in [17]. The above expression yields the energy flux measured by a detector at a large and constant distance to the horizon, which corresponds to a comoving detector with respect to laboratory coordinates. A detector sitting at the far end of the waveguide (i.e., at rest with respect to the laboratory) would correspond to an in-falling observer far away, and thus see the Hawking radiation Doppler shifted, such that the created power in the laboratory frame is even larger, see also [13]. (Close to the horizon, on the other hand, the response of the comoving detector would diverge, whereas the in-falling observer cannot detect anything special since the available observation time is too short.)

The $1 + 1$ dimensional character of the above expression reflects the fact that we have excluded transverse modes via assumption (1). With appropriate generalizations (e.g., $\lambda \not\gg \Delta y$), we could also reproduce higher (e.g., $2 + 1$) dimensional behavior [11]. There are basically two major possibilities for the geometry of the setup—a line or a circle. In both cases one may accumulate energy over some time by building a very long line (e.g., as a spiral) or using many revolutions in the circle. In the latter scenario, however, the problem of how to deal with the white hole horizon has to be solved.

The control of the space-time dependence of $c(t, x)$ can be achieved actively or passively: An active control could be realized via sweeping the laser beam externally (e.g., shining through holes in the lower capacitor plate), for example. Alternatively (passive control), one could arrange optical fibers filling the capacitors—either wound up around the capacitor plates or aligned along the waveguide—in a way such that the linear or nonlinear (e.g., self-focusing) optical pulse propagation exactly generates the desired behavior $c(t, x)$.

The Hawking effect could be measured by connecting an amplifying circuit (e.g., field effect transistors) to the end of the waveguide (an alternative method would be a bolometer) subject to impedance matching—which can be achieved by manipulating μ/ε, for example. In case the measurement is not fast enough, that amplifier should be disconnected (switched off) before the arrival of the horizon—inducing additional noise, etc., The thermal spectrum of the Hawking radiation could be determined with successive band passes.

In summary, the present proposal for an experimental verification of the Hawking effect (in black hole analogues) appears to be just at the edge of the present experimental capabilities—but not far beyond them (as is some other scenarios).

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[11] In view of the assumptions made, $A(t, x)$ does not depend on y, i.e., the perpendicular wave number k_y vanishes. Similar to an optical fiber based on total reflection, the set of k_y modes is discrete and the energy of the modes with $k_y > 0$ is so large (for small Δy) that they can be neglected. But taking into account the y dependence, Eq. (4) generalizes to $\partial_t c_x^{-2} \partial_t A = \partial_x^2 A + \xi^2 \partial_y^2 A$ where ξ^2 denotes the ratio of the (constant) inductances in the x and y directions. After a simple rescaling $y \to y\xi$, we reproduce Eq. (8). Since Hawking radiation is basically a $1 + 1$ dimensional effect, with the Hawking temperature being independent of the transversal dimensions and conformal factors, etc., the existence of the silent y dimension does not alter the conclusions.

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